Two Views on Unification:
Terms as Strategies

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joint work with

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We give a novel criterion to verify if two algebraic terms *unify* by viewing the terms to be unified as *strategies*, *i.e.* sets of pairs of occurrences of the same variable, and verifying the termination of the game obtained by playing the two strategies against each other. We prove that such a criterion of unification is equivalent to the standard one arising from Robinson algorithm. It can be viewed as a local, bottom-up criterion compared to Robinson's global top-down. The criterion is dual to the interpretation of Girard's Geometry of Interaction semantics for λ-calculus in terms of *resolution* between *principal types* of linear λ-terms, given by the authors in earlier work.

In particular, we show that two terms σ and τ unify if and only if

$$\mathcal{R}(\sigma) \mathrel{\widehat{\subseteq}} \mathcal{R}(\tau) \mathbin{\widehat{;}} (\mathcal{R}(\sigma) \mathbin{\widehat{;}} \mathcal{R}(\tau))^* \text{ and } \mathcal{R}(\tau) \mathrel{\widehat{\subseteq}} \mathcal{R}(\sigma) \mathbin{\widehat{;}} (\mathcal{R}(\tau) \mathbin{\widehat{;}} \mathcal{R}(\sigma))^*$$

where $\mathcal{R}(\sigma)$ denotes the set of pairs of paths leading to the same variable in the term σ, $\widehat{\subseteq}$ denotes "inclusion up to substitution" and $\widehat{;}$ denotes "composition up to substitution".