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BOOK of ABSTRACTS

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მეორე საერთაშორისო კონფერენცია

„გამოყენებითი მათემატიკის თანამედროვე პრობლემები“

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საერთაშორისო კონფერენცია
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სპონსორი

შოთა რუსთაველის ეროვნული სამეცნიერო ფონდი

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I. VEKUA INSTITUTE OF APPLIED MATHEMATICS OF TSU – 45

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The talk concerns 45 years long history of Ilia Vekua Institute of Applied Mathematics of Ivane Javaxishvili Tbilisi State University [1]. The Institute was founded by Georgian mathematician and mechanist Ilia Vekua on October 29, 1968. The aim of the Institute was to carry out research on important problems of applied mathematics, to involve University professors, teachers and students in research activities on topical problems of applied mathematics in order to integrate mathematics into the study processes and research, and to implement mathematical methodologies and calculating technology in the non-mathematical fields of the University. In 1978, the Institute was named after its founder and first director Ilia Vekua. At present, the Institute successfully continues and develops activities launched by his founder in four scientific directions:

- Mathematical problems of mechanics of continua and related problems of analysis;
- Mathematical modelling and numerical mathematics;
- Discrete mathematics and theory of algorithms;
- Probability Theory and mathematical Statistics.

The institute sees its mission as threefold:

- Carrying out fundamental and practical scientific research in applied mathematics, mathematical and technical mechanics, industrial mathematics and informatics, undertaking state and private sector contracts to provide expert services;
- Offering the university a high-level computer technology base for University professors and teachers, research employees and students undertaking their scientific research activities;
- Supporting PhD and post-graduate students to attain scientific grants, as well as through employment within the Institute and participation in scientific conferences.

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MATHEMATICS AT I. JAVAKHISHVILI TBILISI STATE UNIVERSITY

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The talk deals with the history of mathematics at the I.Javakhishvili Tbilisi University. On January 26, 1918 (February 8 by new style) on the day of memory of David the Builder (Georgian King) the Georgian State University - the first university in the Caucasus was opened. From the very beginning scientific work started in mechanics and mathematics. Georgian University became the centre of the national mathematical education. To a small group of scientists who founded the University belonged also Andria Razmadze, graduate student and private-docent of the Mathematical Department of the Physics and Mathematics Faculty of Moscow University. In September 1918 the integrated faculty of natural sciences, mathematics and medicine was founded (Dean – V. Moseshvili, secretary-A. Kharadze). In 1919 this faculty was divided into two faculties, A. Razmadze and A. Kharadze were elected as dean and secretary of the faculty of natural and mathematical sciences, respectively. At the end of 1919 Andria Benashvili became dean of this faculty. Along with Andria Razmadze a crucial role in the founding of the Georgian Mathematical School was played by N. Muskhelishvili, G. Nikoladze, A. Kharadze and later I. Vekua and V.Kupradze. N. Muskhelishvili created very important works in the theory of elasticity and integral equations. Very remarkable is I. Vekua's immeasurable contribution to the development of mathematics. He initiated the establishment of the Institute of Applied Mathematics. Sh. Mikeladze created the basis for the scientific research work in computational mathematics. L Gokieli laid the basis for studying foundation and methodological problems of mathematics. V. Chelidze's works gave the basis of the Georgian school in the theory of functions. G. Chogoshvili created the school of topology and abstract algebra. The high-quality works in differential and integral equations are associated with the name of A. Bitsadze. I. Kiguradze created the Georgian school of ordinary differential equations. R. Gamkrelidze and G. Kharatishvili founded the Georgian school in optimal control and G. Mania established school of Probability and Statistics. In 1951 the Faculty of Mechanics and Mathematics was isolated as an independent structural unit. The first dean of this faculty was N.Vekua. The scientific research work was carried out at ten chairs: Mathematical Analysis, Differential and Integral Equations, Theory of Functions and Functional Analysis, Probability Theory and Mathematical Statistics, Algebra and Geometry, Foundation of Mathematics and Methods, Computational Mathematics and Informatics, Common Mathematics, Theoretical Mechanics and Astronomy. Presently, the great scientific research traditions and pedagogical work are going on successfully in the Department of Mathematics of the Faculty of Exact and Natural Sciences.

Mathematical Problems of Mechanics of Continua and Related Problems of Analysis

Functional Classes of the Solutions of Elliptic Systems on the Plane

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The classes of generalized solutions of elliptic systems on the plane are introduced and studied. They are the analogues of classical Hardy and Smirnov spaces of holomorphic functions.

Neumann-Type Boundary Value Problem for Second Order Partial Differential Equations on the Unit Ball in Clifford Analysis

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Using a Cauchy-Pompeiu representation in complex Clifford algebra $\mathbb{C}_m$ and an explicit representation formula for Neumann function for Laplacian over the unit ball in $\mathbb{R}^m$ ($m \geq 3$), we give the solution of the Neumann problem for Poisson equation. We introduce some integral operators in Clifford analysis and making use of them, we study a Neumann-type problem for second order equations.

About Genetic Algorithm Approach for One Dynamic Model of Communication Market

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This talk considers one dynamic model of communication market using the example of the behavior of two competitor companies. The model is described by the system of ordinary differential equations with delays. Delays have the potential to better describe the process. The optimal control problem is considered for this model.

The application of genetic algorithms especially those for solving optimal control problems is a relatively new area of research [1], [3]. According to real data the solution of this problem is obtained by genetic algorithm, using Mathcad-15. The results of genetic algorithm obtained here are compared with numerical solution of the same problem, obtained in [2].

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On some Non-Standard Problems in the Theory of Thin-Walled Structures

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Thin-walled structures are up to now in the focus of researchers. The reason is very simple: even in the case of advanced computers and software sometimes problems arises, for example, if the thickness is very thin. In this case two-dimensional theories can be helpful.

What is the best way to formulate two-dimensional theories - this is an open question. On different approaches and the state of the art is reported, for example, in Altenbach et al. (2010). The main directions are

- the application of hypotheses on the stress, strain and/or displacement state,
- the use of mathematical approaches (power series, special functions, asymptotic integration, etc.) and
- direct formulations.

All approaches have advantages and disadvantages which will be discussed in detail.

Summarizing the different approaches some practical problems will be discussed. The examples are coming from different branches of the shell and plate theory considering, for example,

- the size effect (theory of nanoplates),
- the influence of inhomogeneous thickness properties or
- the problems related to the inelastic behavior.

It will be shown that for solving the above-mentioned non-standard problems it is not necessary to develop new theories. It is enough to apply the basics of continuum mechanics more carefully and understand deeper the meaning and consequences of the introduced hypotheses, assumptions, etc.

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On the Main Boundary Value Problems of the Theory of Elasticity for Some Non-Homogeneous Plane Bodies

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The main plane boundary value problems of the mathematical theory of elasticity are considered for the special non-homogeneous body, for which it is possible to construct in terms of Bessel functions the general solution of the corresponding system of partial differential equations.

Extensions of the Dirac Factorization and Applications

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We show that the Dirac factorization method can be successfully employed to treat problems involving operators raised to a fractional power. The technique we adopt is based on an extension of the Pauli matrices and the properties of the roots of unity. We also comment about the possibility of using the method to linearize evolution equations containing the n-th root of differential operators.

Green Functions for Irregular Plane Domains

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The conformal invariance of the harmonic Green function for plane domains guarantees the existence of the Green function for any simply connected domain conformal to the unit disc even when the boundary fails to be smooth. The explicit construction of the conformal mapping however is often not easy to achieve. Although for polygonal domains there exists the Schwarz-Christoffel formula, in general it is too involved for being used for practical problems. Instead for certain even not necessarily simply connected domains the parqueting-reflection principle works out. It can be applied to domains the boundary of which consists of segments from straight lines and circles. Moreover, reflecting the domain at its boundary parts the entire plane has to be covered eventually up to some singular points. If this is not the case, continued reflections might serve to achieve this covering. Examples of such domains are circles, equilateral triangles, right angle triangles, rectangles, hexagons, half hexagons, disc sectors, half planes, cones, concentric rings, half rings, quarter rings, some lunes and lenses. The Green function then is attained as $\log|P(\cdot,z)|$ with a meromorphic function P depending on the parameter z. Choosing z in the domain as a simple pole of P and taking any reflection of z as a simple zero and if further reflections are needed taking any reflection of a zero as a simple pole and any reflection of a pole as a simple zero, P is found by multiplying these respective factors. In case when singular points are not covered some extra investigations are needed. As all the zeros and poles of P lie outside the domain up to the point z itself, $\log|P|$ is harmonic in the domain up to the characteristic singularity. The basic idea behind this construction is that if z approaches a boundary point of the domain its respective reflection being a zero of P approaches the same boundary point. Hence both factor out in the product P. Because this happens with all the other factors at the same time, $|P|$ becomes 1 at the boundary. At the same time the harmonic Neumann function is attained. Instead of distinguishing the reflection points into two classes of poles and zeroes all these points are taken as poles. In case of convergence log of this product $|P|$ is a Neumann function. If the product attained fails to converge additional factors are to be added to achieve convergence. They just contribute a harmonic alteration of the Neumann function. This general relation between Green and Neumann functions have turned out in all cases. However, there should be a general proof for this. In the case of the unit disc there is an interpolation function between the Green and the Neumann function, a kind of harmonic Robin function. It depends on two parameters. For particular choices of them the Robin function becomes Green for others Neumann.

On some Solutions of Equations of the 2D Dynamical Theory of Thermoelasticity with Microtemperatures

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In the present talk the mathematical model of the linear 2D dynamical theory of thermoelasticity with microtemperatures is considered [1-5]. The representation of regular solutions of the equations of dynamic in this theory is obtained, the fundamental and singular solutions for a governing system of equations of the linear 2D dynamical theory of thermoelasticity with microtemperatures in the Laplace transform space are constructed. Finally, the single-layer, double-layer and volume potentials are presented.

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An Application of Theory of Self-Conjugate Differential Forms to the Dirichlet Problem for Cimmino System

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In the paper "S. Dragomir and E. Lanconelli *On the first order linear PDE systems all of whose solutions are harmonic functions* Tsukuba J.Math. 30 (2006), No. 1, 149–170" the authors proved that if there exists a solution of the following Dirichlet problem for the Cimmino system

$$\begin{cases} u_{\overline{z}} + \overline{v}_w = f, \qquad u_{\overline{w}} - \overline{v}_z = g & \text{in } \Omega \\ u = \phi \qquad\qquad v = \psi & \text{on } \partial\Omega \end{cases} \tag{1}$$

(Ω being a sufficiently smooth subset of $\mathbb{C}^2$), then (f, g, ϕ, ψ) satisfies the compatibility conditions

$$Re\left\{2\int_\Omega (f\overline{h} + g\overline{k})dV - \int_{\partial\Omega} \left\{\phi\left[(n_1 + in_2)\overline{h} + (n_3 + in_4)\overline{k}\right]\right.\right.$$
$$\left.\left. +\psi\left[(n_3 + in_4)h - (n_1 + in_2)k\right]\right\}\right\} = 0 \tag{2}$$

for any solutions h, k of

$$h_z + k_w = 0, \qquad h_{\overline{w}} - k_{\overline{z}} = 0 \qquad \text{in } \Omega.$$

Here (n_1, n_2, n_3, n_4) is the outward unit normal to $\partial\Omega$. In the same work they pose the problem whether such conditions are also sufficient for the existence of a solution of problem (1).

By means of the concept of self-conjugate differential forms we were able to solve this problem. A non-homogeneous differential form $U = \sum_{k=0}^n u_k$, u_k being a $k-$differential form, is self-conjugate if

$$dU = \delta U \tag{3}$$

(d and δ being the differential and the co-differential operator respectively), i.e. if

$$\delta u_1 = 0; \quad du_{k-1} = \delta u_{k+1}, \quad (k = 1, \dots, n-1); \quad du_{n-1} = 0.$$

Systems providing analogues of Cauchy-Riemann system in $\mathbb{R}^n$, like Moisil-Theodorescu system, Fueter system, Cimmino system etc. can be considered as particular cases of (3). Hinging on some existence theorems for self-conjugate differential forms (see "A. Cialdea *On the theory of self-conjugate differential forms* Atti Sem. Mat. Fis. Univ. Modena, XLVI (1988) 595–620") we showed that conditions (2) are also sufficient for the existence of the solution of problem (1).

On a Boundary Value Problem for Non-Linear Elliptic Complex Partial Differential Equations in the Upper Half Plane

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We consider non-linear elliptic complex partial differential equations with Schwarz boundary conditions in the upper half plane. Using the integral representation formulas for the solutions of the model equations, some classes of singular integral operators are introduced together with some of their properties. Such operators are used in transforming the boundary value problem into an integro-differential system and then solvability of the problem is discussed.

Hierarchical Models for Biofilms Occupying Thin Prismatic Domains with Variable Thickness

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The present talk is devoted to the construction and investigation of two-dimensional hierarchical models for biofilms occupying thin prismatic domains with variable thickness ([1], [2]). We investigate the existence and uniqueness of solutions of the reduced two-dimensional problems in suitable weighted Sobolev spaces. For the sake of simplicity we state the zero approximation; in the N-th approximation similar but more complicated assertions can be carried out.

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L^p-Dissipativity of the Lamé Operator

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In this talk I will discuss some results concerning the L^p-dissipativity. In particular a necessary and sufficient condition for the L^p-dissipativity of the Lamé operator in the two-dimensional case and necessary and sufficient conditions for the weighted L^p-dissipativity in the class of rotationally invariant vector functions (in any dimension) will be given.

This is a joint work with Vladimir Maz'ya.

On the Potential Theory in Cosserat Elasticity

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Let us consider the Dirichlet problem for the Laplace equation

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ u = f & \text{on } \Sigma \end{cases}$$

where Ω be a bounded domain of $\mathbb{R}^n$. In the classical indirect method, the solution is given in the form of a double layer potential and the boundary condition gives an integral equation which can be reduced to a Fredholm one. If we look for the solution in the form of a simple layer potential, we are led to an integral equation of the first kind:

$$\int_\Sigma \varphi(y) s(x,y) d\sigma_y = f(x), \qquad x \in \Sigma. \tag{1}$$

If $n = 2$, Muskhelishvili solved the problem differentiating on Σ both sides of (1) with respect to the arc length. In [1] Cialdea extended the method of Muskhelishvili to $\mathbb{R}^n$. The main idea was to replace the derivative with respect to the arc length with the exterior differential. In this way he obtained and solved a singular integral equation in which the unknown is a scalar function while the data is a differential form of degree one. This methods hinges on the theory of reducible operators and the theory of differential forms and it uses neither the theory of pseudodifferential operators nor the concept of hypersingular integrals. This approach has been applied to different boundary value problems for several partial differential equations, also in multiply connected domains.

In this talk I will present some applications of this method to the main four boundary value problems of the three-dimensional Cosserat elasticity.

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On the One Method of Constructing Optimal Mixed Strategies in Fuzzy Antagonistic Matrix Games

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In classical problems of game theory usually there is no consideration of personal, psychological data of the opponent. Such information is very useful when concrete decision must be made. They are described by fuzzy linguistic variables. Decisions are made by use of a base of fuzzy rules and rules of fuzzy logic. For example: let S_i, $i = 1, 2, \cdots, k$ be the base of fuzzy rules. $z_i, i = 1, 2, \cdots, l$, is a game rule, criterion. The certainty α_0 of selecting criterion $z_0 \in \{z_i, i = 1, 2, \cdots, l\}$ can be calculated as follows

$$\alpha_0 = \min\{\mu_A(x), \mu_B(y)\},$$

if $S_0 \in \{S_i, i = 1, 2, \cdots, k\}$ is a logical conjunction $\{S_0 : \text{if } (x \text{ is } A) \text{ and } (y \text{ is } B), \text{ then } z_0\}$;

$$\alpha_0 = \max\{\mu_A(x), \mu_B(y)\},$$

if S_0 is a logical disjunction $\{S_0 : \text{if } (x \text{ is } A) \text{ or } (y \text{ is } B), \text{ then } z_0\}$, where $\mu_A(x), \mu_B(y)$ are the membership functions of fuzzy sets A, B, respectively.

The method of constructing optimal mixed strategies on a base of fuzzy rules is offered. One may note that in creation of the base of fuzzy rules and membership function the decisions are made by subjective judgments. Note also that for logical operations there are different ways to represent a membership function, which causes, as a rule, a certain effect of variability. The study of the sensitivity and stability of a solution is also significant. In this aspect the works [1,2,3] must be mentioned.

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Second-Order Internal Constraints in Theories of Shear-Deformable Beams and Plates

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In linear elasticity, a simple internal constraint is a restriction on the values that the displacement gradient can assume in deformations of a body. An internal constraint is accompanied by a reactive stress that adds to the ordinary active stress given by the constitutive equation, and has the role of maintaining the constraint. The reactive stress does no work in any possible deformation and, since it is not constitutively determined, is available to be used for assuring the equilibrium of an internally constrained body.

The form of displacement field in the classical theory of plates can be regarded as the effect of the presence of simple internal constraints. Such interpretation allows to remove the inconsistency with three-dimensional elasticity usually attributed to that theory, because the reactive stress makes possible to satisfy the three-dimensional equilibrium equations in the body modeled by the plate (cf. [1], [2]).

We discuss how this approach is extended to the theories of shear-deformable beams and plates, in which the forms of displacement field can be viewed as results of “a priori” restrictions on both the first and the second displacement gradients. The latter prescriptions are non-simple, second-order, internal constraints that are admissible in bodies made of non-simple, second-grade, materials in which, in addition to the usual stress, also hyperstress is present (cf. [3], [4]).

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Modeling Laminated Plates

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The talk focuses on recent developments in the modeling of linearly elastic laminated plates.

We work under the following assumptions: in each layer, the material is transversely isotropic in any plane parallel to the mid-plane, and deformations are of Reissner-Mindlin (R-M) type; across planes separating adjacent layers, displacement and traction vectors are both continuous. The governing equations are deduced from a 2D formulation of the Virtual Power Principle, a formulation obtained by thickness integration of the general 3D expression, where the admissible virtual displacements are consistent with R-M representation and are compatible with the assumptions of continuity across the interfaces.

Explicit analytical solutions are furnished, for both circular and rectangular plates [1], which turn out to be in good agreement with the predictions of other plate theories, such as those due to Yang et al. [2] and Seide [3], and with 3D Finite Element simulations obtained by COMSOL Multiphysics.

Moreover, it is shown that exact 3D solutions of Levinson-type can be derived by exploiting the same assumptions [4]. In addition to serve as benchmarks for plate theories, these 3D solutions may serve to check how the approximation of the 3D stress field achievable by the use of standard plate theories can be improved by taking into account the reactive stresses associated with the internal constraints implicit in the kinematical assumptions of the R-M theory [5].

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Samuelson's Model Realization Problem for Georgia

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In modern conditions the adequate enough econometric models construction problem allowing with a necessary accuracy to predict rates of economic growth is very important for our (and not only our!) economy. Thus, as we know, it is frequent that in this point of view, models of small dimensions don't concede to models of big dimension. Having this in mind, in this work the realization problem of Samuelson model for the Georgian economy is considered. As it is well known, Samuelson's models initial equations have the form

$$C_t = \beta_1 Y_{t-1},$$

$$I_t = \beta_2 (C_t - C_{t-1}),$$

$$Y_t := C_t + I_t + G_t,$$

where Y_t denotes volume of gdp, I_t stands for the volume of investment, while C_t and G_t are denoting, correspondingly, private and government sectors consumer expenditures. Resulting from this, Samuelson's model has the form

$$Y_t = G_t + \beta_1(1 + \beta_2)Y_{t-1} + \beta_1\beta_2 Y_{t-2}.$$

However, in view of the fact that available for us the data don't allow creation of classical variant of Samuelson's model, we made an attempt to create some modification of this model. As a result we obtained the model

$$Y_t = 1.1571Y_{t-1} - 0.3236Y_{t-2} + G_t + 320.8t,$$

which gives us some forecast estimations, for 2013-2017. As this forecast estimations probably are very optimistic, the model option which turns out by "direct" identification of this modification of Samuelson's model is constructed. The latter model has the form

$$Y_t = 1.0617Y_{t-1} - 0.1715Y_{t-2} + 299.82t - 328.70.$$

Finally, the comparative analysis of accuracy of these models and the forecast estimations received on their basis is carried out.

Minimal Vector Fields on Riemannian Manifolds

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We consider a vector field V defined on a manifold M, of dimension n, as being a map from M into the tangent manifold TM. Given a Riemannian metric g on M, there is a natural metric on TM, known as the Sasaki metric. The submanifold $V(M) \subset TM$, diffeomorphic to M, is endowed with the corresponding induced metric and we can consider the n-dimensional volume of this submanifold. It is easy to see that if the vector field V is parallel, with respect to the Levi-Civita connection of the metric g, then $V(M)$ is a totally geodesic submanifold of TM, isometric to (M, g), that has the minimum volume among all submanifolds obtained from smooth vector fields of the same lenght that V. If the Riemannian manifold (M, g) does not admit parallel vector fields, a natural question is to compute the value of the infimum of the volume of smooth unit vector fields and to find those vector fields determining submanifolds of TM of less possible volume. This is known as the Gluck and Ziller problem; it was proposed in a basic paper of these authors in 1986 and it is far for being solved.

Regular Systems of ODE on Riemann Surfaces and Riemann-Hilbert Boundary Value Problem

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We give a brief of survey of recent results on the classical Riemann-Hilbert problem for differential equations on a Riemann surfaces. We emphasize geometrical aspects of the problem involving the notion of stability of vector bundles with connections. To a system of Fuchsian differential equations $df = \omega f$ defined on a Riemann surface X of genus g, where ω is a meromorphic 1-form with the set of singular points $S = s_1, s_2, \dots, s_m$, we will associate certain numerical invariants [1]. These invariants are numerical characteristics of algebraic, analytic and topological objects obtained from the system. The above Fuchsian system determines the monodromy representation $\rho : \pi_1(X - S, z_0) \to GL(n, C)$ from which on the non-compact Riemann surface $X - S$ one obtains the holomorphically trivial vector bundle $E_\rho \to X - S$ with the holomorphic connection ∇, whose connection matrix is the 1-form ω. It is known that the bundle (E_ρ, ∇_ρ) may be extended to the whole Riemann surface X so that one obtains a family of holomorphically nontrivial bundles with logarithmic connections, which does not in general contain any semistable bundles. If (E^0, ∇^0) is the canonical extension of the given bundle [2], then using the monodromy representation one can compute the Chern number, Fuchs weight, and the moduli space of holomorphic deformations of this bundle. We will use these invariants to investigate the s. c. inverse problem (see [1] or [2]). The latter purports construction, from a given representation as above, of a Fuchs system whose monodromy representation coincides with ρ. Along with this problem we will also consider the linear conjugation problem with the piecewise-constant boundary condition which is constructed from the monodromy matrices.

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Absolute Convergence of Multiple Fourier Series

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Let $f \in L(T^s),\ s \geq r,\ T = [-\pi, \pi]$ and let

$$\sum_{n=-\infty}^{\infty} C_n(f) e^{int}$$

be its multiple trigonometric Fourier series, and let (a_n) be a sequence of non-negative S-multiple numbers that satisfies sufficiently general conditions.

The sufficient conditions are found which should be satisfied by the best approximations of the function by trigonometric quasi-polynomials so that the series

$$\sum_{n=-\infty}^{\infty} |C_n(f)|^r a_n \qquad r \in (0, r)$$

may be convergent.

On Partial Sums of Fourier Series of Functions of Boundary Variation

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Banach [1] has proved that for any function $f \in l_2(I),\ \ I = [0, 1],\ \ f \sim 0$ there exists an orthonormal system $(\varphi_n(x))$ such that $\overline{\lim\limits_{n\to\infty}} |S_n(f, x)| = +\infty$ almost everywhere on I, where $S_n(f, x)$ are partial sums of Fourier series of the function $f(x)$ with respect to the system $(\varphi_n(x))$.

Necessary and sufficient conditions are found which should be satisfied by the system $(\varphi_n(x))$ so that the partial sums of any function of boundary variation may be uniformly bounded on I.

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Additive Difference Schemes for Solution of Problems of Thermo-Elasticity in Micro Temperature Field

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The present talk deals with research of finite-difference schemes and models constructed by additive method for some initial-boundary value problems of thermo-elasticity taking into account micro temperature field.

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Well-Possedness of the Cauchy Problem for One Class of Neutral Functional Differential Equation Taking into Account Delay Perturbation

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The talk deals with the neutral functional differential equations

$$\dot{x}(t) = A(t)\dot{x}(\sigma(t)) + f_0(t, x(t), x(\tau_0(t))), t \in [t_0, t_1] \subset I = [a, b]$$

with the initial condition

$$x(t) = \varphi_0(t), t \in [\tau_0(a), t_0), x(t_0) = x_0; \dot{x}(t) = h_0(t), t \in [\sigma(a), t_0].$$

Here, $\sigma(t)$ and $\tau_0(t)$ are continuous differentiable functions on the interval I satisfying the conditions:

$$\sigma(t) < t, \dot{\sigma}(t) > 0, \tau(t) < t, \dot{\tau}_0(t) > 0;$$

the initial function $\varphi_0(t)$ is piecewise-continuous with finitely many discontinuity points of the first kind; the initial function $h_0(t)$ is measurable and bounded.

In this work, a theorem is proved on the continuous dependence of a solution with respect to perturbations of initial moment t_0, initial vector x_0, variable delay $\tau_0(t)$, initial functions $\varphi_0(t)$ and $h_0(t)$, righthand side f_0. We note that the perturbations of function f_0 are small in the integral sense.Analogous theorems without perturbation of delay are proved in [1-3].

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On Construction of Approximate Solutions of Equations of the Non-Shallow Spherical Shell for the Geometrically Nonlinear Theory

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I. Vekua constructed several versions of the refined linear theory of thin and shallow shells, containing the regular processes by means of the method of reduction of 3-D problems of elasticity to 2-D ones [1].

In the present paper we consider non-shallow spherical shells [2]. The components of the deformation tensor have the following form:

$$e_{ij} = \frac{1}{2}(\mathbf{R}_j \partial_i \mathbf{u} + \mathbf{R}_i \partial_j \mathbf{u} + \partial^k \mathbf{u} \partial_k \mathbf{u}),$$

where $\mathbf{R}_i$ are covariant basis vectors, $\mathbf{u}$ is the displacement vector.

By means of I. Vekua method the systems of two-dimensional equations are obtained. Using the method of the small parameter, approximate solutions of these equations are constructed [3], [4]. The small parameter $\varepsilon = \frac{h}{R}$, where $2h$ is the thickness of the shell, R is the radius of the middle surface of the spherical shell. Some boundary value problems are solved.

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Stability, Bifurcation and Post-Critical Behavior of a Flexible Missile under an End Thrust

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Elastic beam stability under follower forces has been extensively studied as models for such problems as a flexible missile under an end thrust, a cantilever column with impinging fluid of free end, a cantilever rod with distributed tangential load, a cantilever pipe conveying fluid which loses stability by flutter, and others. The study of the dynamic stability of an elastic beam under a harmonically varying load with a pinned end and an end constrained to move axially has been presented by Mettler, Bolotin, Blekhman and others. Approximate techniques using numerical methods are often required to solve the linear and nonconservative problems. Galerkin's technique was first applied by Leipholz to calculate bifurcation loads for a nonconservative system. Mettler and Bolotin have used Galerkin's method to convert the equations of motion to time differential equation which are studied for their stability using Floquet theory. Levinson showed that the conventional Hamilton's principle with nonconservative forces may be treated as a variational problem with some constraints, and then the Ritz method can be employed. Leipholz used the adjoint system to establish a well posed variational problem without any constraint conditions. The finite element method has been used based on the variational approach suggested by Levinson, and based on Leipholz's variational principle. The ordinary finite difference method was used with the concept of a transfer matrix to solve a non-conservative problem by Leipholz. Guran investigated the dynamic stability of a uniform free rod under an end thrust using finite difference techniques with the concept of a transfer matrix. In nonconservative stability problems, the dynamic stability criteria must be applied. The bifurcation or critical load is the smallest load which, when slightly disturbed, causes a change in the equilibrium configuration of the system. A forced vibration analysis must be performed to determine this load. In reality, in all above mentioned problems, we are faced with nonlinear non-conservative phenomena. In order to obtain a tractable solution, the equations governing such phenomena are very frequently linearized. Linearization as a means of approximation is valuable for many purposes but often new phenomena occur in nonlinear systems which cannot in principle occur in linear systems. A standard elastic system used to study the features of flutter is known as Beck's column. It consists of a uniform elastic cantilever subjected to a follower (i.e., tangential) load at its tip. Postcritical behaviour of elastic beams under non-conservative forces has been studied more recently in the context of Beck's column by Chen. In this talk attention is focused on the nonlinear dynamic stability and post-critical behavior of a uniform free-free rod (idealizing a flexible missile) under an end thrust is investigated using a discrete model.

On Some Bicriterial Problems of the Discrete Optimization

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Issues of solving some binary optimization problems are presented. It is shown that the sentence below stands true for a bicriteria assignment problem: There exist some instances, where we get $|P(Y)| = n!$. here n is a size of an instance, Y is a feasible set in criterion space, $P(Y)$ is a set of Pareto optimal solutions. Numerical test results are included for the low dimension assignment problem. One practical method is considered for solving bicriteria version of project selection problem.

Dolbeaut's Lemma for the Functions of the Class $L_p^{loc}(\mathbb{C}), p > 2$

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This talk is devoted to the existence of $\dfrac{\partial}{\partial \overline{Z}}$ - primitive of the function of the class $L_p^{loc}(\mathbb{C}), p > 2$.

The following theorem is valid (see [1]):

Theorem. Every function $a(z)$ of the class $L_p^{loc}(\mathbb{C}), p > 2$, has $\dfrac{\partial}{\partial \bar{z}}$ – primitive function $Q(z)$ on the whole complex plane satisfying the Hölder condition with the exponent $\dfrac{p-2}{p}$ on each compact subset of the complex plane $\mathbb{C}$; moreover if $q(z)$ is one $\dfrac{\partial}{\partial \bar{z}}$ – primitive of the function $a(z)$ then all $\dfrac{\partial}{\partial \bar{z}}$ – primitives of this function are given by the formula

$$Q(z) = q(z) + \Phi(z),$$

where $\Phi(z)$ is an arbitrary entire function.

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On Improved Algorithm for Analytical Solution of the Heat Conduction Problem in Periodic 2D Composites

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We consider a boundary value problem in unbounded 2D doubly periodic composite with circular inclusions having arbitrary constant conductivities. By introducing complex potentials, the boundary value problem for the Laplace equation is transformed to a special R-linear BVP for doubly periodic analytic functions. This problem is solved with use of the method of functional equations. The R-linear BVP is transformed to a system of functional equations. A new improved algorithm for solution of the system is proposed. It allows one not only to compute the average property but to reconstruct the solution components (temperature and flux) at an arbitrary point of the composite. Several computational examples are discussed in details demonstrating high efficiency of the method. Indirect estimate of the algorithm accuracy has been also provided.

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On One Boundary Value Problem for a Wave Equation with Power Nonlinearity

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In the plane of independent variables x and t we consider a wave equation with power nonlinearity of the following form

$$u_{tt} - u_{xx} + \lambda |u|^{\alpha} u = f(x,t), \tag{1}$$

where λ and α are given real constants and $\lambda\alpha \neq 0,\ \alpha > -1;\ f$ is a given real function, while u is an unknown real function.

Denote by $D := \{(x,t) \in \mathbb{R}^2 : 0 < x < kt,\ t > 0;\ 0 < k := const < 1\}$ angular domain lying within the characteristic angle $t > |x|$ and bounded by straight beams $\gamma_1 :\ x = kt,\ t > 0$ and $\gamma_2 :\ x = 0,\ t > 0$.

For equation (1) we considered the problem of Cauchy-Darboux on finding in the domain D a solution $u(x,t)$ of this equation by the boundary conditions

$$u|_{\gamma_1} = 0, \quad u_x|_{\gamma_2} = 0. \tag{2}$$

Theorem. Let $\alpha > 0$ and $\lambda > 0$. Then for any $f \in C^1(\overline{D})$ problem (1), (2) has a unique global classical solution u in the domain D.

When the conditions of the theorem are violated the questions of existence of local and nonexistence of global solutions of problem (1), (2) are also considered.

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The Solution of Three Auxiliary Problems and Saint-Venant's Problems for Three-Layer Con Focal Elliptic Tube for Elastic Materials with Differents Poisson Ration

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Solution of three auxiliary problems with the help of Faber-Suetin polynomials are given for the indicated problem in closed form and the Saint-Venant's problem is solved with their help for three layers on focal elliptic isotropic tube.

Analytical Solution of Classical and Non-Classical Boundary Value Contact Problems of Thermoelasticity for a Rectangular Parallelepiped Consisting of Contractible and Non-Contractible Elastic Layers and a Corresponding Program

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The talk is devoted to analytical solution construction of some application problems of thermoelasticity for a multilayer rectangular parallelepiped. There are solutions of some boundary value problems for a rectangular parallelepiped known in literature. In contrast with those works, our paper deals with a class of boundary value and boundary value contact problems, the great majority of which have been solved for the first time. It should be noted that some of the layers may consist of a non-contractible elastic material. Analytical solutions have been obtained and on their basis corresponding illustration are given. A description of a computer program is also presented by means of which computations and graph construction are performed.

Static thermoelastic equilibrium is considered for a rectangular parallelepiped. On the lateral facets of the parallelepiped boundary conditions of symmetrical or anti-symmetrical solution continuation are imposed. Condition of rigid, sliding or other type of contact between layers may be given. On the upper and lower facets of the parallelepiped arbitrary boundary conditions are defined. The stated problem are solved analytically using separation of variable technique while the used general solution is represented by means of harmonic functions. The solution of the problems is reduced to the solution of a system of linear algebraic equations with block-diagonal matrices. The asymptotic behavior of these matrices is studied and by means of necessary transformations matrices convenient for numerical implementation are obtained.

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Can Mathematics Help to Control and Avoid Environmental Stress?

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How much time will be used by a tsunami to reach the shoreline? Which airport have to be closed if a volcano on Iceland blows out? Which region will be flooded if the water level will increase by 1 meter? How to control the speed and density of cars on a highway to avoid traffic jams?

All these questions and many others are related to the safeness of people on the planet earth and mathematics can help to answer these questions. In this contribution we will show the underlying mathematical models of these problems, we shortly discuss the theoretical and numerical background and present some results obtained by numerical simulations. The related mathematical challenges for these computations are accuracy, efficiency, local dynamical grid adaption, parallel computation, load balancing, artificial boundary conditions, validation and visualization. Furthermore we will demonstrate by an interactive installations of the vulcano problem, how we can interest non-experts for mathematics.

On the Existence of Bounded and Vanishing at Infinity Solutions of Linear Ordinary Differential Equations of Third Order

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Let us consider the linear differential equation of third order

$$u''' + p_1(t)u'' + p_2(t)u' + p_3(t)u = 0, \tag{1}$$

where $p_k : R_+ \to R$ $(k = 1, 2, 3)$ are continuous functions.

In this work, we establish conditions for the existence of oscillatory solutions of equation (1) which are bounded and tend to zero at infinity.

The equation

$$v'' + p_1(t)v' + p_2(t)v = 0 \tag{2}$$

is called an equation without conjugate points if each of its nontrivial solutions has at most one zero on the interval R_+.

Theorem 1. *Let $p_3(t) \le 0$ for $t \ge 0$, $|p_3(t)| \exp\big(\int\limits_0^t p_1(\tau)\,d\tau\big)$ be non-decreasing, let equation (1) be oscillatory, and let (2) be the equation without conjugate points. Then for any $t_0 > 0$, $c_0, c_1 \in R$, equation (1) has an oscillatory solution satisfying the conditions*

$$u(t_0) = c_0, \quad u'(t_0) = c_1, \quad \limsup_{t\to+\infty} |u(t)| < +\infty.$$

Theorem 2. *Let $p_1(t) \equiv 0$ and let for some $t_0 > 0$, $\beta \in [0, \frac{1}{2}]$ the following inequalities be fulfilled $0 \le p_2(t) \le \frac{\beta(1-\beta)}{t^2}$, $p_3(t) \le 0$ for $t \ge t_0$. Then for any $c_0, c_1 \in R$, equation (1) has a unique solution satisfying the conditions*

$$u(t_0) = c_0, \quad u'(t_0) = c_1, \quad \limsup_{t\to+\infty} \frac{u'(t)}{t^\beta} < +\infty.$$

Theorem 3. *Let $p_1(t) \equiv 0$ and let for some $t_0 > 0$, $\beta \in [0, 1]$, $c > 0$ the following inequalities be fulfilled $-\frac{\beta(\beta+1)}{t^2} \le p_2(t) \le 0$, $p_3(t) \le -ct^{\frac{3}{2}\beta}$ for $t \ge t_0$. Then for any $c_0, c_1 \in R$, equation (1) has a unique solution satisfying the conditions*

$$u(t_0) = c_0, \quad u'(t_0) = c_1, \quad \lim_{t\to+\infty} u(t) = 0, \tag{3}$$

and at the same time the solution of problem (1), (3) is oscillatory and

$$\limsup_{t\to+\infty} t^{-\frac{1}{2}} |u'(t)| < +\infty.$$

Oscillation Criteria for Differential Equations with Several Delays

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Consider the first-order differential equation with several retarded arguments

$$u'(t) + \sum_{i=1}^{m} p_i(t)u(\tau_i(t)) = 0,$$

where the functions $p_i, \tau_i \in C(R_+; R_+)$, $\tau_i(t) \le t$ for $t \ge 0$ and $\lim\limits_{t\to+\infty} \tau_i(t) = +\infty$ $(i = 1, \dots, m)$. Integral conditions of the oscillation of all solutions of this equation are established.

Fast Cubature of Volume Potentials Over Rectangular Domains by Approximate Approximations

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We present an algorithm for computing volume potentials over rectangular domains in $\mathbb{R}^n$. We follow ideas of the method of approximate approximations, which provides high-order semi-analytic cubature formulas for many important integral operators of mathematical physics. They are based on the use of approximating functions with the property that, on the one hand, simple linear combinations provide high-order approximations up to a small negligible saturation error and, on the other hand, the action of the integral operators on these functions can be taken analytically. We choose as basis functions products of Gaussians and special polynomials, for which the action of integral operators can be written as one-dimensional integrals with a separable integrand, i.e., a product of functions depending only on one of the variables. Then the approximation of a separated representation of the density combined with a suitable quadrature of the one-dimensional integrals leads to a separated approximation of the integral operator. Since the separated representation of the density can be approximated with high order and only one-dimensional operations are used, the resulting method is effective also in high dimensional cases. In this talk we consider cubature formulas for advection-diffusion operators over rectangular boxes in $\mathbb{R}^n$. Numerical tests show that these formulas are accurate and provide approximation of order $\mathcal{O}(h^6)$ up to dimension 10^8.

This is a joint work with V. Maz'ya (University of Liverpool,UK, and Linköping University, Sweden) and G. Schmidt (Weierstrass Institute for Applied Analysis and Stochastics, Berlin).

On One Method of Construction the Nonlinear Theory of Non-Shallow Shells

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In this talk, the 3-D nonlinear theory of non-shallow shells are considered, when, so-called "basis vectors" have the form:

$$\mathbf{R}_\alpha = (a_\alpha^\beta - x_3 b_\alpha^\beta)\mathbf{r}_\beta, \quad \mathbf{R}_3 = \mathbf{n}, \quad (-h \le x_3 \le h,\ \alpha, \beta = 1,\ 2),$$

$$\mathbf{R}^\alpha = \frac{a_\beta^\alpha + x_3(b_\beta^\alpha - 2Ha_\beta^\alpha)}{1 - 2Hx_3 + Kx_3^2}\mathbf{r}^\beta, \quad \mathbf{R}^3 = \mathbf{n}, \quad (a_\beta^\alpha = \mathbf{r}^\alpha \mathbf{r}_\beta = \delta_\beta^\alpha,\ b_\beta^\alpha = -\mathbf{n}_\beta \mathbf{r}^\alpha)$$

or

$$\mathbf{R}^\alpha \cong (a_\beta^\alpha + x_3 b_\beta^\alpha)\mathbf{r}^\beta, \quad \mathbf{R}_3 = \mathbf{n}, \quad (Coiter - Naghdi,\ A.\ Lurie, ...)$$

$$(2H = b_1^1 + b_2^2,\ K = b_1^1 b_2^2 - b_1^2 b_2^1).$$

Using the reduction method of I. Vekua, the 2-D system of equations is obtained.

Regularity of Solutions to Mixed Interface Crack Problems

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Keywords: Mixed transmission problems, Interface crack, Singularities of solution.

We investigate asymptotic properties of solutions to mixed boundary value problems of thermo-piezoelectricity (thermo-electro-elasticity) for piece wise homogeneous anisotropic elastic solid structures with interior and interface cracks. Using the potential method and theory of pseudodifferential equations on manifolds with boundary we prove the existence and uniqueness of solutions. The singularities and asymptotic behaviour of the mechanical, thermal and electric fields are analyzed near the crack edges and near the curves, where different types of boundary conditions collide. In particular, for some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and demonstrate their dependence on the material parameters. The questions related to the so called oscillating singularities are treated in detail as well.

The contribution extends the results obtained in the reference [1] to more complex problems.

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Mellin-Edge Quantisation of Operaotrs in Mixed Boundary Problems

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Mixed elliptic boundary value problems, such as the Zaremba problem for the Laplacian, say, with a smooth interface on the boundary, induce elements in algebras of pseudo-differential operators. They may appear as parametrices, or within the solvability process via the reduction of operators to the boundary. In order to represent the operators, for instance, in edge algebras, there have to be established suitable quantisations, partly based on the Mellin transform or in terms of operator-valued symbols along the interface, formulated by means of twisted symbolic estimates. For instance, if in the case of the Zaremba problem the interface on the boundary is smooth, the operators together with the parametrices belong to the edge algebra, and we are in the context of the Mellin edge quantisation, where the interface is the edge, and the Mellin transform acts in the distance variable to the edge. This is the approach in the monographs

[1] D. Kapanadze and B.-W. Schulze, *Crack theory and edge singularities*, Kluwer Academic Publ., Dordrecht, 2003,

[2] G. Harutjunjan and B.-W. Schulze, *Elliptic mixed, transmission and singular crack problems*, European Mathematical Soc., Zürich, 2008,

and in a number of papers since then. It has always been a problem to smoothly organize within the edge algebras the relationship between the mixed problems themselves and the operators obtained by reducing the problems to the boundary, although [2] deals with elliptic mixed problems in general, combined with methods from Eskin's book

[3] G.I. Eskin, *Boundary value problems for elliptic pseudodifferential equations*, Transl. of Nauka, Moskva, 1973, Math. Monographs, Amer. Math. Soc. 52, Providence, Rhode Island 1980

and the paper [4] S. Rempel and B.-W. Schulze, *Parametrices and boundary symbolic calculus for elliptic boundary problems without transmission property*, Math. Nachr. **105** (1982), 45-149,

see also the monograph [5] B.-W. Schulze, *Pseudo-differential boundary value problems, conical singularities, and asymptotics*, Akademie Verlag, Berlin, 1994.

Now in a new paper [6] Der-Chen Chang, N. Habal and B.-W. Schulze, *Quantisation on a manifold with singular edge*. NCTS Preprints in Mathematics, Hsinchu, Taiwan, 2013; we organised a completely general machinerie, entirely working in the edge calculus of boundary value problems, which is doing the above-mentioned job. Besides the algebraic methods we obtain new quantisations for the Dirichlet-to-Neumann operator and various charactersisations of ellipticity with extra edge conditions along the interface. The talk will present details of these ideas.

Effective Solution of the Basic Mixed Boundary Value Problem of Statics of the Elastic Mixture in a Circular Domain

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Let us assume that an elastic mixture occupies the circular domain D centred at the coordinates origin and radius $R = 1$. Denote by L the boundary of the D. Let on L nonintersecting arcs $L_j = a_j b_j$, $i = \overline{1,n}$, $(a_{j+1} \neq b_j, a_{n+1} \equiv a_1)$ be given. Suppose that $L^{'} = \bigcup\limits_{j=1}^{n} L_j$, and $L^{''}$ are remaining parts of L.

In the work by the method of N. Muskhelishvili we obtain an explisit solution to the basic mixed boundary value problem for homogeneous equation of statics of the linear theory of elastic mixture in a circular domain.

The problem deals with the finding of elastic equilibrium of the plate D for the conditions: on $L^{'}$ the displacement vector and on $L^{''}$ the stress vector are given.

Boundary Value Problems of the Linear Theory of Viscoelasticity for Kelvin-Voigt Materials with Voids

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Viscoelastic materials play an important role in many branches of civil engineering, geotechnical engineering, technology and, in recent years, biomechanics. In the last decade there has been an interest in formulation of the mechanical theories of viscoelastic materials with voids of differential type. In this connection, Iesan [1] extends theory of elastic materials with voids, the basic equations of the theory of thermoviscoelasticity for Kelvin-Voigt materials with voids are established.

In this talk the linear theory of viscoelasticity for Kelvin-Voigt materials with voids [1] is considered and some basic results of the classical theory of elasticity are generalized. Indeed, the basic properties of plane harmonic waves are established. The explicit expression of fundamental solution of the system of equations of steady vibrations is constructed by means of elementary functions. Green's formulas in the considered theory are obtained. The uniqueness theorems of the internal and external basic boundary value problems (BVPs) are proved. The representation of Galerkin type solution is obtained and the completeness of this solution is established. The formulas of integral representations of Somigliana type of regular vector and regular (classical) solution are obtained. The Sommerfeld-Kupradze type radiation conditions are established. The basic properties of elastopotentials and singular integral operators are given. Finally, the existence theorems for classical solutions of the internal and external basic BVPs of steady vibrations are proved by using the potential method (boundary integral method) and the theory of singular integral equations (for details see [2]).

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Variation Formulas of Solution for a Class of Neutral Functional Differential Equation Taking into Account Delay Function Perturbation and the Continuous Initial Condition

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For the neutral functional differential equation

$$\dot{x}(t) = A(t)\dot{x}(\sigma(t)) + f_0(t, x(t), x(\tau_0(t))), t \in [t_0, t_1] \subset [a, b]$$

with the continuous initial condition

$$x(t) = \varphi_0(t), t \in [\tau_0(a), t_0], \dot{x}(t) = h_0(t), t \in [\sigma(a), t_0),$$

linear representations of the main part of a solution increment (variation formulas) with respect to perturbations of initial moment t_0, initial functions $\varphi_0(t)$ and $h_0(t)$, delay function $\tau_0(t)$ and righthand side f_0 are proved. In this work, the essential novelty is an effect of delay function perturbation in the variation formulas. The variation formula plays the basic role in proving of the necessary optimality conditions [1]. Variation formulas for various classes of neutral functional differential equations without perturbation of delay are given in [2,3].

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Nonself-Adjoint Degenerate Differential-Operator Equations of Higher Order

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The main object of the present talk is the nonself-adjoint differential-operator equation

$$Lu \equiv (-1)^m (t^\alpha u^{(m)})^{(m)} + A(t^{\alpha-1}u^{(m)})^{(m-1)} + Pt^\beta u = f(t), \qquad (1)$$

where $m \in \mathbb{N}, t \in (0,b), \alpha \geq 0, \alpha \neq 1,3,\ldots,2m-1$, $\beta \geq \alpha - 2m$, A and P are linear operators (in general unbounded) in the separable Hilbert space H and have common complete system of eigenfunctions $\{\varphi_k\}_{k=1}^\infty$, i.e., $A\varphi_k = a_k\varphi_k, P\varphi_k = p_k\varphi_k, k \in \mathbb{N}$, which forms Riesz basis in H, $f \in L_{2,-\beta}((0,b),H)$, i.e., $\|f\|^2_{L_{2,-\beta}((0,b),H)} = \int_0^b t^{-\beta}\|f(t)\|^2_H\,dt < \infty$. Let $\dot{W}^m_\alpha(0,b)$ be the completion of $\dot{C}^m := \{u \in C^m[0,b], u^{(k)}(0) = u^{(k)}(b) = 0, k = 0,1,\ldots,m-1\}$ in the norm $\|u\|^2_{\dot{W}^m_\alpha(0,b)} = \int_0^b t^\alpha |u^{(m)}(t)|^2\,dt$. It is known that for $\beta \geq \alpha - 2m$ there is continuous embedding $\dot{W}^m_\alpha(0,b) \hookrightarrow L_{2,\beta}(0,b)$, which is compact for $\beta > \alpha - 2m$.

Using equalities $u(t) = \sum_{k=1}^\infty u_k(t)\varphi_k, f(t) = \sum_{k=1}^\infty f_k(t)\varphi_k$ the operator equation (1) can be decomposed into an infinite chain of degenerate ordinary differential equations

$$L_k u_k \equiv (-1)^k (t^\alpha u_k^{(m)})^{(m)} + a_k(t^{\alpha-1}u_k^{(m)})^{(m-1)} + p_k t^\beta u_k = f_k, \quad f_k \in L_{2,-\beta}(0,b), k \in \mathbb{N}. \quad (2)$$

First we define the generalized solution for the ordinary differential equation (2) in the weighted Sobolev space $\dot{W}^m_\alpha(0,b)$ and then using general method of A.A. Dezin pass to the operator equation (1). Under some conditions we prove existence and uniqueness of the generalized solution for the operator equation (1).

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The Solutions of Dynamical Problems for Elastic Bodies with Microstructures

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We consider for an elastic bodies the dynamic boundary-value problems of the theory of consolidation with double porosity. The problems are reduced to the corresponding problems for systems of equations for pseudooscillation by Laplace transformation relative to time. The solutions are represented in terms of metaharmonic functions. It is proved that the problem of pseudooscilation has a unique solution. Conditions are given for existence of inverse transformations that provide solutions for the initial problem

Some Issues of Conducting Fluid Unsteady Flows in a Circular Tube

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The developed flows of conducting fluid in an annular pipe are recently studied in detail [1-4], and the possibility to obtain new exact analytical exact solutions seems quite limited. However, such opportunities do exist and it is possible, as will be shown below, to find even simple new solutions that however, have rather interesting qualitative features.

In this article we considered the unsteady flow of a viscous incompressible electrically conducting fluid in annular pipe under external radial magnetic field. An exact solution of the problem in the general form and its extreme case are obtained.

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Multiscale Acoustic Modeling of Porous Media

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In this talk we present some recent results in mathematical modeling of acoustics of porous media with multi-scale structure. In particular, we derive effective acoustic properties of cancellous bone: the spongy, porous inner layer of bone filled with bone marrow-blood mixture. Using two-scale convergence and other homogenization tools we derive effective material properties of the fluid-filled porous matrix.

Theoretical and Experimental Detection of Nonlinear Electromagnetic Waves as the Elements of Turbulence in the Geospace Environment

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The physical and mathematical model of generation and further linear and nonlinear evolution of ultra low frequency (ULF) electromagnetic waves is investigated in the ionosphere at interaction with inhomogeneous winds (shear flows), which represents a new mechanism of energy redistribution in the medium. Self organization of the considered waves into nonlinear strongly localized vortex structures (monopole and dipole vortices, vortex chains, vortex streets) is studied theoretically in the different regions of the ionosphere and magnetosphere. Plasma vortices are often detected by spacecraft in the geospace (atmosphere, ionosphere, magnetosphere) environment, for instance, in the magnetosheath and in the magnetotail region. Large scale vortices may correspond to the injection scale of turbulence, so that understanding their origin is important for understanding the energy transfer processes in the geospace environment. In a recent work, the THEMIS mission has detected vortices in the magnetotail in association with the strong velocity shear of a substorm plasma flow (Keiling et al., J. Geophys. Res., 114, A00C22 (2009), doi:10.1029/2009JA014114), which have conjugate vortices in the ionosphere. By analyzing the THEMIS data for that event, we find that several vortices in the magnetotail can be detected together with the main one, and that the vortices indeed constitute a vortex chain. Such vortices can cause the strong turbulent state in the different media. The strong magnetic turbulence is investigated in the ionsophere as an ensemble of such strongly localized (weakly interacting) vortices. The diffusive processes have been studied too. An effective coefficient of the diffusion was estimated as a square root of the stationary level of noise.

Assessment of Risk Factors of Emergency Cases at Georgian Section of Transport Corridor Europe-Caucasus-Asia by Means of Mathematical Modelling

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The oil spillage caused by oil transportation through pipeline and railway results in serious deterioration of soil, and as a consequence subsurface waters pollution. All these processes have very seriously impact on environment, and therefore, on human health. So it is very important to carry out preventive model studies of possible emergency situations. In this talk on the basis of nonlinear filtration equation of a liquid, petroleum and mineral oil distribution into the soils in case of their emergency spilling on the flat surface containing pits is studied and analyzed. Results of calculations have shown that risk of surface and subsurface waters pollution owing to oil emergency spilling is high. For detection of hydrates origination placement in the gas pipeline for gas non-stationary flow the system of partial differential equations is investigated. Numerical calculations have shown efficiency of the suggested method. A mathematical model (an algorithm) for definition a placement of a gas accidental escape in a simple inclined main gas pipeline is suggested. The algorithm required knowledge of corresponding initial hydraulic parameters at entrance and ending points of the inclined pipeline. The algorithm is based on mathematical model describing gas stationary movement in the simple gas pipeline. A method and a formula for the determination a location of the gas accidental escape in simple inclined main gas pipeline is suggested. Spilled oil's spreading in the eastern coastal zone of the Black Sea is simulated on the basis of a 2-D numerical model describing oil distribution in the seawaters. Numerical experiments have been carried out for different hypothetical sources of pollution in case of different sea circulation regimes dominated for the four seasons in eastern coastal zone of the Black Sea. Some results of numerical calculations have shown that risk of surface and subsurface waters pollution owing to oil emergency spilling on the territory of Georgia is high.

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Operator Splitting for Quasi-linear Evolution Problem with Variable Operator

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We considered the following nonlinear evolution problem with variable operator:

$$u'(t) + b(t)\,Au(t) + M(u(t)) = f(t), \quad t > 0, \quad u(0) = \varphi. \tag{1}$$

Here A is a self-adjoint positive definite (generally unbounded) operator in the Hilbert space H and is represented as a sum of two addends, where each is a self-adjoint positive definite operator ($A = A_1 + A_2$), scalar function $b(t) \geq b_0 > 0$ satisfies Helder inequality, φ is a given element from $D(A)$, $f(t)$ is a continuous and continuously differentiable function, nonlinear operator $M(\cdot)$ satisfies Liptschitz condition. Let the solution $u(t)$ of problem (1) be such that the function $M(u(t))$ is continuous and continuously differentiable with respect to t. Let us introduce the following difference net domain $\overline{\omega}_\tau = \{t_k = k\tau, k = 0, 1, ..., \tau > 0\}$. For the solution of problem (1) the following third order of accuracy decomposition scheme is constructed:

$$u_{k+2} = V(t_{k+2}, t_{k-1})\,u_{k-1} + \frac{3}{4}\tau\left(3V(t_{k+2}, t_{k+1})\,\widetilde{f}(t_{k+1}, u_{k+1}) + V(t_{k+2}, t_{k-1})\,\widetilde{f}(t_{k-1}, u_{k-1})\right),$$

where

$$V(t,s) = \frac{1}{2}\left(W(t,s,\alpha A_1)\,W(t,s,A_2)\,W(t,s,\overline{\alpha}A_1) + W(t,s,\alpha A_2)\,W(t,s,A_1)\,W(t,s,\overline{\alpha}A_2)\right),$$

$$W(t,s,A_i) = (I - \lambda_{0,t,s}(t-s)A_0)\,(I + \lambda_{1,t,s}(t-s)A_0)^{-1}\,(I + \lambda_{2,t,s}(t-s)A_0)^{-1},$$

and where

$$\lambda_{0,t,s} = \frac{6\gamma_{1,t,s}^3 - 6\gamma_{1,t,s}\gamma_{2,t,s} + \gamma_{3,t,s}}{6\gamma_{1,t,s}^2 - 3\gamma_{2,t,s}}, \quad \lambda_{l,t,s} = \frac{1}{2}\left(d_{t,s} + (-1)^l\, i\sqrt{4e_{t,s} - d_{t,s}^2}\right), \ l = 1, 2,$$

$$d_{t,s} = \frac{3\gamma_{1,t,s}\gamma_{2,t,s} - \gamma_{3,t,s}}{6\gamma_{1,t,s}^2 - 3\gamma_{2,t,s}}, \quad e_{t,s} = \frac{3\gamma_{2,t,s}^2 - 2\gamma_{1,k}\gamma_{3,t,s}}{2\left(6\gamma_{1,t,s}^2 - 3\gamma_{2,t,s}\right)},$$

$$\gamma_{1,t,s} = \frac{3b(t) + b\left(s + \frac{t-s}{3}\right)}{4}, \quad \gamma_{2,t,s} = b^2\left(\frac{t+s}{2}\right), \quad \gamma_{3,t,s} = b^3\left(\frac{t+s}{2}\right).$$

The stability of the constructed scheme is investigated and the error of the approximate solution is estimated.

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Numerical Simulation of Some Complex Mesometeorological Processes

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Two-dimensional (in a vertical plane x-z) non-stationary problem about a mesometeorological boundary layer of atmosphere (MBLA) is posed and solved numerically. In it ecologically actual processes, such as a full cycle of development of a cloud and a fog and aerosol distribution are considered against MBLA thermohydrodynamics.

A number of abnormal meteoprocesses are simulated:

Simultaneous existence of a stratus cloud and radiation fog;

An incorporated vertical complex of a stratus cloud and radiation fog;

daily continuous overcast;

Ensemble of humidity processes, particularly, three clouds and a fog which then were transformed to four clouds are simultaneously simulated.

To the roles of horizontal and vertical turbulence in formation of a tropical cyclone and a tornado and in mutual transformation of humidity processes are considered.

Influence of some meteoparameters on aerosol distribution is investigated. Besides, such problems are resulted in a stage of computer realization, as:

"secondary" pollution MBLA (capture already sedimented aerosols and its repeated carrying over);

the account of cooling process, available on cloud and fog border;

influence of cloudy shades on MBLA processes;

the account of difficult temperature heterogeneity of an underlying surface.

On Solution of One Nonlocal Boundary Value Contact Problem

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In the present talk nonlocal boundary contact problem is stated and solved for two-dimensional elliptic equations; uniqueness of solution is proved; iteration process is constructed by means of which solution of initial problem is reduced to solution of two, more simple classical problems and by means of which uniqueness of solution is proved.

Difference Scheme of Variable Directions and Averaged Model of Sum Approximation for One Nonlinear System

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The following multi-dimensional system of nonlinear partial differential equations is considered:

$$\frac{\partial U}{\partial t} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left(V_i \frac{\partial U}{\partial x_i}\right), \tag{1}$$

$$\frac{\partial V_i}{\partial t} = -V_i + g_i\left(V_i \frac{\partial U}{\partial x_i}\right), \tag{2}$$

$$i = 1, ..., n,$$

where g_i are given functions of their arguments.

If $n = 2$ and $0 < \gamma_0 < g_i(\xi_i) \leq G_0, i = 1, 2$, where γ_0, G_0 are constants, then the corresponding two-dimensional analog of system (1), (2) describes the vein formation in meristematic tissues of young leaves (Mitchison G.J. A Model for Vein Formation in Higher Plants. Proc. R. Soc. Lond. B., 207, 1166, 1980, 79-109).

Difference scheme of variable directions and averaged model of sum approximation are considered for system (1), (2). Stability and convergence of these models are studied. Numerical experiments verifying theoretical results for Mitchison's system are carried out.

On the Mathematical Model of Artificial Crystals Growth

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The single crystal growth at the crystallizer, when the permanent temperature is keeping is considered. During the crystallization process the solid crystal is formed from a supersaturated solution by means of the chemical reaction. This phenomena is described by the reaction-diffusion equation with the appropriate initial-boundary conditions, where the unknown function is supersaturation.

In this talk the linear reaction-diffusion equation with the specific initial-boundary conditions is considered. By the separation of variables the effective solutions are obtained.

The case of growth of crystals of the hexagonal configuration is considered. The graphics of supersaturation for the different parameters are constructed by using Maple.

Semi-Discrete Scheme for One System of Nonlinear Integro-Differential Equations

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The following system of nonlinear integro-differential equations arises for mathematical modeling of the process of penetrating of electromagnetic field in the substance:

$$\frac{\partial U}{\partial t} = a(S)\frac{\partial^2 U}{\partial x^2}, \quad \frac{\partial V}{\partial t} = a(S)\frac{\partial^2 V}{\partial x^2}, \tag{1}$$

where

$$S(t) = \int_0^t \int_0^1 \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2\right] dx d\tau. \tag{2}$$

The convergence of the semi-discrete scheme for initial-boundary value problems with different kinds of boundary conditions is studied for the system (1), (2) in case $a(S) = (1+S)^p$, $0 < p \leq 1$. Many numerical experiments have been carried out and compared with theoretical results.

Numerical Investigation of Some Peculiarities of Hydrological Mode in the Upper Layer of the Black Sea for Spring Season

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By means of the 3-D baroclinic model of the Black Sea dynamics developed at M. Nodia Institute of Geophysics (Tbilisi, Georgia) some peculiarities of the forming of the vertical structure of the Black Sea hydrological regime for transitive (April) climatic conditions are investigated. Investigations are carried out with consideration of both the nonstationary atmospheric wind and thermohaline forcing. Herewith the atmospheric thermohaline action is tested in the model by both the Dirichlet conditions through setting the temperature and salinity at the sea surface and the Neumann conditions through setting the heat fluxes, evaporation, and atmospheric precipitation. In the performed numerical experiments wind driven action is reduced to alternation of different climatic wind fields.

The performed numerical experiments have promoted the primary role of the thermohaline impact on formation of the vertical structure of the Black Sea circulation within upper 0 - 136 m layer for transitive climatic conditions. Besides the thermohaline action plays an important role on the horizontal heat transport intensity within an upper 2-26 m layer.

On Some Methods of Approximate Solution of Antiplane Problems of Elasticity Theory for Composite Bodies Weakened by Cracks

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In the present talk antiplane problems of elasticity theory for composite bodies weakened by cracks is presented. The above mentioned problem is studied by the following methods: a) integral equations method [1] and b) finite-differense method [2].

In the first case the body represents a piece-wise homogeneous plane. For tangential jumps of pressure are obtained: the system of the singular integral equations with motionless features when a creak crosses dividing border and one singular integral equation when the crack leaves on dividing border. The aforementioned integral equations are solved by a collocation method, in particular, a method of discrete features.

In the second case the body has the rectangular form. The bending harmonious differential equations were approximated by the difference scheme with the account of corresponding boundary conditions. Such statement of the problem gives possibility to find directly numerical values of function of moving in grid knots.

The suggested calculation algorithms have been tested for the concrete practical tasks.

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From the Uniform Approximation of a Solution of the PDE to the L^2-Approximation of the Gradient of the Solution

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We establish a new energy inequality for the difference of two semiconvex functions in a bounded open convex set D of R^n. This inequality is applied to the L^2-approximation problem of the gradient of the unknown solution of the nonlinear elliptic partial differential equation provided that the latter solution is a semiconvex function in D. Our method of approximation essentially uses the convex envelope $\Gamma(v)$ of a given bounded continuous function v in a domain D. The construction of the convex envelope for piecewise linear functions reduces to one of the core problems in computational geometry: the construction of the convex hull of a finite set of points of R^n.

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A Linear Generalized Central Spline Algorithm of Computerized Tomography

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The main problem of computerized tomography is contained in the reconstruction of function by its integral over hyperplanes. We study the problem of approximate inversion of Radon transform $\mathfrak{R}$ in the n-dimensional Euclidean space $\mathbb{R}^n$, i.e. the problem of construction of an algorithm for the approximate solution of the equation $\mathfrak{R}u = f$, where $\mathfrak{R}$ acts from the weighted space $H_\nu = L_2(\Omega^n, W_\nu^{-1})$ in the weighted space $K_\nu = L_2(Z, w_\nu^{-1})$. Here $\Omega^n = \{x \in \mathbb{R}^n : |x| \le 1\}$, $W_\nu(x) = (1-|x|^2)^{\nu-n/2}$, $w_\nu(s) = (1-s^2)^{\nu-1/2},\ s \in [-1,1]$ and Z is the cylinder $Z = \{x \in \mathbb{R}^n : |x| = 1\} \times \mathbb{R}$. Let $\mathfrak{R}^*$ be the adjoint for the operator $\mathfrak{R}$. The above equation is ill-posed and we transfer this one in the Fréchet space $D((\mathfrak{R}^*\mathfrak{R})^{-\infty})$, where it stands well-posed. Here $(\mathfrak{R}^*\mathfrak{R})^{-\infty}u = \{u, (\mathfrak{R}^*\mathfrak{R})^{-1}u, \cdots\}$ and the space $D((\mathfrak{R}^*\mathfrak{R})^{-\infty})$ is isomorphic to a subspace M of the Fréchet-Hilbert space $H_\nu^{\mathbb{N}}$ due to the mapping $D((\mathfrak{R}^*\mathfrak{R})^{-\infty}) \ni x \to \mathrm{Orb}((\mathfrak{R}^*\mathfrak{R})^{-1}, x) = (\mathfrak{R}^*\mathfrak{R})^{-\infty}x \in M \subset H^{\mathbb{N}}$. If $(\mathfrak{R}^*\mathfrak{R})_\infty u = \{(\mathfrak{R}^*\mathfrak{R})^{-1}u, u, (\mathfrak{R}^*\mathfrak{R})u, \cdots\}$, the transfered equation becomes the form $(\mathfrak{R}^*\mathfrak{R})_\infty u = f$ and has an unique and stable solution. We introduce the notations of generalized spline and generalized central algorithms and also of the solutions operator. The following theorem is proved.

Theorem. *Let $\{v_{rlk}^\nu, u_{rlk}^\nu, \sigma_{rl}\},\ l \le r,\ 1 \le k \le N(n,l)$, where $N(n,l) = (2l+n-2)!(n+l-3)!/(l!(n-2)!)$, is a singular system for the Radon transform $\mathfrak{R}$, which acts from $H_\nu,\ \nu > n/2-1$, in the space K_ν. Then the algorithm*

$$\varphi^s(I(f))(x) = \Sigma_{k=0}^m \Sigma_{l\le r}{}' \sigma_{rl} \Sigma_{k=1}^{N(n,l)} (f, u_{rlk}^\nu)_{L_2(Z, w_\nu^{-1})} v_{rlk}^\nu(x),\ x \in \Omega^n,$$

where Σ' means that the summability takes place only for even $m+l$, is the linear generalized spline and the generalized central for the solution operator $S = (\mathfrak{R}^*\mathfrak{R})_\infty^{-1}$ and nonadaptive information $I(f) = [(f, u_{001}^\nu), \cdots, (f, u_{mmN(n,m)}^\nu)]$. Moreover, these approximative solutions converges to the solution of equation $\mathfrak{R}u = f$ (in the sense of Moorie-Penrose) in the energetic space of the operator $(\mathfrak{R}^*\mathfrak{R})_\infty$, and also in the space $D((\mathfrak{R}^*\mathfrak{R})^{-\infty})$.

To Computational Modeling and Numerical Mechanics of Solids

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The approximate solution of some BVPs for spatial theory of elasticity cylindrical region, ordered by Committee on Science and Technology of FSU is considered (named "The Applied Program of Calculating of Spatial Structures", N0.80.14.09.20.). This package contains the manager program and modules. The full software was elaborated by T. Vashakmadze and his group. Here we should remark, that he developed new fast stable convergent modification of variotional-discrete methods of approximated solution of BVPs for quadrangular, band and quadrant regions.

Between effective methods of solution approximately BVPs as direct methods are variational and finite-difference methods. In [1], points 12, 17.2, T. Vashakmadze considered modification of variational methods selecting coordinate functions so that moments of unknown vector would satisfy a system of algebraic equations which has the structure by small perturbation of the finite-difference scheme, appropriate to a two-dim BVP. As different from pure variational methods this one gives possibility to study the properties of coordinate functions using some results of numerical analysis and investigated problem of stability of corresponding numerical processes. For example, if we used the first order differences with respect to Legendre polynomials, then the variational-discrete schemes for Laplace operator has a five-point structure, for biharmonic operator-13 piont one as pure finite-difference algorithms. This technology is essentially convenient if we consider unbounded domains [1, pp. 10-13, 30]. Today there are reliable algorithms for calculation of orthogonal polynomials, which have degree no more than 10000. We created algorithms and realized programs with R. Chikashua when degree of ultraspherical polynomials are 1.000.000 (million) with no less than 1.000 decimal points.

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On Solving the Dirichlet Generalized Problem for a Harmonic Function in the case of an Infinite Plane with Holes

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Let a domain D be the infinite plane $z = x + iy \equiv (x, y)$ with the holes B_i $(i = 1, 2, \cdots, m)$, which are bounded by closed piecewise smooth contours S_i $(S_i = \sum_{j=1}^{l} S_i^j;\ i = 1, 2, \cdots, m)$, respectively, having no multiple points. It is evident that the whole boundary of domain D will be $S = \bigcup_{i=1}^{m} S_i$. Moreover, we assume that parametric equations of the smooth curves S_i^j are given and $S_k \bigcap S_j = \emptyset$ for $k \neq j$. For domain D the following Dirichlet generalized problem is considered.

On the boundary S of the domain D a function $g(\tau)$ is given which is continuous everywhere, except a finite number of points $\tau_1, \tau_2, \ldots, \tau_n$ at which it has first kind break points. It is required to find a function $u(z) \equiv u(x, y) \in C^2(D) \bigcap C(\overline{D} \setminus \{\tau_1, \tau_2, \cdots, \tau_n\})$ satisfying the conditions

$$\Delta u(z) = 0, \qquad z \in D,$$

$$u(\tau) = g(\tau), \qquad \tau \in S, \quad \tau \neq \tau_k \ (k = 1, 2, \ldots, n),$$

$$u(z) = c + O\left(\frac{1}{|z|}\right) \qquad \quad for \quad |z| \to \infty,$$

where Δ is the Laplace operator and c is a real constant provided $|c| < \infty$.

An algorithm of approximate solution of the formulated problem is given, which consists of the following stages: 1) reduction of the Dirichlet generalized problem to an ordinary new (auxiliary) problem; 2) approximate solution of the new problem by the modified version of MFS (the method of fundamental solutions); 3) definition of the approximate solution of the posed generalized problem by the solution of the new problem. Examples of application of the proposed algorithm and the results of numerical experiments are given.

Programming Means for Modeling of Natural and Formal Languages

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We have developed programming means, by which it is possible to modeling of natural or formal language. In case of Chomsky approach, natural or formal language is considered as a subset of universal set of objects. An object from universal set of objects is taken and then we should state whether the object is element of the subset or not. For this we should construct a formal grammar of searched subset, define an inference for this grammar and if by inference we can obtain the object, then the object is an element of the subset. In our case, formal grammar is an auxiliary mean, by which we construct logical expression. It is named by program file. We pass program file with given object to programming means. If logical expression is true, then the object is element of the subset. In the process of calculation, to the object is added some features and their values, which give object description. Definition of logical expression and rules of its construction will be given on presentation.

Such approach has some advantages, as compared to approach of Chomsky, which will be considered at presentation.

Morphological Analysis of Georgian Words Using Formal Grammar

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In order to make morphological analysis of Georgian words, it is necessary to create formal grammar based on the rules of Georgian language grammar. We have created the above mentioned grammar and it can solve the task. In particular, its use can decompose given words by morpheme. It implies all parts of speech.

Definitions of used rules in the created formal grammar may still be rules, but finally definition of rule is totality of terminal symbols, which in particular occasion is a sequence of pre-verbs, person sign, vowel prefix or root of verbs, etc.

Software is also developed, using "Visual Studio C Sharp", which can decompose given words by morpheme and explain each of them. The program uses translated version of formal grammar on "C Sharp" language.

Given formal grammar is created using GOLD Parser Builder, where it is possible to create context-free grammars, it also gives possibility to make their translated versions for a desired programming language.

The Fractal Structures and Their Properties

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A fractal is a mathematical object, a geometric pattern that is repeated at ever smaller scales to produce (self similar) irregular shapes and surfaces that cannot be represented by classical (Euclidian) geometry. Fractals are used especially in computer modeling of irregular patterns and structures found in nature. The Sierpinski triangle is the classic example of an orbital fractal. Mandelbrot and Sierpinski are two mathematicians who made important contributions in the field of fractals. The Sierpinski triangle has all the properties of a fractal: A fractal is a geometric shape which is selfsimilar and which has a fractal dimension.

On Multi-Criteria Algorithms and the Analysis for Some Discrete Optimization Problems

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A person in his activity constantly comes up against situations in which he has to make a choice. It is usually assumed, that such a possible decision is selected, which most fully satisfies to desires, interests or the purposes of this decision-maker. The desire of DM to achieve a particular goal quite often can be expressed with mathematical terms in the form of maximizing (or minimizing) of some numerical function defined on the set X. However in more difficult situations it is necessary to deal not with one but with several functions.The problem of one-parametrical optimization always has the best decision, which gives the maximum possible value of optimized function under the given restrictions. Probably, the maximum is reached on several various configurations of system. In any case, the best value can be found by full sorting of all options.

In case of optimization on several criteria, considered in the work, a situation is different: if parameters are considered equivalent, then there can be no mutually comparable decisions, such that one is better on one parameter, the second – on another.

Pareto optimality - state of the system under which, the value of each particular criterion describing the state of the system cannot be improved without deterioration of other elements.

The situation when reached Pareto efficiency - a situation in which all benefits from an exchange are exhausted. Pareto efficiency is one of the central concepts of modern science.

This paper discusses issues related to the selection decisions in the presence of a number of criteria for some discrete tasks.

The Technological Alphabet of the Georgian Language – Aims, Methods and Results

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The elaboration of the technological alphabet of the Georgian language aims at a mechanical foundation of the Georgian language. This means to build up a talkie Georgian computer system, i.e. it means to provide full and free spoken human-machine interaction by means of the Georgian language.

The methodology of our research follows Leibniz's epistemic view about mathematical foundation of the lingually given and logically developable knowledge. In our case, this means: Firstly, to found the natural Georgian language as a formally developable intellectual mathematical theory. Here we are mainly based on the methods elaborated within Sh. Pkhakadze's Notation Theory and his semantic program of the foundation of mathematics, and, also, within the researches pursued for the elaboration of the Logical Grammar of the Georgian language by the leadership of K. Pkhakadze; and after, to software a computer with the already founded formally developable intellectual mathematical theory of the Georgian language system.

During the talk, we will shortly overview above-mentioned methods of the elaboration of the technological alphabet of the Georgian language. In particular, we will present the fundamental questions of the historically first mathematical theory of the Georgian language; also, we will present those systems, which are already built up in our group with the purposes of creation of the Georgian technological alphabet. They are as follows: Georgian Grammar Checker (http://translate.dmind.net/Login.aspx), Georgian Logical Task Solver-Checker, Georgian-Mathematical and Georgian-English-German Translator, Georgian Speech Recognizer, and Georgian Text Reader (http://www.geotextreader.com).

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One Algorithmic Process for Deciding Validity of some Unranked τSR Logic

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An algorithmic process to establish the validity of the formulae with some property of the Unranked τSR logic is constructed. They are directed to the mechanical theorem proving.

If the associated formula of the given formula of the Unranked τSR logic includes only meta-variables from the formulas $A, B, A_1, B_1 \dots$ and the symbols $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$ we call it the propositional meta-formula associated to the given formula.

The propositional meta-formula with is considered as a formula of the propositional logic and represents tautology is called propositional tautology form.

The following theorems are immediate consequence of the algorithmic process described by us.

Theorem 1. Existence of tautology form associated with a formula A of Unranked τSR logic is sufficient for validity of the formula A.

Theorem 2. A formula A of Unranked τSR logic is an instance of a tautology of the Unranked propositional logic if there exists a propositional tautology form associated with the formula.

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On the Problem of Optimal Stopping with Incomplete Data

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We consider the Kalman-Bucy model of partially observable random sequence (θ_n, ξ_n), $n = 0, 1, ...$:

$$\theta_{n+1} = a_0(n) + a_1(n)\theta_n + b(n)\eta_1(n+1), \tag{1}$$

$$\xi_{n+1} = \theta_{n+1} + \epsilon\eta_2(n+1), \tag{2}$$

where $\eta_1(n), \eta_2(n)$ are independent standard normal random variables, $n = 0, 1, ...;$ $\epsilon > 0$ [1]. Consider the linear gain function $g(n, x) = f_1(n) + f_2(n)x$ and define payoffs [2]:

$$S^0 = sup_{\tau\in\Re^\theta} Eg(\tau, \theta_\tau), \quad S^\epsilon = sup_{\tau\in\Re^\xi} Eg(\tau, \theta_\tau). \tag{3}$$

The following notations are introduced: $m_n = E(\theta|\Im_n^\xi)$, $\gamma_n = E[(\theta_n - m_n)^2]$.

Theorem 1. Let partially observable sequence (θ_n, ξ_n), $n = 0, 1, ...,$ be given by (1), (2). Then

$$S^\epsilon = sup_{\tau\in\Re^\xi} Eg(\tau, m_\tau) = sup_{\tau\in\Re^{\widetilde{\theta}}} Eg(\tau, \widetilde{\theta}_\tau),$$

where

$$\widetilde{\theta}_{n+1} = a_0(n) + a_1(n)\widetilde{\theta}_n + \frac{a_1(n)b^2(n) + a_1^3(n)\gamma_n}{\sqrt{a_1^2(n)b^2(n) + a_1^4(n)\gamma_n + \epsilon^2}}\eta_1(n+1).$$

Theorem 2. Let the payoffs S^0 and S^ϵ be defined by (3). Then the following estimate is valid:

$$0 \le S^0 - S^\epsilon \le c\epsilon,$$

where $c > 0$ is constant.

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Some Limit Properties of Maximal Likelihood Estimation in a Hilbert Space

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Let H be a separable, real Hilbert space and let $X_1, X_2, ..., X_n$ be iid data in H. $\mu = \mu(\theta)$ is the distribution in H of X_1. Here θ is an unknown parameter, $\theta \in \Theta$, Θ is a bounded set of a separable Banach space. Suppose the probability measure μ has the logarithmic derivative - $\lambda_\theta(x, h)$ for $h \in H_0$, where H_0 is a dense subspace of H. Then we may define maximal likelihood estimation $\widehat{\theta}_n$ of θ as a solution of equation (see [1])

$$\sum_{k=1}^{n} \lambda_\theta(X_k, h) = 0.$$

Our aim is to study limit properties of MLE estimator. In particular we proved asymptotically normality of this one.

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Clark Representation and Independence and Dependence of Brownian Functionals

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Consider the Brownian motion $B = \{(B_t),\ t \in [0,T]\}$ and the natural filtration $(\Im_t^B)$, $t \in [0,T]$, generated by B. Let X_T and Y_T are the Brownian functionals, i.e. the random variables that are $(\Im_T^B)$-measurable.

Theorem 1. The Brownian functionals X_T and Y_T are independent if and only if for all $(x,y) \in R^2$:

$$cov(I_{\{X_T \le x\}}, I_{\{Y_T \le y\}}) = \int_0^T E[u_s(x)u_s(y)]ds = 0,$$

where $u_s(x)$ and $u_s(y)$ are integrand in Clark integral representations of $I_{\{\omega : X_T(\omega) \le x\}}$ and $I_{\{\omega : Y_T(\omega) \le y\}}$.

The problem is: how to find these integrands. The directly using of Clark-Ocone formula it is impossible. Therefore we represent here

Theorem 2. Suppose that $g_t = E[F_T|\Im_t^B]$, $(t \in [0,T])$ are Malliavin differentiable $(g_t(\cdot) \in D_{1,2})$. Then

$$g_T = F_T = E[F_T] + \int_0^T v_s dB_s,$$

where

$$v_s := lim_{t \to T} E[D_s g_t | \Im_s^B] \quad in\ \ the\ \ L_2([0,T] \times \Omega)$$

(here D_s - is Malliavin derivative).

Also we study the problem of characterization of dependence of random variables X_T and Y_T.

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The Results of Investigation of Statistical Hypotheses Testing Methods

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Investigation focuses on the consideration of basic approaches to hypotheses testing, which are Fisher, Jeffreys, Neyman, Berger approaches and a new one proposed by the author of this work and called the constrained Bayesian method (CBM). Wald and Berger sequential tests and the test based on CBM are presented also. The positive and negative aspects of these approaches are considered on the basis of computed examples. Namely, it is shown that CBM has all positive characteristics of the above-listed methods. It is a data-dependent measure like Fisher's test for making a decision, uses a posteriori probabilities like the Jeffreys test and computes error probabilities Type I and Type II like the Neyman-Pearson's approach does. Combination of these properties assigns new properties to the decision regions of the offered method. In CBM the observation space contains regions for making the decision and regions for no-making the decision. The regions for no-making the decision are separated into the regions of impossibility of making a decision and the regions of impossibility of making a unique decision. These properties bring the statistical hypotheses testing rule in CBM much closer to the everyday decision-making rule when, at shortage of necessary information, the acceptance of one of made suppositions is not compulsory. Computed practical examples clearly demonstrate high quality and reliability of CBM. In critical situations, when other tests give opposite decisions, it gives the most logical decision. Moreover, for any information on the basis of which the decision is made, the set of error probabilities is defined for which the decision with given reliability is possible.

On Exponential Integrability of Subgaussian Series

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A real valued random variable ξ is *Subgaussian* [1, 2] if there exists $a \geq 0$ such that

$$\mathbf{E}\, e^{t\xi} \leq e^{\frac{1}{2}t^2a^2}, \quad \forall t \in \mathbf{R}\,. \tag{1}$$

If ξ is Subgaussian and (1) holds for some $a \geq 0$, then necessarily $\mathbf{E}\xi = 0$ and $(\mathbf{E}\xi^2)^{\frac{1}{2}} \leq a$. Hence, to a Subgaussian random variable ξ it can be associated with a quantity $\tau(\xi)$ defined by the equality:

$$\tau(\xi) := \inf\{a \geq 0 : \mathbf{E}\, e^{t\xi} \leq e^{\frac{1}{2}t^2a^2}, \quad \forall t \in \mathbf{R}\,\},$$

which is called *the Gaussian deviation* [1] or *the Gaussian standard* [2] of ξ.

Theorem. *Let $\xi_n, n = 1, 2, \ldots$ be Subgaussian random variables. If*

$$\sum_n \tau(\xi_n) < \infty\,,$$

then

$$\sum_n |\xi_n| < \infty \quad a.s. \tag{2}$$

and

$$for\ some\ \varepsilon > 0, \quad \mathbf{E}\, e^{\varepsilon\left(\sum_n |\xi_n|\right)^2} < \infty. \tag{3}$$

In this theorem the condition (2) may not imply even that $\mathbf{E}\left(\sum_n |\xi_n|\right) < \infty$ [3], while if $\xi_n, n = 1, 2, \ldots$ are Gaussian, then condition (2) alone implies (3) (Vakhania N.N.).

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New Approach in the Development of Stochastic Differential Equations in a Banach Space

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In developing the stochastic differential equations in a Banach space, we introduce the corresponding generalized stochastic differential equation. It is possible to solve this equation by traditional methods and we receive the generalized stochastic process as a solution. If there exists the Banach space valued random process corresponding to this generalized random process, it will be solution of the main stochastic differential equation. Using this approach we consider the question of the existence and uniqueness of the solution and receive the solutions of the linear stochastic differential equations in a Banach space.

On the Properties of the Dynamic Value Functions in the Problem of Optimal Investing in Incomplete Markets

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We consider an incomplete financial market model, where the dynamics of asset prices are described by a continuous R^d-valued semimartingale S satisfying the structure condition

$$S_t = M_t + \int_0^t d\langle M\rangle_s \lambda_s, \qquad \int_0^t \lambda_s' d\langle M\rangle_s \lambda_s < \infty, \qquad t \in [0,T]$$

where M is a continuous local martingale and λ is a predictable process.

Let $U = (U(x) : x \in R)$ be a utility function defined on all real line and denote by $X_t^{x,\pi}$ the wealth process $x + \int_0^t \pi_u dS_u$ determined by a self-financing trading strategy π and initial capital x.

We consider the utility maximization problem, i.e., the problem to find a trading strategy $(\pi_t, t \in [0,T])$ such that the expected utility of terminal wealth $X_T^{x,\pi}$ becomes maximal.

Our goal is to study properties of the dynamical value function of the problem defined as

$$V(t,x) = \mathrm{esssup}_{\pi\in\Pi_x} E(U(x + \int_t^T \pi_u dS_u)/\mathcal{F}_t), \tag{1}$$

where Π_x is a class of admissible strategies.

We call a family of processes $Y(t,x), x \in R$ a regular family of semimartingales, if a) $Y(t,x)$ is two-times continuously differentiable at x for all $t \in [0,T]$ P-a.s. and b) for any $x \in R$ the processes $Y(t,x)$ and $Y_x(t,x)$ are special semimartingales and their bounded variation parts are absolutely continuous with respect to the increasing process $\langle M\rangle$.

Under some regularity assumptions on the utility function, we show that $V(t,x)$ is a regular family of semimartingales. We use these properties to show that the value function satisfies a certain backward stochastic partial differential equation, which enables us to characterize the optimal wealth process.

On Functionals From Gasser–Müller Estimators

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Suppose we have n measurements taken at the points

$$0 \le t_1 \le t_2 \le \cdots \le t_n \le 1.$$

In Gasser–Müller scheme the model is the following

$$X(t_i) = a(t_i) + \varepsilon_i, \quad i = 1, 2, \dots, n,$$

$$\varepsilon_i \ \text{ i.i.d.with } \ E\varepsilon_i = 0, \ \ Var(\varepsilon_i) = \sigma^2 < \infty.$$

The estimate of unknown regression function $a(\cdot)$ proposed in [1] (Gasser–Müller estimator) is defined as follows:

$$\widehat{a}_n(t) = \frac{1}{h_n} \sum_{j=1}^{n} \int_{s_{j-1} K\left(\frac{t-u}{h_n}\right) du \cdot X(t_j)}^{s_j},$$

where K is a non-negative kernel function, h_n is a sequence of positive bandwidths such that $h_n \to 0$, $nh_n \to \infty$ as $n \to \infty$, $0 = s_0 \leq s_1 \leq \ldots \leq s_n = 1$, $t_j \leq s_j \leq t_{j+1}$, $j = 1, 2, \ldots, n-1$, $\max |s_j - s_{j-1}| = O\left(\frac{1}{n}\right)$.

Let us have the integral functional

$$I(a) = \int_{-\infty}^{\infty} \Phi\left(t, a(t), a'(t), \ldots, a^{(k)}(t)\right) dt,$$

where $a^{(j)}(t)$ is the jth derivative of function $a(\cdot)$. We consider "plug-in estimator" of the integral functional $I(a)$:

$$I(\widehat{a}_n) = \int_{-\infty}^{\infty} \Phi\left(t, \widehat{a}(t), \widehat{a}'(t), \ldots, \widehat{a}^{(k)}(t)\right) dt.$$

In this paper we investigate rate of convergence $I(\widehat{a}_n)$ to $I(a)$ and proved some asymptotic properties of limit distribution of $I(\widehat{a}_n)$.

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On a Certain Problem of H. Shi in Solovay's Model

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We show that in Solovay's model an arbitrary non-trivial closed ball in an infinite - dimensional non-separable Banach space of all real valued sequences ℓ^∞ is an infinite-dimensionally Haar null set. This answers positively to the Problem 8 stated in [Shi H., *Measure-Theoretic Notions of Prevalence,* Ph.D.Dissertation (under Brian S.Thomson), Simon Fraser University, October 1997, ix+165] for non-separable Banach space ℓ^∞.

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On Statistical Estimation in Infinite Dimensional Spaces

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We investigate general case of parameter and nonparametric statistical estimation in infinite dimensional spaces (see [1-2]). Hypotheses testing procedure are considered.

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Professor Levan Magnaradze – 100

President of the Georgian Mathematical Union over the years, Professor L. Magnaradze was born on June 22, 1913, in New Senaki in Georgia. In 1934, he graduated from the Tbilisi State University (TSU) as a mathematician. His career was closely connected with the Tbilisi State University, where he passed a way from assistantship to professorship. Besides, he worked at the Institute of Geophysics and A. Razmadze Institute of Mathematics, was the Dean of the faculty of Mechanics and Mathematics of TSU, head of departments at I. Vekua Institute of Applied Mathematics of TSU. L. Magnaradze's professional scientific interests were significantly influenced by works of N. Muskhelishvili and V. Kupradze. L. Magnaradze's professional scientific scope was wide. He studied the vibration problem for the elastic half-space, the basic problems of plane elasticity for domains bounded by contours with angular points, problems of wing theory; the systems of linear differential and integro - differential equations, regular solutions of some nonstationary equations in a Banach space, explored the Riemann-Hilbert boundary value problem, constructed an effective solution of the Cauchy problem for the hyperbolic type linear partial differential equation, obtained asymptotic expressions of the solutions of such equations for large values of the parameter. He generalized Abel and Plemeli-Privalov theorems for the Laplace double transformation. He constructed a general solution of a linear differential equation of nonstationary and inhomogeneous fields by means of harmonic and analytic functions. He generalized Zygmund's inequality for generalized conjugated functions, which can be represented by the Stieltjes singular integrals, whose principal value is understood in a generalized sense.

L. Magnaradze's work "On an integral equation of airplane wings theory" brought him great popularity. In 1959 G. Miles, based on this paper, in his monograph "Potential theory of nonstationary supersonic flows" titled one of the chapters as "Method of Magnaradze". In 1939 L. Magnaradze defended a candidate's thesis on the theme "Some problem of mathematical physics for contours with angular points", and in 1986 defended a doctoral thesis on the theme "Singular Integrals, special functional spaces, boundary problems for analytical functions and their application for boundary value problems of inhomogeneous physical fields".

L. Magnaradze was known as a highly educated person with ranging erudition. He knew well the achievements and problems of modern mathematics. Great is his professional contribution to publishing scientific works at I.Vekua Institute of Applied Mathematics.

L. Magnaradze passed away on February 6, 2002.

He was a remarkable scientist, a very original and creative personality, public figure with versatile activities and a teacher, greatly loved and respected by whole generations of mathematicians in Georgia.

Jondo Sharikadze, Tamaz Tadumadze, Tsitsino Gabeskiria

Professor Gvanji Mania – 95

Professor Gvanji Mania was a well-known Georgian mathematician and the founder of Georgian school of Probability Theory and Mathematical Statistics. He was born on May 29, 1918 in the village of Etsery to a family of a teacher. In 1935 he became a student of the faculty of Physics and Mathematics of the Tbilisi State University TSU. In 1940-1945 he worked as an assistant at the Pedagogical Institute of Zugdidi. In 1943-1946 he worked as an assistant at Railway Engineering Institute of Tbilisi. From 1945 to 1946 he was inspector of higher education school of the Ministry of Education of Georgia.

During 1946-1949 Mania was a postgraduate student of Moscow Potiomkin State Pedagogical Institute where he defended his thesis and received a candidate's degree. His scientific supervisor was a famous mathematician, professor N. Smirnov and official opponents were academician B. Gnedenko and professor Lyapunov. After the candidate’s speech academician Gnedenko said: “Glivenko, Kolmogorov and Smirnov permanently pointed out the shortcomings of their theorems and it should be noted that G. Mania worked exactly on these interesting and important problems. The results obtained in the thesis have the first-class value”. Professor Lyapunov said: “It is obvious that these results will be included into the “gold fund” of mathematical statistics”. No less successful was Mania’s doctoral dissertation in 1963. His official opponents were academicians B. Khvedelidze, Y. Prokhorov and S. Syradjinov. In 1964 he was elected professor at Tbilisi State University.

In 1949-1950 Professor Mania worked at Nikoloz Baratashvili Gori Pedagogical Institute as an assistant professor. In 1950-1953 he was assistant professor at Georgian Polytechnic Institute and in 1950-1953 senior researcher at Razmadze Tbilisi Mathematical Institute.

Professor Mania made a great contribution to the creation of new research centres in Georgia such as Computational Centre and Institute of Applied Mathematics. In 1956-1964, Mania worked as a deputy director of Muskhelishvili Computational Centre of Georgian Academy of Sciences and in 1966-1972 at Vekua Institute of Applied Mathematics of TSU. In 1968 Mania founded the chair of Probability Theory and Mathematical Statistics at Tbilisi State University. He led this chair until his decease. Between 1973 and 1983 he was head of the department of Probability Theory and

Mathematical Statistics at the Institute of Law and Economics and in 1983-1985 he was in charge of the same department at the Razmadze Mathematical Institute of the Georgian Academy of Sciences.

Professor Mania worked productively as a tutor and a researcher. He is the author of many scientific papers, textbooks and monographs. He was a member of various boards and societies, including that of the Bernoulli Society of International Statistical Institute since 1965 and member of the American Mathematical Society. He was also member of the board of International journal *Statistics*. In 1969, he was delegated to 37th session of International Statistical Institute in London and in 1970 he participated in International congress of mathematicians in Nice. He was awarded two state orders and Javakhishvili medal.

Professor Mania has founded limiting distributions of several important statistics and tabulated these distributions. The proofs of these statements are based on Abel and Tauber type theorems proved by Mania, where errors made in Feller's similar theorems were eliminated. These results became important for many scientists who used them in their studies. These statistics are known as Mania’s Statistics in scientific literature. In 1961 he represented homogeneity criteria for two normal independent samples. Later, Mania investigated the properties of nonparametric estimations of normal distribution density, which shows that it is impossible to generalize famous theorems of Boid and Still. In 1974 the monograph “Statistical Estimations of Probability Distributions” was published, which was highly appreciated by colleagues.

Professor Mania founded a Georgian scientific school of Probability Theory and Mathematical Statistics which covered a wide range of sub-disciplines. He made an invaluable contribution to the education of a new generation of Georgian mathematicians.

G. Mania died at the age of 67, on March 16th, 1985.

Elizbar Nadaraya, Omar Purtukhia

CONTENTS

Author's Index