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ON THE SPLITTING TYPE OF VECTOR BUNDLE ON THE RIEMANN SPHERE

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Abstract. We discuss splitting type of holomorphic vector bundle induced from Fuchsian system. We give an explicit form of the transformation matrices.

Keywords and phrases: Holomorphic bundle, Fuchsian systems, splitting type.

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Introduction

Consider the Fuchsian system with singular points $a_1, ..., a_n \in \mathbb{CP}^1$ of the form

$$\frac{dy}{dz} = \sum_{i=1}^{m} \frac{B_i}{z - a_i} y, \tag{1}$$

where the B_i are $p \times p$ constant matrices. System (1) gives the monodromy representation

$$\rho : \pi_1(\mathbb{CP}^1 - \{a_1, ..., a_n\}, z_0) \to GL(n, \mathbb{C}).$$

Let E be associated from the monodromy representation ρ a vector bundle E on $\mathbb{CP}^1 - \{a_1, ..., a_n\}$ and let F be a vector bundle on the Riemann sphere $\mathbb{CP}^1$, obtained from the extension of E on the whole sphere [1], [2]. Then by the Birkhoff-Grothendieck Theorem

$$F \cong \mathcal{O}(k_1) \oplus ... \oplus \mathcal{O}(k_p)$$

and the collection of the numbers $k_1, , k_p$ is called a *splitting type* of the vector bundle F.

In [1] the possibility of calculations of numbers $k_1, ..., k_p$ algorithmically is indicated. In [3] the realization of the algorithm for determining the splitting type is presented. Here we give detailed calculation of the transformation matrices.

Let T be a nondegenerate matrix such that $B_1' = TB_1T^{-1}$ is a Jordan canonical form of B_1. Consider the transformation $y_1 = Ty$. We obtain for y_1 the following system of equations

$$\frac{dy_1}{dz} = \sum_{i=1}^{m} \frac{B_i'}{z - a_i} y_1 \tag{2}$$

where $B_i' = TB_1T^{-1}$, $i = 1, ..., m$. (2) is the Fuchsian system and B_1 has a Jordan canonical form. Denote by $\lambda_1, ..., \lambda_n$ eigenvalues with multiplies $p_1, ..., p_n$.

Proposition. *Let* $(\mu_1, ..., \mu_n) \in \mathbb{Z}^n$ *be a given integer valued vector. There exists the transformation* T_1 *of the unknown function* y_1*, such that the system of differential equations*

$$\frac{dy_2}{dz} = B(z)y_2 \tag{3}$$

for $y_2 = T_1 y_1$ *function is a Fuchsian system with singular points* $a_1, ..., a_n$ *and a residue matrix*

$$\tilde{B}_1 = Res_{z=a_1} B(z)$$

with eigenvalues $\mu_1, ..., \mu_1, \tilde{\mu}_{p+1}, ..., \tilde{\mu}_n$. *Besides,* $T_1(z)=(z-a_1)^D$*, where*

$$D = diag(\underbrace{1, 1, ..., 1}_{q}, 0, 0, ..., 0)$$

if $[Re(\lambda_1)] < \lambda_1$ *and*

$$D = diag(\underbrace{-1, -1, ..., -1}_{q}, 0, 0, ..., 0),$$

if $[Re(\lambda_1)] > \lambda_1$, $1 \leq q \leq p_1$.

Proof. Direct calculation shows that

$$\frac{dT_1(z)}{dz} T_1^{-1}(z) = \frac{D}{z - a_1}$$

and

$$\frac{dy_2}{dz} = \sum_{i=1}^{m} \frac{T_1(z) B_i' T_1^{-1}(z)}{z - a_i} \frac{dT_1(z)}{dz} T_1^{-1}(z) y_2$$

$$= \sum_{i=1}^{m} \frac{B_i''(z)}{z - a_i} + \frac{D}{z - a_1} y_2 = \frac{B_1''(z) + D}{z - a_1} + \sum_{i=2}^{m} \frac{B_i''(z)}{z - a_i} y_2$$

where $T_1(z) B_i' T_1^{-1}(z) = B_i''$.

Let

$$B(z) = \frac{B_1''(z) + D}{z - a_1} + \sum_{i=2}^{m} \frac{B_i''(z)}{z - a_i}.$$

The proof this proposition is completed in the following Lemma.

Lemma. *The system (2) gauge equivalence the following Fuchsian system*

$$\frac{df}{dz} = \sum_{i=1}^{m} \frac{A_i}{z - a_i} f,$$

where $A_1 = \mathrm{Res}_{z=a_1} B(z)$, $A_2 = \mathrm{Res}_{z=a_2} B(z)$, ... , $A_m = \mathrm{Res}_{z=a_m} B(z)$.

The proof of Lemma follows from calculation of residue matrices. First we suppose $p = 1$. Then $D = diag(1, 0, ..., 0)$ and therefore

$$T_1(\mathrm{z})=(z-a_1)^D=\begin{pmatrix} z-a_1 & 0 & . & . & 0\\ 0 & 1 & . & . & 0\\ . & . & . & . & .\\ 0 & 0 & . & . & 1\end{pmatrix},\ T_1^{-1}(z)=\begin{pmatrix} \frac{1}{z-a_1} & 0 & . & . & 0\\ 0 & 1 & . & . & 0\\ . & . & . & . & .\\ 0 & 0 & . & . & 1\end{pmatrix}$$

and $B_j'=\begin{pmatrix} b_{11}^j & b_{12}^j & . & . & b_{1n}^j\\ b_{21}^j & b_{22}^j & . & . & b_{2n}^j\\ . & . & . & . & .\\ b_{n1}^j & b_{n2}^j & . & . & b_{nn}^j\end{pmatrix}$.

For each fixed $j\in\{2,...,m\}$ we have

$$\frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}=\begin{pmatrix} \frac{b_{11}^j}{z-a_j} & \frac{b_{12}^j(z-a_1)}{z-a_j} & . & . & \frac{b_{1n}^j(z-a_1)}{z-a_j}\\ \frac{b_{21}^j}{(z-a_1)(z-a_j)} & \frac{b_{22}^j}{z-a_j} & . & . & \frac{b_{2n}^j}{z-a_j}\\ . & . & . & . & .\\ \frac{b_{n1}^j}{(z-a_1)(z-a_j)} & \frac{b_{n2}^j}{z-a_j} & . & . & \frac{b_{nn}^j}{z-a_j}\end{pmatrix}.$$

We used the following identities

$$\frac{z-a_1}{z-a_j}=1+(a_j-a_1)\frac{1}{z-a_j},$$

$$\frac{1}{(z-a_1)(z-a_j)}=\frac{1}{(a_j-a_1)(z-a_j)}-\frac{1}{(a_j-a_1)(z-a_1)}.$$

The expression above obtained the form:

$$\frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}$$

$$=\begin{pmatrix} \frac{b_{11}^j}{z-a_j} & . & . & b_{1n}^j+b_{1n}^j(a_j-a_1)\frac{1}{z-a_j}\\ \frac{b_{21}^j}{a_j-a_1}\frac{1}{z-a_j}-\frac{b_{21}^j}{a_j-a_1}\frac{1}{z-a_1} & . & . & \frac{b_{2n}^j}{z-a_j}\\ . & . & . & .\\ \frac{b_{n1}^j}{a_j-a_1}\frac{1}{z-a_j}-\frac{b_{n1}^j}{a_j-a_1}\frac{1}{z-a_1} & . & . & \frac{b_{nn}^j}{z-a_j}\end{pmatrix}.$$

Denote by $\{z:|z-x|=r\}=B(x,r)$. Then

$$A_j=\frac{1}{2\pi i}\int\limits_{\{z:|z-a_j|=r\}}\frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}dz=\frac{1}{2\pi i}\int\limits_{B(a_j,r)}\frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}dz=$$

$$= \frac{1}{2\pi i}\begin{pmatrix} b^j_{11}2\pi i & b^j_{12}(a_j-a_1)2\pi i & . & b^j_{1n}(a_j-a_1)2\pi i \\ \frac{b^j_{21}}{a_j-a_1}2\pi i & b^j_{22}2\pi i & . & b^j_{2n}2\pi i \\ . & . & . & . \\ \frac{b^j_{n1}}{a_j-a_1}2\pi i & b^j_{n2}2\pi i & . & b^j_{nn}2\pi i \end{pmatrix}$$

$$= \begin{pmatrix} b^j_{11} & b^j_{12}(a_j-a_1) & . & b^j_{1n}(a_j-a_1) \\ \frac{b^j_{21}}{a_j-a_1} & b^j_{22} & . & b^j_{2n} \\ . & . & . & \\ \frac{b^j_{n1}}{a_j-a_1} & b^j_{n2} & . & b^j_{nn} \end{pmatrix}.$$

Indeed,

$$\frac{1}{2\pi i}\int\limits_{\{z:|z-a_1|=r\}} \frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}dz = \frac{1}{2\pi i}\int\limits_{B(a_1,r)} \frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}dz$$

$$= \frac{1}{2\pi i}\begin{pmatrix} 0 & 0 & . & 0 \\ -\frac{b^j_{21}}{a_j-a_1}2\pi i & 0 & . & 0 \\ . & . & . & . \\ -\frac{b^j_{n1}}{a_j-a_1}2\pi i & 0 & . & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & . & 0 \\ -\frac{b^j_{21}}{a_j-a_1} & 0 & . & 0 \\ . & . & . & . \\ -\frac{b^j_{n1}}{a_j-a_1} & 0 & . & 0 \end{pmatrix},$$

$$\frac{1}{2\pi i}\int\limits_{\{z:|z-a_t|=r\}} \frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}dz = \frac{1}{2\pi i}\int\limits_{B(a_t,r)} \frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}dz = 0.$$

For $j=1$ we have

$\frac{T_1(z)B_1'T_1^{-1}(z)+D}{z-a_1} = \frac{B_1''(z)+D}{z-a_1} =$

$$=\begin{pmatrix} \frac{\lambda_1+1}{z-a_1} & 1 & . & 0 & & & & \\ . & . & . & . & & & & \\ 0 & 0 & . & \frac{1}{z-a_1} & & 0 & & \\ 0 & 0 & . & \frac{\lambda_1}{z-a_1} & & & & \\ & & & & & . & & \\ & & & & \frac{\lambda_n}{z-a_1} & \frac{1}{z-a_1} & . & 0 \\ & & & & . & . & . & . \\ & 0 & & & 0 & 0 & . & \frac{1}{z-a_1} \\ & & & & 0 & 0 & . & \frac{\lambda_n}{z-a_1} \end{pmatrix}.$$

and

$$\frac{1}{2\pi i}\int\limits_{\{z:|z-a_1|=r\}}\frac{T_1(z)B_1'T_1^{-1}(z)+D}{z-a_1}\mathrm{dz}=\frac{1}{2\pi i}\int\limits_{\{z:|z-a_1|=r\}}\frac{B_1''(z)+D}{z-a_1}\mathrm{dz}$$

$$=\begin{pmatrix} \lambda_1+1 & 0 & 0 & . & 0 & 0 & & & & & & & \\ 0 & \lambda_1 & 1 & . & 0 & 0 & & & & & & & \\ . & . & . & . & . & . & & & & 0 & & & \\ 0 & 0 & 0 & . & \lambda_1 & 1 & & & & & & & \\ 0 & 0 & 0 & . & 0 & \lambda_1 & & & & & & & \\ & & & & & & . & & & & & & \\ & & & & & & . & & & & & & \\ & & & & & & . & & & & & & \\ & & & & & & & \lambda_n & 1 & 0 & . & 0 & 0 \\ & & & & & & & 0 & \lambda_n & 1 & . & 0 & 0 \\ & & 0 & & & & & . & . & . & . & . & . \\ & & & & & & & 0 & 0 & 0 & . & \lambda_n & 1 \\ & & & & & & & 0 & 0 & 0 & . & 0 & \lambda_n \end{pmatrix}.$$

Therefore,

$$A_1=\text{Res}_{z=a_1}\text{B(z)}=\begin{pmatrix} \begin{matrix}\lambda_1+1 & 0 & 0 & . & 0 & 0\\ 0 & \lambda_1 & 1 & . & 0 & 0\\ . & . & . & . & . & .\\ 0 & 0 & 0 & . & \lambda_1 & 1\\ 0 & 0 & 0 & . & 0 & \lambda_1\end{matrix} & & 0\\ & \begin{matrix}.\\.\\.\end{matrix} & \\ 0 & & \begin{matrix}\lambda_n & 1 & 0 & . & 0 & 0\\ 0 & \lambda_n & 1 & . & 0 & 0\\ . & . & . & . & . & .\\ 0 & 0 & 0 & . & \lambda_n & 1\\ 0 & 0 & 0 & . & 0 & \lambda_n\end{matrix}\end{pmatrix}$$

$$+\sum_{j\in\{2,\dots,m\}}\begin{pmatrix} 0 & 0 & . & 0\\ -\dfrac{b_{21}^j}{a_j-a_1} & 0 & . & 0\\ . & . & . & .\\ -\dfrac{b_{n1}^j}{a_j-a_1} & 0 & . & 0\end{pmatrix}.$$

In the general case, i.e. $D = diag(\underbrace{1,1,...,1}_{\text{q}},0,0,...,0)$ then

$$A_1 = \text{Res}_{z=a_1} B(z)$$

$$=\begin{pmatrix} \underbrace{\begin{matrix}\lambda_1+1 & . & 0\\ . & . & .\\ 0 & . & \lambda_1+1\\ 0 & . & 0\\ . & . & .\\ 0 & . & 0\\ 0 & . & 0\end{matrix}}_{\text{q}} \; \underbrace{\begin{matrix}0 & 0 & . & 0 & 0\\ . & . & . & . & .\\ 0 & 0 & . & 0 & 0\\ \lambda_1 & 1 & . & 0 & 0\\ . & . & . & . & .\\ 0 & 0 & . & \lambda_1 & 1\\ 0 & 0 & . & 0 & \lambda_1\end{matrix}}_{p_1\text{-q}} & & 0\\ & \begin{matrix}.\\ & .\end{matrix} & \\ 0 & & \begin{matrix}\lambda_n & 1 & . & 0 & 0\\ . & . & . & . & .\\ 0 & 0 & . & \lambda_n & 1\\ 0 & 0 & . & 0 & \lambda_n\end{matrix}\end{pmatrix}+$$

$$+\sum_{j\in\{2,...,m\}}\begin{pmatrix} 0 & . & 0 & 0 & . & 0\\ . & . & . & . & . & .\\ 0 & . & 0 & 0 & . & 0\\ -\frac{b^j_{(q+1)1}}{a_j-a_1} & . & -\frac{b^j_{(q+1)q}}{a_j-a_1} & 0 & . & 0\\ . & . & . & . & . & .\\ -\frac{b^j_{n1}}{a_j-a_1} & . & -\frac{b^j_{nq}}{a_j-a_1} & 0 & . & 0 \end{pmatrix}$$

and

$$A_j=\text{Res}_{z=a_j}\text{B(z)}=\begin{pmatrix} b^j_{11} & . & b^j_{1q} & b^j_{1(q+1)}(a_j-a_1) & . & b^j_{1n}(a_j-a_1)\\ . & . & . & . & . & .\\ b^j_{q1} & . & b^j_{qq} & b^j_{q(q+1)}(a_j-a_1) & . & b^j_{qn}(a_j-a_1)\\ \frac{b^j_{(q+1)1}}{q-a_1} & . & \frac{b^j_{(q+1)q}}{a_j-a_1} & b^j_{(q+1)(q+1)} & . & b^j_{(q+1)n}\\ . & . & . & . & . & .\\ \frac{b^j_{n1}}{a_j-a_1} & . & \frac{b^j_{nq}}{a_j-a_1} & b^j_{n(q+1)} & . & b^j_{nn} \end{pmatrix}$$

for each $j\in\{2,...,m\}$.

Let $[Re(\lambda_1)]>\lambda_1$, then as mentioned above $D=diag(-1,0,...,0)$ if $p=1$. It means that we have the following identities:

$$T_1\text{(z)}=(z-a_1)^D=\begin{pmatrix} \frac{1}{z-a_1} & 0 & . & . & 0\\ 0 & 1 & . & . & 0\\ . & . & . & . & .\\ 0 & 0 & . & . & 1 \end{pmatrix},\ T_1^{-1}(z)=\begin{pmatrix} z-a_1 & 0 & . & . & 0\\ 0 & 1 & . & . & 0\\ . & . & . & . & .\\ 0 & 0 & . & . & 1 \end{pmatrix}$$

and $B_j'=\begin{pmatrix} b^j_{11} & b^j_{12} & . & . & b^j_{1n}\\ b^j_{21} & b^j_{22} & . & . & b^j_{2n}\\ . & . & . & . & .\\ b^j_{n1} & b^j_{n2} & . & . & b^j_{nn} \end{pmatrix}$.

From here it follows that for each fixed $j\in\{2,...,m\}$ we have:

$$\frac{T_1(z)B_j'T_1^{-1}(z)}{z-a_j}=\begin{pmatrix} \frac{b^j_{11}}{z-a_j} & \frac{b^j_{12}}{(z-a_1)(z-a_j)} & . & . & \frac{b^j_{1n}}{(z-a_1)(z-a_j)}\\ \frac{b^j_{21}(z-a_1)}{z-a_j} & \frac{b^j_{22}}{z-a_j} & . & . & \frac{b^j_{2n}}{z-a_j}\\ . & . & . & . & .\\ \frac{b^j_{n2}(z-a_1)}{z-a_j} & \frac{b^j_{n2}}{z-a_j} & . & . & \frac{b^j_{nn}}{z-a_j} \end{pmatrix}.$$

By using the following identities

$$\frac{z-a_1}{z-a_j}=\frac{z-a_j+a_j-a_1}{z-a_j}=1+\frac{a_j-a_1}{z-a_j}=1+(a_j-a_1)\frac{1}{z-a_j}.$$

$$\frac{1}{(z-a_1)(z-a_j)}=\frac{(z-a_1)-(z-a_j)}{(a_j-a_1)(z-a_1)(z-a_j)}=\frac{1}{a_j-a_1}\left(\frac{1}{z-a_j}-\frac{1}{z-a_1}\right)$$

$= \dfrac{1}{(a_j - a_1)(z - a_j)} - \dfrac{1}{(a_j - a_1)(z - a_1)}$

we obtain:

$$\frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j}$$

$$= \begin{pmatrix} \dfrac{b_{11}^j}{z - a_j} & \cdot & \cdot & \dfrac{b_{1n}^j}{a_j - a_1}\dfrac{1}{z - a_j} - \dfrac{b_{1n}^j}{a_j - a_1}\dfrac{1}{z - a_1} \\ b_{21}^j + b_{21}^j (a_j - a_1)\dfrac{1}{z - a_j} & \cdot & \cdot & \dfrac{b_{2n}^j}{z - a_j} \\ \cdot & \cdot & \cdot & \cdot \\ b_{n1}^j + b_{n1}^j (a_j - a_1)\dfrac{1}{z - a_j} & \cdot & \cdot & \dfrac{b_{nn}^j}{z - a_j} \end{pmatrix}.$$

Calculation of the integrals

$$\int\limits_{\{z:|z-a_j|=r\}} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j} dz, \int\limits_{\{z:|z-a_1|=r\}} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j} dz$$

and

$$\frac{1}{2\pi i} \int\limits_{\{z:|z-a_t|=r\}} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j}$$

gives

$$\frac{1}{2\pi i} \int\limits_{\{z:|z-a_j|=r\}} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j} \mathrm{dz} = \begin{pmatrix} b_{11}^j & \dfrac{b_{12}^j}{a_j - a_1} & \cdot & \dfrac{b_{1n}^j}{a_j - a_1} \\ b_{21}^j (a_j - a_1) & b_{22}^j & \cdot & b_{2n}^j \\ \cdot & \cdot & \cdot & \cdot \\ b_{n1}^j (a_j - a_j) & b_{n2}^j & \cdot & b_{nn}^j \end{pmatrix},$$

$$\frac{1}{2\pi i} \int\limits_{\{z:|z-a_1|=r\}} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j} \mathrm{dz} = \begin{pmatrix} 0 & -\dfrac{b_{12}^j}{a_j - a_1} & \cdot & -\dfrac{b_{1n}^j}{aj - a_1} \\ 0 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 0 \end{pmatrix},$$

$$\frac{1}{2\pi i} \int\limits_{\{z:|z-a_t|=r\}} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j} \mathrm{dz} = \frac{1}{2\pi i} \int\limits_{B(a_t,r)} \frac{T_1(z) B_j' T_1^{-1}(z)}{z - a_j} \mathrm{dz} = 0.$$

Finally we obtain

$A_1 = \mathrm{Res}_{z=a_1} \mathrm{B(z)} =$

$$\begin{pmatrix} \lambda_1-1 & 0 & . & 0 & & & & & & & \\ 0 & \lambda_1 & . & . & & & & 0 & & & \\ . & . & . & . & . & . & & & & & \\ & & & & & & . & & & & \\ & & & & & & . & & & & \\ & & & & & & . & & & & \\ & & & & & & \lambda_n & 1 & 0 & . & 0 & 0 \\ & & & & & & 0 & \lambda_n & 1 & . & 0 & 0 \\ & 0 & & & & & . & . & . & . & . & . \\ & & & & & & 0 & 0 & 0 & . & \lambda_n & 1 \\ & & & & & & 0 & 0 & 0 & . & 0 & \lambda_n \end{pmatrix}$$

$$+\sum_{j\in\{2,...,m\}}\begin{pmatrix} 0 & -\dfrac{b^j_{12}}{a_j-a_1} & . & -\dfrac{b^j_{1n}}{aj-a_1} \\ 0 & 0 & . & 0 \end{pmatrix}$$

and

A_j=Res$_{z=a_j}$B(z)= $\begin{pmatrix} b^j_{11} & \dfrac{b^j_{12}}{a_j-a_1} & . & \dfrac{b^j_{1n}}{a_j-a_1} \\ b^j_{21}(a_j-a_1) & b^j_{22} & . & b^j_{2n} \\ . & . & . & . \\ b^j_{n1}(a_j-a_j) & b^j_{n2} & . & b^j_{nn} \end{pmatrix}$ for each $j \in \{2, ..., m\}$.

When $p > 1$, then $D = diag(\underbrace{-1,-1,...,-1}_{\text{q}},0,0,...,0)$ and residue matrices have the form:

$$A_1 = \text{Res}_{z=a_1} B(z)$$

$$= \begin{pmatrix} \underbrace{\begin{matrix} \lambda_1-1 & . & 0 \\ . & . & . \\ 0 & . & \lambda_1-1 \\ 0 & . & 0 \end{matrix}}_{\text{q}} & \underbrace{\begin{matrix} \lambda_1 & 0 & . & 0 & 0 \\ 1 & \lambda_1 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & 1 & \lambda_1 \end{matrix}}_{p_1-q} & & 0 \\ & & . & \\ & & & . \\ & 0 & & \begin{matrix} \lambda_n & 1 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & \lambda_n & 1 \\ 0 & 0 & . & 0 & \lambda_n \end{matrix} \end{pmatrix} +$$

$$+\sum_{j\in\{2,...,m\}}\begin{pmatrix} 0 & . & 0 & -\dfrac{b^j_{1(q+1)}}{a_j-a_1} & . & -\dfrac{b^j_{1n}}{aj-a_1} \\ . & . & . & . & . & . \\ 0 & . & 0 & -\dfrac{b^j_{q(q+1)}}{a_j-a_1} & . & -\dfrac{b^j_{qn}}{aj-a_1} \\ 0 & . & 0 & 0 & . & 0 \\ . & . & . & . & . & . \\ 0 & . & 0 & 0 & . & 0 \end{pmatrix}$$

$$A_j = \mathrm{Res}_{z=a_j} B(z)$$

$$= \begin{pmatrix} b^j_{11} & . & b^j_{1q} & \dfrac{b^j_{1(q+1)}}{a_j-a_1} & . & \dfrac{b^j_{1n}}{a_j-a_1} \\ . & . & . & . & . & . \\ b^j_{q1} & . & b^j_{qq} & \dfrac{b^j_{q(q+1)}}{a_j-a_1} & . & \dfrac{b^j_{qn}}{a_j-a_1} \\ b^j_{(q+1)1}(a_j-a_1) & . & b^j_{(q+1)q}(a_j-a_1) & b^j_{(q+1)(q+1)} & . & b^j_{(q+1)n} \\ . & . & . & . & . & . \\ b^j_{n1}(a_j-a_j) & . & b^j_{nq}(a_j-a_j) & b^j_{n(q+1)} & . & b^j_{nn} \end{pmatrix}$$

for each $j \in \{2, ..., m\}$.

Lemma is proved.

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