On an alternative approach for mixed boundary value problems for the Lamé system

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We consider a special approach to investigate a mixed boundary value problem (BVP) for the Lamé system of elasticity in the case of three-dimensional bounded domain $\Omega \subset \mathbb{R}^3$, when the boundary surface $S = \partial\Omega$ is divided into two disjoint parts, S_D and S_N, where the Dirichlet and Neumann type boundary conditions are prescribed respectively for the displacement vector and stress vector. Our approach is based on the potential method. We look for a solution to the mixed boundary value problem in the form of linear combination of the single layer and double layer potentials with densities supported respectively on the Dirichlet and Neumann parts of the boundary. This approach reduces the mixed BVP under consideration to a system of pseudodifferential equations which do not contain neither extensions of the Dirichlet or Neumann data, nor the Steklov-Poincaré type operator. Moreover, the right hand sides of the resulting pseudodifferential system are vectors coinciding with the Dirichlet and Neumann data of the problem under consideration. The corresponding pseudodifferential matrix operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies the unconditional unique solvability of the mixed BVP in the Sobolev space $[W_2^1(\Omega)]^3$ and representability of solutions in the form of linear combination of the single layer and double layer potentials with densities supported respectively on the Dirichlet and Neumann parts of the boundary. Using a special structure of the obtained pseudodifferential matrix operator, it is also shown that the operator is invertible in the L_p-based Besov spaces with $\frac{4}{3} < p < 4$, which under appropriate boundary data implies C^α-Hölder continuity of the solution to the mixed BVP in the closed domain $\overline{\Omega}$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.