XXXVIII International Enlarged Sessions of the Seminar of Ilia Vekua Institute of Applied Mathematics of Ivane Javakhisvili Tbilisi State University

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The present book of abstracts contains abstracts of talks given at XXXVIII Enlarged Sessions (April 22-24, 2024) of the Seminar of I. Vekua Institute of Applied Mathematics of I. Javakhishvili Tbilisi State University.

Each Section (there are 10 ones) is presented as separate Chapter of the book. The responsibility for the contents of each Chapter lies with leaders together with speakers.

SECTION OF MATHEMATICAL LOGIC AND FOUNDATIONS

Chairs: Alexander Kharazishvili, Roland Omanadze
Co-chair: Archil Kipiani

ON TOPOLOGICAL AND MEASURE-THEORETICAL PROPERTIES OF SOME POINT SETS

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As is known, a Luzin space (I) is an uncountable topological T_1-space without isolated points in which every nowhere dense subset is at most countable. According to another definition, a Luzin space (II) is a topological space X, such that there exists no nonzero σ-finite Borel measures on X vanishing at all singletons in X. Let us remark several properties of Luzin spaces:

a) It is consistent with ZFC theory that every metrizable Luzin set (I) is a Luzin space (II); the converse assertion is not true in general;
b) Every subspace of a Luzin space (II) is a Luzin space (II);
c) Every topological product of finitely many Luzin spaces (II) is a Luzin space (II);
d) There is a topological product of countable many Luzin spaces (II), which is not a Luzin space (II);
e) If $\{X_i\}_{i\in I}$ is a family of Luzin spaces (II) such that $\mathrm{card}(I) = \kappa$, where κ is not a real-valued measurable cardinal, then the topological sum of $\{X_i\}_{i\in I}$ is also a Luzini space (II).

In the present talk we will discuss some connections between Luzin spaces (I, II) and other classical point sets such as: Bernstein set, weakly Luzin set, and Cantorval sets.

ON K-SEMI-REGULAR POLYHEDRA

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It is known, that semi-regular polyhedra play an important role in various questions of combinatorial geometry (see e.g. [1], [2]).

Let P be an m-dimensional polyhedron in R^m ($m \geq 2$) space and let k be a fixed natural number such that $1 \leq k \leq m-1$. We say that P is a k-semi-regular polyhedron in R^m space if all k-dimensional faces of P are pairwise congruent.

In the present talk, we discuss k-semi-regular polyhedra and their relationships to k-isosceles simplexes.

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SOME NOTES ON $Q_{1,N}$-REDUCIBILITY

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Tennenbaum (see [2, p.159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is Q-reducible to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \Leftrightarrow W_{f(x)} \subseteq B.$$

We say in this case that $A \leq_Q B$ via f. If $A \leq_Q B$ via a computable function f such that for all $x, y, x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset$ and $\bigcup_{x \epsilon \omega} W_{f(x)}$ is computable, then we say that A is $Q_{1,N}$ -reducible to B, and denote $A \leq_{Q_{1,N}} B$. The notion of $Q_{1,N}$ -reducibility was introduced by Bulitko in [1].

A set A is hemi r-maximal if there are an r-maximal set M and a nontrivial splitting M_0, M_1 of M such that $A = M_0$.

Our notations and terminology are standard and can be found in [2] and [3]. In this talk we will present the following results:

Theorem 1. Let A be a hemi r-maximal set and B be a noncomputable c.e. set such that $B \leq_{Q_{1,N}} A$. Then B is a hemi r-maximal set.

Theorem 2. Let a be a $Q_{1,N}$-degree of any hemi r-maximal set. Then for all c.e. sets $A, B \in a$, we have $A \equiv_1 B$.

Acknowledgments. This work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number: **STEM-22-1837,** Recursive Functions and Probabilistic Ontology Engineering].

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FINITELY GENERATED FREE AND PROJECTIVE MV(C)-ALGEBRAS

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MV-algebras are algebraic models of infinitely valued Lukasiewicz logic. MV(C)-algebras are proper subvariety of the variety **MV** of all MV-algebras which selected by the identity $2(x^2)=(2x)^2$, that is a special extension of Boolean algebras. The first order Lukasiewicz logic is complete with respect to all perfect chain MV(C)-algebras.

Finitely, generated free and projective MV-algebras are described in the variety **MV(C)** generated by perfect MV-algebras that in turn generated Chang algebra C.

ABOUT SOME FINITE SYSTEMS OF VECTORS

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A problem concerning finite systems of vectors in the Euclidean n-dimensional space R^n is considered and solved. Namely, it is shown that if we have m many radius-vectors in R^n, which satisfy the conditions

1. $|x_1| \le 1, |x_2| \le 1, \ldots, |x_m| \le 1$ and $0 \in conv\{x_1, \ldots, x_m\}$, then $|x_1 + x_2 + \cdots x_m| \le m-1.$

Analogously, if

2. $|x_1| = 1, |x_2| = 1, \ldots, |x_m| = 1$ and $0 \in conv\{x_1, \ldots, x_m\}$, then $|x_1 + x_2 + \cdots x_m| \le m-2.$

These estimates are precise in a certain sense.

PARTIAL MONO-UNARY ALGEBRAS AND CONTINUUM HYPOTHESES

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We consider some simply formulated statements about partial mono-unary algebras, which are associated with the Continuum Hypothesis and Generalized Continuum Hypotheses.

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ON THE STABILITY OF CERTAIN TYPES OF SETS IN COMMUTATIVE GROUPS WITH RESPECT TO SURJECTIVE HOMOMORPHISMS OF THESE GROUPS

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For nonzero invariant (quasi-invariant) σ-finite measures on uncountable commutative groups, the stability of absolutely negligible sets with respect to surjective homomorphisms and the algebraic sums is studied.

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BI-MODAL LOGICS OF MAPPINGS

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Structures that are described by modal languages are often found in applied and fundamental mathematics. The typical examples are directed graphs (finite-state machines) and topological spaces. The study of such structures and corresponding modal systems is of current importance for the last three decades and can be applied in such areas as verification and control of programs, databases, geo-information systems, etc. Maps that preserve logical validity between the mentioned structures play the crucial role. In its turn, these maps produce more complex, multi-structural sets where not only the domain and codomain are fixed, but also the map between them. The aim of our research is to study these multi-structural sets using languages containing two or more modalities. The corresponding logical structures have a multi-modal type. The relevance of studying such multi-modal systems is confirmed by several fundamental monographs and a number of scientific articles published over the last decade. Our task is to study multi-modal logical systems that are generated by modal maps, and to establish their

basic logical properties, such as: axiomatization, decidability, the finite model property, computational complexity.

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ON MAZURKIEWICZ TYPE SETS FROM THE MEASURE-THEORETICAL AND BAIRE PROPERTY VIEWPOINT

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In the talk, some subsets of the Euclidean plane $\mathbf{R}^2$ that are Mazurkiewicz type sets with respect to the family of all straight lines lying in $\mathbf{R}^2$ are considered. Analogously, subsets of $\mathbf{R}^2$ that are Mazurkiewicz type sets with respect to the family of all circumferences lying in $\mathbf{R}^2$ are studied. Several statements highlighting the above-mentioned sets from the viewpoint of the Baire property and various measures on $\mathbf{R}^2$ are presented.

SECTION OF APPLIED LOGICS AND PROGRAMMING

Chair: Matthias Baaz (Austria)
Co-chair: Jemal Antidze, Besik Dundua, Mikheil Rukhaia

DATA REPRESENTATION BY INDUCTIVE LOGIC PROGRAMMING

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Inductive logic programming is a combination of logic programming and machine learning. It attempts to provide a formal framework and also practical algorithms for inductively learning relational descriptions from examples and background knowledge [1]. Machine learning (ML) automates induction. It induces a hypothesis that generalizes training examples and Inductive logic programming (ILP) is a form of Machine learning [2].

The main difference is that whereas most forms of ML use tables to represent data, ILP uses logic programs. Moreover, whereas most forms of ML learn functions, ILP learns relations. The main building block of a logic program is an atom and the atom is of the form $p(x_1, ..., x_n)$ [2].

In this talk, we will speak about the difference between how Machine Learning and Inductive logic programming represent data with examples and scenarios.

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PROBABILISTIC PRIMITIVES IN LOGIC

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During the initial stages of the probability theory development, the fundamental concepts of probability theory, such as probability, conditional probability, independence, and others, lacked standardization.

Moreover, the interest in probability logic started massively growing because of the development of many fields of application for the representation of and reasoning about uncertain knowledge (in economics, artificial intelligence, computer science, philosophy, etc.) resulting in numerous publications.

Two types of probabilistic logic were used which are Probabilistic Operators and Probabilistic Quantifiers.

In the realm of compositional semantics, probability operators pose unique and compelling challenges to straightforward analysis, which can be formulated as $P \geq_s \alpha$ [1].

While, the Probabilistic Quantifiers. Instead of classical universal and existential quantifiers, Keisler [2] introduced probability quantifiers, exemplified by expressions such as $Px > r$ [3][4].

In this talk, we mention the importance of these probabilistic logics while demonstrating a few examples with their formulas.

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PATTERN CALCULUS WITH FUZZY SIMILARITY MATCHING

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Pattern calculus extend the λ-calculus with pattern matching capabilities. Instead of abstracting from a variable, they permit abstractions from a pattern: a λ-term which specifies the form of the argument. The more flexible the patterns are, the more powerful the calculus is. Patterns are the most expressive ones: They can be instantiated, generated, and reduced. Pattern calculus is expressive, but there is also a price to pay for that: confluence is lost and various restrictions have to be imposed to recover it.

Pattern calculus is an expressive formalism that supports exact computations. However, many problems require computations with some approximation. Several approaches use similarity relations to express computation with approximation. In this talk we propose fuzzy extension of the pattern calculus, where term reduction is parameterized by fuzzy similarity matching, which means a term (\P.M)N reduces to Mσ by similarity degree d if Pσ is similar to N with degree d. We discuss sufficient conditions that guarantee confluence of the fuzzy pattern calculus.

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ABOUT VERIFICATION TECHNIQUES OF CYBER-PHYSICAL SYSTEMS

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Cyber-Physical Systems, shortly CPS, are networks of controllers that interact or control physical environments. Some examples of CPS are cars, aircrafts, railway systems, smart traffic systems, and the like. Such systems are becoming very complex and hard to get right, and are very expensive to develop and certify. It is very important, that CPS behaves correctly and securely, and only formal mathematical methods can provide strong guarantees about system correctness and security. These features must be established as early as possible in the CPS development process, it should be part of the design. Implementing a system with a flawed design can have catastrophic consequences. A recent example of this is the Boeing 737 max design flaw, that caused several aircraft crashes.

The mathematical modeling and analysis of modern CPS is very challenging for the following reasons:

- the need for advanced control programs combined with physical environments
- CPS are real-time systems whose controllers have to interact with each other and their environments in a timely manner
- distributed nature of CPS leads to combinatorial explosion in the number of different system behaviors that must be analyzed
- the need to deal with network delays, imprecise local clocks, etc.

It is obvious that an expressive mathematical formalism is needed to model modern CPS with their advanced control programs. A candidate for such formalism can be rewriting logic – a simple, general and expressive logic for distributed systems, whose modeling and analysis is supported by the Maude language and tool. In this talk we will speak about different approaches to use the Maude programming language and the Lingua Franca framework for verification of CPS. We underline some difficulties in this process and techniques to overcome them.

THE IMPACT OF ARTIFICIAL INTELLIGENCE IN THE EDUCATIONAL PROCESS

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In the modern world, the interest in artificial intelligence is increasing more and more, every day we see impressive examples of the use AI in various fields, including in the field of education. Artificial intelligence can transform education system by improving teaching and learning outcomes but, as any new technology, there are also risks associated with its use. In

this paper we talk about AI's benefits and risks in education, including personalized learning, improved assessment, reduced planning time for teachers, and the risk of cheating. While AI has specific risks, it's using in educational process is very important.

Acknowledgements. The work was supported by the Georgian National University SEU.

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FIRST-ORDER UNRANKED PROBABILISTIC LOGIC

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Probability theory deals with the challenges posed by uncertainty, while logic is more used for reasoning with perfect knowledge. First-order combines capability of probability and logic. It gives expressive and flexible platform to model and reason problems coming from with Artificial Intelligence (AI). Unranked First-order logic is variant of First-order logic with function symbols having flexible arity. Such an extension brings flexibility and expressiveness in the language to model and reason with unstructured data. In this talk we propose probabilistic extension of unranked First-order logic. In particular, we discuss syntax, semantic and inference mechanism of the extended formalism – probabilistic unranked logic.

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SECTION OF ALGEBRA, GEOMETRY AND NUMBER THEORY

Chairs: Mikhail Amaghlobeli, George Khimshiashvili, Teimuraz Vepkhvadze, Malkhaz Bakuradze
Co-chair: Ketevan Shavgulide

VARIETIES OF EXPONENTIAL R-GROUPS

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The notion of an exponential R-group, where R is an arbitrary associative ring with unity, was introduced by R. Lyndon [1]. A. Myasnikov and V. Remeslennikov [2] retired the notion of an R-group by introducing an additional axiom. In particular, the new concept of an exponential MR-group (R-ring) is a direct generalization of the concept of an R-module to the case of noncommutative groups. In this report we introduce the notion of a variety of exponential MR-groups and tensor completion of groups in varieties. We study relationships between free groups of a given variety under different of scalars and describe varieties of Abelian MR-groups. Moreover, in the category of MR-groups, we consider several analogs of n-class nilpotent groups. We get that the completion of a 2-class nilpotent group is a 2-class nilpotent [3].

Acknowledgements. The work is supported by the Shota Rustaveli National Science Foundation (SRNSF grant # FR 21-4713).

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A NOTE ON APPLICATIONS AND EXTENSIONS OF R-FUNCTIONS

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R-functions, named after V.L. Rvachev [1], translate Boolean operations into geometric domains, and they have found various applications in technology. In this contribution we discuss recent work on extensions to n-valued R-functions [2, 3], applications to Generalized Möbius-Listing bodies and their cutting [4], extensions of Boolean operations [4], and relationships to modeling of natural shapes [5].

Acknowledgements. To Yohan Fougerolle, for his seminal work in this field**.** To Revaz Grigolia, for work on n-valued R-functions and to Ilia Tavkelidze for the joint work on Generalized Möbius-Listing bodies.

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FACTORIZATION, SEMI-DIRECT PRODUCT AND NON-ABELIAN COHOMOLOGY OF MONOIDS

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Factorizations of mathematical objects is an important topic in mathematics, whose basic underlying idea is to represent a mathematical object as a product of two (usually simpler) sub-objects with minimal intersection. We describe the relationships between monoid factorizations, non-abelian cohomology of monoids and semi-direct product of monoids, and give examples for calculating how many different ways a monoid can be factorized.

THE SPACES OF GENERALIZED THETA-SERIES WITH SPHERICAL POLYNOMIALS OF SECOND END FOURTH ORDER

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F. Gooding [1] constructed the spaces of spherical polynomials with respect to some positive definite binary quadratic forms and calculated the dimensions of the spaces of corresponding generalized theta-series. In [2], the upper bounds of the dimension of the space $T(\nu,Q)$ for some quadratic form of r variables are obtained. In this paper, some positive diagonal and non-diagonal quadratic forms are considered; the basis of the spaces of spherical polynomials of second and fourth order with respect to these forms are constructed and corresponding generalized theta series are considered.

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GEOMETRIC FIGURES WHICH APPEAR AFTER SS CUTTING OF THE GML BODIES IN THE RADIAL CROSS SECTION

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In the previous works, we were able to calculate all possible, and various options that appear after VV, VS or SS cuts of the GML bodies with the help of the so-called straight chordal knives. Then we did not specify how many and what traces of flat figures appear on the radial

cross section of the body, depending on: m-number of polygon vertices, n-number of twist and a parameter showing which vertices (sides) are connected by this knife!? In this article, a regularity is given with the help of which it is possible to calculate the number and nature of flat figures appearing after an arbitrary SS cut. This work is another step towards resolving the question whether it is possible to unequivocally restore the GML body knowing the information about the traces left on the radial cross section. It is worth noting that such a regularity for VS cutting has been published in previous 2023 reports [3].

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STRUCTURAL EQUATIONS OF THE TANGENT SPACE T(T(Vn))

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Consider a tangent bundle $\mathrm{T}(\mathrm{T}(\mathrm{Vn}))$ with the local coordinates $(x^i, y^{\bar{\imath}}, y^i, z^{\bar{\imath}})$ where $(x^i, y^{\bar{\imath}})$ are the coordinates of the basis T(Vn), and $(y^i, z^{\bar{\imath}})$ are those of the layer $T_z, z \in T(Vn)$, local coordinates of the point of the space $\mathrm{T}(\mathrm{T}(\mathrm{Vn}))$ are transformed as follows:

$$\bar{x}^i = \bar{x}^i(x^k), \bar{y}^i = x^i_k y^k, \bar{x}^{\bar{\imath}} = x^{\bar{\imath}}_{\bar{k}} y^{\bar{k}}, \bar{z}^{\bar{\imath}} = x^{\bar{\imath}}_{\bar{k}} z^{\bar{k}} + x^{\bar{\imath}}_{\bar{k}j} y^{\bar{k}} y^j.$$

The structural equations of the space T(T(Vn)) have been found:

$$D\omega^i = \omega^k \wedge \tilde{\omega}^i_k,\ D\tilde{\theta}^{\bar{\imath}} = \tilde{\vartheta}^{\bar{k}} \wedge \omega^{\bar{\imath}}_{\bar{k}} + R^{\bar{\imath}}_{pq} \omega^p \wedge \omega^q,$$

$$D\tilde{\theta}^i = \tilde{\theta}^k \wedge \tilde{\omega}^i_k + \mathbb{R}^i_{pk} \omega^p \wedge \omega^k + \mathbb{R}^i_{\bar{j}k} \tilde{\theta}^{\bar{\jmath}} \wedge \omega^k + \mathbb{S}^i_{pk} \tilde{\theta}^p \wedge \omega^k,$$

$$D\tilde{\vartheta}^{\bar{\imath}} = \tilde{\vartheta}^{\bar{\jmath}} \wedge \tilde{\omega}^{\bar{\imath}}_{\bar{\jmath}} + \mathbb{A}^{\bar{\imath}}_{pk} \omega^p \wedge \omega^k + \mathbb{B}^{\bar{\imath}}_{\bar{j}k} \tilde{\vartheta}^{\bar{\jmath}} \wedge \omega^k + \mathbb{M}^{\bar{\imath}}_{\bar{j}\bar{k}} \theta^{\bar{\jmath}} \wedge \theta^{\bar{k}} +$$

$$+ \mathbb{C}^{\bar{\imath}}_{\bar{j}k} \tilde{\vartheta}^{\bar{\jmath}} \wedge \omega^k + \mathbb{D}^{\bar{\imath}}_{\bar{j}k} \tilde{\theta}^i \wedge \theta^{\bar{k}} + \mathbb{E}^{\bar{\imath}}_{\bar{j}k} \tilde{\vartheta}^{\bar{\jmath}} \wedge \omega^k.$$

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PRIME NUMBERS REPRESENTED BY BINARY FORMS WITH ODD DISCRIMINANT

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The formulae for the average number of representations of positive integers by the genus of positive binary forms with odd discriminant are obtained. It gives us the opportunity to characterize the primes which can be reprezented by binary forms with the odd discriminant.

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SECTION OF REAL ANALYSIS

Chairs: Ushangi Goginava, Leri Gogoladze
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SCHWARZ GRADIENTS AND DIFFERENTIABILITY FOR FUNCTIONS OF TWO VARIABLES

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The Schwarz derivative is generalized for functions of two variables. For this, we introduce the Schwarz concepts of differentiability and differential, an ordinary gradient, an angular gradient, a strong gradient, a generalized angular gradient and a generalized strong gradient.

It is proved that the Schwartz differentiability is equivalent to the existence of a generalized angular Schwarz gradient. It is established that the Schwarz differentiability of a function at some point implies the smoothness of this function at the same point. The set of smooth functions contains both a set of ordinary differentiable functions and a set of Schwarz-differentiable functions.

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ON LUZIN'S PROBLEM ON THE CONVERGENCE OF DOUBLE FUNCTIONAL SERIES TO $+\infty$

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The question whether the trigonometric series converges to $+\infty$ on the set of positive measure was posed by N.N. Luzin (see [1, p. 239]) in 1915. Yu.B. Germier [2] showed that the trigonometric series cannot be summable by the Riemmann method to $+\infty$ on the set of positive measure. At the same time N.N. Luzin and I.I. Privalov [3] have constructed an example of the trigonometric series summable almost everywhere to $+\infty$ by the Abel method. P.L. Ulyanov [4] has proved that if E is a subset of the second category of the non-degenerate interval $(a,b)\subset[-\pi,\pi]$ such that $(E\cap(c,d))>0$ for every nondegenerate interval $(c,d)\subset(a,b)$, then there does not exist the trigonometric series, converging to $+\infty$ or to $-\infty$ when $x\in E$.

D.E. Menshov [5] has shown that there exists the trigonometric series converging in measure to $+\infty$ on $[-\pi,\pi]$. A.A. Talalyan and F.G. Arutyunyan [6] have shown that the series with respect to the Haar system cannot converge to $+\infty$ on the set of positive measure. V.A. Skvortsov [7] has given a simpler proof of this result. At the same time P.L. Ul'yanov [8] and R.I. Ovsepyan and A.A. Talalyan [9] have shown that there exist uniformly bounded orthonormal systems of functions such that the series with respect to them converge to $+\infty$ on the set of positive measure.

In 1988, S.V. Konyagin [10] proved

Theorem (S. V. Konyagin). For every trigonometric series the equality

$$\operatorname{mes}\{x : x \in [-\pi,\pi],\ -\infty < \underline{\lim}_{m\to\infty} S_m(x) \le \overline{\lim}_{m\to\infty} S_m(x) = +\infty\} = 0$$

is true, where $S_m(x)$ are partial sums of this series.

It is clear that this gives a negative answer to N. N. Luzin's problem.

G. G. Gevorkyan [11] has proved that every Franklin series cannot converge to $+\infty$ on the set of positive measure. The similar result, in particular, for multiple Franklin series was obtained by G. G. Gevorkyan and M. G. Grigoryan[12]. G. G. Gevorkyan [13] has obtained the criterion for almost everywhere convergence of Franklin series on a set. G. G. Gevorkyan, K. A. Keryan, and M. P. Poghosyan [14] have proved that there does not exist a series with respect to orthogonal splines and Ciesielskie series converging to $+\infty$ on the set of positive measure. After S.Konyagin had proved his theorem, there naturally arose N.N.Luzin's problem for double trigonometric series. In [15] I have proved

Theorem 1. Every double trigonometric series cannot converge to $+\infty$ on the set E, $\mu_2 E > 0$.

Consider the systems of functions

$$\left\{\varphi_{n_j}^{(j)}(x_j)\right\}_{n_j=1}^{\infty}, \quad x_j \in [0,1], \quad j = 1,2, \tag{1}$$

where every function $\varphi_{n_j}^{(j)}(x_j)$, $j = 1,2,$ is measurable and takes finite values, and the series

$$\sum_{n_j=1}^{\infty} a_{n_j}^{(j)} \varphi_{n_j}^{(j)}(x_j), \quad j = 1,2, \tag{2}$$

$$\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} c_{n_1,n_2} \varphi_{n_1}^{(1)}(x_1)\varphi_{n_2}^{(2)}(x_2). \tag{3}$$

Let

$$S_{m_j}^{(j)} = \sum_{n_j=1}^{m_j} a_{n_j}^{(j)} \varphi_{n_j}^{(j)}(x_j), \quad S_{m_1,m_2}(x_1,x_2) = \sum_{n_1=1}^{m_1}\sum_{n_2=1}^{m_2} c_{n_1,n_2} \varphi_{n_1}^{(1)}(x_1)\varphi_{n_2}^{(2)}(x_2)$$

be, respectively, partial sums of series (2) and (3).

Definition. It is said that system $\{\psi_n(x)\}$ has the property K if the following equality holds

$$\operatorname{mes}\{x \in [0,1] : -\infty < \underline{\lim}_{m\to\infty} S_m(x) \le \overline{\lim}_{m\to\infty} S_m(x) = +\infty\} = 0$$

for each series $\sum_{n=1}^{\infty} a_n \psi_n(x)$, where

$$S_m(x) = \sum_{n=1}^{m} a_n \psi_n(x).$$

Theorem 2 (see [15])**.** Let the systems (1) have the property K, then every series (3) will not be convergent to $+\infty$ on the set $E \subset [0,1]^2$, $\mu_2 E > 0$.

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A NOTE ON HIGHER MODULUS OF CONTINUITY AND FOURIER SERIES

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It is well known, that trigonometric system does not derive a basis in the space of continuous functions. Moreover, Lebesgue has constructed an example of continuous function with unboundedly divergent Fourier series at point. Hence, the question arised to characterize the subsets of continuous functions space that provide uniform convergence of Fourier series. There are two approaches to this direction: 1) characterizing of subclasses using the variation of function (Jordan, Wiener, Young, Chanturia, Oskolkov, Karchava, Akhobadze , Goginava); 2) On this point it should be noted the following estimation of Lebesgue:

$$\|f - S_n(f)\| \le c\omega\left(\frac{1}{n}, f\right)\log(n+2),$$

where $S_n(f,x)$ is the n-th partial sums of the trigonometric Fourier series of the function f and w is the modulus of continuity of function.

The main aim of the paper is an improvement of Lebesgue estimate.

Let $\omega_p(\delta, f)$ is a continuity modules of higher order, which is defined in the following way:

$$\omega_p(\delta, f) = \sup_x \sup_{|h|\le\delta} |\Delta_p(x,h,f)|, \qquad \omega_1(\delta, f) = \omega(\delta, f),$$

where $\Delta_1(x,h,f) = f(x+h) - f(x)$; $\Delta_{p+1}(x,h,f) = \Delta_p(x+h,h,f) - \Delta_p(x,h,f)$.

We prove that the following is true:

Theorem 1. Let $f \in C([0;2\pi])$. Then

$$\|f - S_n(f)\|_c \le c\sum_{k=1}^{n} \frac{\omega_k\left(\frac{1}{n}, f\right)}{k2^k}$$

Corollary 1. Let $f \in C([0;2\pi])$. Then

$$\|f - S_n(f)\|_c \le c \max_{1\le k\le[\log\log n]} \omega_k\left(\frac{1}{n}, f\right).$$

Corollary 2. Let $f \in C([0;2\pi])$ and $\omega_k\left(\frac{1}{n}, f\right) \le \frac{2^k}{l(k)}$, where $\sum_{k=1}^{\infty} \frac{1}{l(k)k} < \infty$.

Then $$\|f - S_n(f)\|_c \to 0 \quad \text{as } n \to \infty.$$

ON SOME PROPERTIES OF DOUBLE FOURIER TRIGONOMETRIC SERIES WITH GAPS

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The question of the generalized absolute convergence of double Fourier trigonometric series with gaps is considered.

The sufficient conditions are obtained for the generalized absolute convergence of double trigonometric Fourier series with gaps in terms of mixed and partial moduli of variation of the function of two variables.

ON UNIVERSAL REPRESENTING FUNCTIONS FOR CERTAIN CLASSES OF FUNCTIONS

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The talk deals with the question of the existence of universal representing functions for certain classes, A, of functions defined over (0,1).

Let us formulate the necessary definitions:

1) We say that a set $E \subset (0,1)$ is a set of uniqueness for the class of functions A if for any functions $f(x)$ and $g(x)$ from the class A, for which $f(x) = g(x)$ when $x \in E$, it follows that $f(x) = g(x)$ for any $x \in (0,1)$.
2) We say that a function $F(x)$ defined over (0,1) is a universal representing function for the class A if for any function $f(x) \in A$, there exists a set of uniqueness E for the class A such that $F(x) = f(x)$ for any $x \in E$.

Note that there exists a class of functions defined over (0,1) for which there is no universal representing function.

In this talk, the class of all continuous functions $A = C(0,1)$ defined over (0,1) is considered. It is clear that a set E is a set of uniqueness for the class $C(0,1)$, if and only if E is a dense set in (0,1).

Taking into account the above remark, it is natural to ask the question: does a universal representing function exist for the class $C(0,1)$?

The answer to this question is positive. Namely, the following statement holds:

Theorem. There exists a function $F(x)$ such that $F(x) \in L_p(0,1)$ for any $p > 0$ and $F(x)$ is a universal representing function for the class $C(0,1)$.

Note that the function $F(x)$ mentioned in the theorem is also a universal representing function for certain classes of discontinuous functions defined over (0,1).

UNCONDITIONAL CONVERGENCE OF GENERAL FOURIER SERIES

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S. Banach, in particular proved that for any function, even $f(x) = 1,$ where $x \in [0,1],$ the convergence of its Fourier series with respect to the general orthonormal systems (ONS) is not guaranteed. In this article, we find conditions for the functions φ_n of an orthonormal system (ONS) $(\varphi_n),$ under which the Fourier series of functions $f \in Lip1$ are unconditionally convergent almost everywhere. The obtained results are the best possible. We also prove that any ONS contains a subsystem such that the Fourier series of any function $f \in Lip1$ are unconditionally convergent a.e. on [0, 1]. Additionally, we have shown that the solutions for these types of problems for the general ONS are trivial for classical ONS trigonometric, Haar and Walsh systems.

SECTION OF COMPLEX ANALYSIS AND APPLICATIONS

Dedicated to the 100th anniversary of Professor Giorgi Manjavidze

Chair: Grigory Giorgadze
Co-chair: George Akhalaia

ON THE RESEARCH OF GIORGI MANJAVIDZE IN THE THEORY OF GENERALIZED ANALYTIC FUNCTIONS

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The main results of George Mandzhavidze in the theory of generalized analytical functions are presented in the monograph [1]. This work was awarded the Nikoloz Maskelishvili prize in 1990.

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PONTRYAGIN MAXIMUM PRINCIPLE FOR THREE-LEVEL QUANTUM SYSTEMS

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We consider modern aspects of optimal control theory in quantum systems [1]. We will set a quantum control problem for a three-level quantum system [2] whose dynamics are governed by the Schrodinger equation and will formulate this problem using Pontryagin Maximum Principle [3]. Besides, we consider the existence of optimal solutions for this optimal control problem.

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ON SOLUTION SPACE OF THE RIEMANN-HILBERT BOUNDARY VALUE PROBLEM WITH SHIFT FOR GENERALIZED ANALYTIC FUNCTIONS

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In the talk we give the generalization of the Riemann-Hilbert boundary value problem with shift for generalized analytic function and prove that the solution of the problem depends on the complex structures of the Riemann surfaces induced from shift function.

ON MONODROMY OF LAME EQUATION

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It is known that the Lame equation is the so-called canonical form of the Heun equation and depends on one accessory parameter. However, computing the monodromy of this equation in the general case is impossible. We characterize the monodromy groups of the Lame equation in some special cases and show that they are generated by reflections (see [1]).

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ON GLOBAL SOLUTIONS OF CONFLUENCE HEUN EQUATION

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The confluence Heun equation has three distinct singular points, one of which is irregular, while the other two are regular. Consequently, the confluence Heun equation is not equivalent to the Riemann (hypergeometric) equation. The structure of local solutions for this equation at all singular points is well-studied, and it is known that solutions at irregular singular points are not represented as convergent power series. From here, it follows that transitioning from local to global, known as the classical connection problem, aids in understanding irregular

critical points. In the report, we discussed the generalization of this problem for the confluence Heun equation. This study represents a continuation of our previous research [1].

Acknowledgements. The research was supported by the Shota Rustaveli National Science Foundation grant STEM-22-308.

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COMPUTATION OF THE DIMENSION OF THE DEFORMATION SPACE OF HOLOMORPHIC STRUCTURES OF THE VECTOR BUNDLES INDUCED FROM THE FUCHSIAN SYSTEM

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In the report we present the algorithm for computing the splitting type of holomorphic vector bundles induced from a Fuchsian system on the Riemann sphere (see [1] and [2]). We consider a general approach to the problem by employing the methods of deformation of complex structures. As a consequence, we calculate the dimension of the deformation space of complex structures of canonical deformation.

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THE REGULAR FOURIER IMAGE OF THE COULOMB WAVE FUNCTION

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The regular Fourier image of the two-particle Coulomb wave function is described. It is rigorously shown that the above function exists in the sense of the generalized functions. This function in the weak representation belongs to the set of the Hilbert space functions and accordingly represents the full set of quantum-mechanical functions of the continuous spectrum. It

has the simple analytical structure and satisfies the two-particle homogeneous equation of the perturbation theory.

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ON THE GENERALIZED MODULUS OF A POLYGON

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In the talk we will introduce a generalized modulus, which we will use to calculate the conformal characteristics of a polygon. For a polygon with n sides, the generalized modulus measures the ratio of any neighboring sides using n-3 parameters and interior angles.

Acknowledgement. The research was supported by the Shota Rustaveli National Science Foundation grant number FR 22-354.

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GLOBAL MONODROMY PROBLEM FOR HYPERGEOMETRIC EQUATIONS

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We construct bases for the linear space of solutions of the Hypergeometric equations at three different singular points. Additionally, we establish an analytic isomorphism between these spaces, connecting their respective bases with each other ([see 1]).

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A NECESSARY AND SUFFICIENT CONDITION FOR THE EXISTENCE OF A SINGULAR INTEGRAL WITH CAUCHY KERNEL

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It is shown that the expression which in the articles [1,2] called by A.P. Calderon as the Cauchy operator is not the Cauchy operator and the results of the above works are erroneously formulated. Let $\Gamma: \zeta = \zeta(t),\ t \in [a,b]$, be a rectifiable Jordan curve on the complex plane E_2. For $F \in L(\Gamma), t_0 \in [a,b]$ and $\zeta_0 = \zeta_0(t_0) \in \Gamma$ the expression

$$K_\Gamma(F,\zeta_0) = \text{v. p.} \int_\Gamma \frac{F(\zeta)}{\zeta - \zeta_0} d\zeta, \quad F \in L(\Gamma),\ \zeta_0 \in \Gamma,$$

Is called a singular integral with Cauchy kernel.

For each $\in E_2, \alpha \in [0,2\pi]$, we denote by $\mu_\Gamma(z,\alpha)$ the number of points, where the half-line $\zeta = z + \rho e^{i\alpha},\ \rho > 0$, meets the curve Γ and

$$R_\Gamma(z) = \int_0^{2\pi} \mu_\Gamma(z,\alpha) d\alpha .$$

Theorem. The condition $\text{mes}\{\zeta:\ \zeta \in \Gamma,\ R_\Gamma(\zeta) = \infty\} = 0$ is necessary and sufficient for the existence of the singular integral with Cauchy kernel for every $F \in L(\Gamma)$ and for almost all points of the curve Γ.

Note that this Theorem is a consequence of the two-sided estimation of the integral $\text{Im}\, K_\Gamma(F,\zeta_0)$ (see theorem 1, [3]), which I didn't notice in that time.

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THE RATING OF GENERALIZED ANALYTIC FUNCTIONS AND SOME OF ITS APPLICATIONS

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The regular generating pair (a,b) on the complex plane and the class of corresponding generalized analytic functions $\mathbb{A}(a,b)$are considered ([1],[2]). Each representative ω of the mentioned class is assigned a rating; The rating is a real non-negative number or infinity, and it gives important information about the behavior of the generalized analytic function ω in the

neigborhood of a point at infinity. Based on the notion of rating, the class $\mathbb{A}(a, b)$can naturally be imagined as a disjunctive union.

A general characteristic property of generalized analytic functions of finite rating has been found. In the class of generalized analytic functions of infinite rating, elements with, in some sense exotic structure, have been searched. It is proved that generalized analytic functions with finite rating do not have a similar structure.

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NEW PHYSICS, SEMI-INCLUSIVE DISTRIBUTIONS AND STATISTICAL POTENTIALS

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Formal definition of the contemporary meaning of the New Physics is proposed. A constituent picture of the W and H bosons is considered. Connection between Hagedorn, Tsallis and Negative binomial distributions for inclusive cross sections are established. Non-Riemannian zeros proposed in [1] are tested by direct calculations.

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A GENERALIZATION OF THE DOLBEAULT LEMMA

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On a complex manifold X, a homomorphism of $\mathcal{O}_X(U)$-modules is defined with respect to a differential 1-form of type $(0,1)$, $\alpha \in \mathcal{A}_X^{0,1}(U)$, where $U \subseteq X$ is an open subset of the manifold, as follows

$$\overline{\partial}_\alpha : \mathcal{A}_X^{p,q}(U) \to \mathcal{A}_X^{p,q+1}(U), \qquad \omega \mapsto \overline{\partial}_\alpha(\omega) := \overline{\partial}\omega - \alpha \wedge \omega,$$

for which, after imposing certain conditions on α, an analogue of the Dolbeault lemma is proven.

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ON THE APPLICATION OF COMPLEX ANALYSIS FOR THE ESSENTIALLY NONLINEAR SYSTEM OF DIFFERENTIAL EQUATIONS

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If we consider the mathematical models corresponding to refined theories for elastic plates, the main part of relevant differential operator contains together with Laplacian and bihatrmonic operator also a composition of Laplacian on Monge-Ampére nonlinear form of second degree. By using complex analysis, we construct a system of integro-differential equations the solution of which we find by Seidel method of successive approximation.

SECTION OF ORDINARY DIFFERENTIAL EQUATIONS AND OPTIMAL CONTROL

Dedicated to the 90th anniversary of Professor Guram Kharatishvili

Chairs: Roman Koplatadze, TamazTadumadze
Co-chair: Tea Shavadze

GURAM KHARATISHVILI-90

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The report deals with aspects of Guram Kharatishvili's life, scientific and pedagogical activities.

THE NECESSARY CONDITIONS OF OPTIMALITY FOR THE OPTIMIZATION PROBLEM OF A MARKET RELATION

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The following optimal problem of market relation with the mixed initial condition is considered

$$\begin{cases} \dot{p}(t) = ap(t) + bq^1(t), \\ \dot{q}^1(t) = cp(t) + dq^1(t), \\ \dot{q}^2(t) = D_1(t, u(t-\rho)) - S_1(t, p(t-\tau), q^1(t-\sigma), u(t)), \\ \dot{q}^3(t) = D_2(t, v(t-\theta)) - S_2(t, p(t-\tau), q^1(t-\sigma), v(t)); \end{cases}$$

$$p(t) = \varphi(t),\ t < t_0,\ p(t_0) = p_0;\ q^1(t) = g(t),\ t \le t_0;$$

$$[q^2(t_1)]^2 + [q^3(t_1)]^2 \to \min .$$

The necessary conditions of optimality for the initial and final moments, delay parameters, the initial vector, the initial and control functions are obtained.

THE CRITERION ON THE EXISTENCE OF BOUNDED SOLUTIONS ON REAL AXIS R OF LINEAR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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Effective necessary and sufficient conditions are established for the existence of bounded solutions of systems of linear ordinary differential equations on the real axis. Sufficient conditions are established for the existence of bounded solutions satisfying the Nicoletti condition. Moreover, the method of the construction of such solutions is given. Sufficient conditions of the existence of unique solution and its positiveness are established as well.

In earlier papers (see [1] and references therein) only sufficient conditions on the existence of bounded solutions are established.

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THE PROBLEM OF FORECASTING RESIDENT REAL ESTATE PRICE INDEX IN GEORGIA

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As is known, the Residential Real Estate Price Index (RPPI) in Georgia reflects the dynamics of residential real estate prices. The index includes the segment of the real estate market of newly built and under-construction housing in Tbilisi City.The paper discusses the problem of forecasting the RPPI index (in terms of quarters), based on modern mathematical models and the well-known computer program EViews-10 ("Econometric Views").

MATHEMATICAL MODELS FOR CREATING THE OPTIMAL STRUCTURE OF KNOWLEDGE VERIFICATION TESTS

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An approach is suggested for determining the test structure, which relies on discrete optimization models and algorithms.

ON THE SPECIFIC PROPERTIES OF SOLUTIONS OF THE FIRST ORDER DIFFERENTIAL EQUATIONS WITH DELAY ARGUMENT

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A first order delayed argument differential equation is considered. Sufficient conditions of the oscillation of proper solutions are established which is a generalization of previously well-known results.

A NONLINEAR MATHEMATICAL MODEL OF THE COURSE AND THERAPY OF RHEUMATOID ARTHRITIS AND ITS REALIZATION

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Based on the models offered by us in [1] and [2], new mathematical models have been obtained to describe the processes of rheumatoid arthritis immunopathogenesis and disease treatment. According to the mentioned models, exist J cells, which for certain reasons the body's immune system perceives as enemy cells and starts to kill them. In the process the connection between the B reactive, T helper, T regulatory cells and the mediator of inflammation IL-6 are foreseen. The system of nonlinear differential equations describes the changes in the population size J cells, B, Th, Treg lymphocytes and IL-6 and the concentration of medicines in the blood. The treatment process goes on combined using of two drugs. According to the model, the disease is originated when the amount of B cells is up to limited standard. The drugs which are parts of the equations helps to multiply the target cells and/or to reduce till limited amount of B lymphocytes, which is completed by the functional effects of

the drugs on Th and Treg cells and IL-6. Is formulated and solved Cauchy's problem to estimate the dosage and effectiveness of medical products.

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ON THE REPRESENTATION OF SOLUTION FOR A CLASS OF PERTURBED CONTROLLED NEUTRAL FUNCTIONAL-DIFFERENTIAL EQUATION WITH THE CONTINUOUS INITIAL CONDITION

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The analytic relation between solutions of the original Cauchy problem and a corresponding perturbed problem is established for the controlled neutral functional-differential equation with the continuous initial condition, whose right-hand side is linear with respect to the prehistory of the phase velocity. In the representation formula of solution the effects of perturbations of the delay parameter containing in the phase coordinates, of the initial and control functions are revealed.

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THE REPRESENTATION FORMULA OF SOLUTION OF THE PERTURBED CONTROLLED FUNCTIONAL DIFFERENTIAL EQUATION TAKING INTO ACCOUNT VARIATION OF THE INITIAL MOMENT AND THE CONTINUOUS INITIAL CONDITION

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Let $t_{01} < t_{02} < t_1, 0 < \tau_{11} < \tau_{12}$ be given numbers and let Φ and Ω be the sets of continuous differentiable initial functions $\varphi(t)$ and piecewise-continuous control functions $u(t)$, respectively. To each element $w = (t_0, \tau, \varphi(t), u(t)) \in W = (t_{01}, t_{02}) \times (\tau_{11}, \tau_{12}) \times \Phi \times \Omega$ we assignthe controlled functional differential equation

$$\dot{x}(t) = f(t, x(t), x(t-\tau), u(t)), t \in [t_0, t_1] \tag{1}$$

with the continuos initial condition

$$x(t) = \varphi(t), t \in [\hat{\tau}, t_0]. \tag{2}$$

we call the solution $x(t; w)$ of problem (1)-(2) the solution corresponding to element w. In the paper, the representation formula of solution $x(t; w_0 + \delta w)$ with the solution $x(t; w_0)$ is established, where $w_0 = (t_{00}, \tau_0, \varphi_0(t), u_0(t)) \in W$ and $\delta w = w - w_0 = (\delta t_0, \delta\tau, \delta\varphi(t), \delta u(t))$. In the formula effects of variation of the initial moment t_{00} and condition (2) are revealed. The analogous problem is proved in [1, 2] when t_0 is fixed.

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SECTION OF PARTIAL DIFFERENTIAL EQUATIONS

Chairs: Temur Jangveladze, Sergo Kharibegashvili, David Natroshvili
Co-chair: Zurab Kiguradze

THE BOUNDARY VALUE PROBLEM FOR ONE CLASS OF HIGHER-ORDER NONLINEAR HYPERBOLIC SYSTEMS

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For one class of higher-order nonlinear hyperbolic systems in the angular domain, a boundary value problem with Dirichlet and Neumann type boundary conditions is considered. The concept of a weak generalized solution of this problem on some subspace of the Sobolev space is introduced. This problem is equivalently reduced to a nonlinear functional equation in the mentioned space. Under certain conditions imposed on the nonlinear terms, an a priori estimate for the solution of a functional equation is obtained, from which its existence follows. The question of uniqueness and nonexistence of a solution to this problem is also discussed.

MATHEMATICAL MODEL OF DETONATION SHOCK WAVE PROPAGATION IN A NONHOMOGENEOUS STAR

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The mathematical modeling of astrophysics processes is one of the most actual problem of modern applied mathematics. To solve many problems of astrophysics, it is necessary to study the dynamics of gaseous bodies interacting with a gravitational field [1, 2]. The work considers a nonavtomodel problem about the central explosion of nonhomogeneous gas body (cubic function of star density drop from central core to the surface) bordering vacuum, which is in equilibrium in its own gravitational field. In [3] proposed an asymptotic method of thin impact layer which is used to solve the problem. The solution of the problem in the vicinity behind the detonation shock wave (the fracturing surface of the first kind) is sought in the form of a singular asymptotic decomposition by a small parameter. Analytically, the main (zero) approximation for the law of motion and the thermodynamic characteristics of the medium was accurately found. The Cauchy problem for zero approximation of the law of motion of the detonation shock wave is solved exactly, with the help of Appell hypergeometric function of two variables. The asymptotics of the zero approximation of the law of the detonation shock wave is found at the moment and during the time coming on the surface of the object. Besides, the time of coming on the surface is found.

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ON THE SEMI-DISCRETE SCHEME CORRESPODING TO THE FOURTH-ORDER NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

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A semi-discrete scheme is built for the initial-boundary problem of the fourth-order nonlinear integro-differential equation. The discussed equation is based on the integro-differential models proposed in [1]. Extensive references to the mentioned models are given in the following monographs [2,3]. In our work, the stability and convergence of the constructed scheme is studied. The stability and uniqueness of the solution for this problem are investigated in [4].

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NUMERICAL SOLUTION OF ONE MULTIDIMENSIONAL SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS USING MACHINE LEARNING

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A multidimensional analogue of Maxwell's system of nonlinear partial differential equations [1] is considered. Numerical experiments have been conducted using decomposition algorithms [2, 3] and machine learning methods [4]. Analysis of the obtained results and comparison of computer experiments with theoretical conclusions is made.

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ON TWO MULTIDIMENSIONAL SYSTEMS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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One multidimensional model based on Maxwell's well-known system of nonlinear partial differential equations [1], describing the process of penetration of a magnetic field in a substance is considered. The uniqueness of the solutions of the corresponding initial-boundary value problems, the convergence of decomposition method and the finite-difference scheme are studied. The results are an extension of some outcomes obtained in [2, 3]. One multidimensional biological model is also considered [4]. Algorithms of sum approximation [5] and variable directions [6,7] have been studied for the corresponding initial-boundary value problem.

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AN ALTERNATIVE POTENTIAL METHOD FOR MIXED STEADY STATE ELASTIC OSCILLATION PROBLEMS

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We consider an alternative approach to investigate three-dimensional exterior mixed boundary value problems (BVP) for the steady state oscillation equations of the elasticity theory for isotropic bodies. The unbounded domain occupied by an elastic body, $\Omega^- \subset R^3$, has a compact boundary surface $S = \partial\Omega^-$, which is divided into two disjoint parts, the Dirichlet part S_D and the Neumann part S_N, where the displacement vector (the Dirichlet type condition) and the stress vector (the Neumann type condition) are prescribed respectively.

Our new approach is based on the classical potential method and has several essential advantages compared with the existing approaches. We look for a solution to the mixed boundary value problem in the form of a linear combination of the single layer and double layer potentials with densities supported on the Dirichlet and Neumann parts of the boundary respectively. This approach reduces the mixed BVP under consideration to a system of boundary integral equations, which contain neither extensions of the Dirichlet or Neumann data nor the Steklov-Poincaré type operator involving the inverse of a special boundary integral

operator, which is not available explicitly for arbitrary boundary surface. Moreover, the right-hand sides of the resulting boundary integral equations system are vector-functions coinciding with the given Dirichlet and Neumann data of the problem in question. We show that the corresponding matrix integral operator is bounded and coercive in the appropriate L_2-based Bessel potential spaces. Consequently, the operator is invertible, which implies unconditional unique solvability of the mixed BVP in the class of vector-functions belonging to the Sobolev space $\left[W^1_{2,\mathrm{loc}}(\Omega^-)\right]^3$ and satisfying the Sommerfeld-Kupradze radiation conditions at infinity. We also show that the obtained matrix boundary integral operator is invertible in the L_p-based Besov spaces and prove that under appropriate boundary data a solution to the mixed BVP possesses C^α-Hölder continuity property in the closed domain $\overline{\Omega^-}$ with $\alpha = \frac{1}{2} - \varepsilon$, where $\varepsilon > 0$ is an arbitrarily small number.

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SOLUTION OF THE CAUCHY PROBLEM IN QUADRATURES FOR ONE CLASS OF STRICTLY HYPERBOLIC EQUATIONS OF HIGH ORDER

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The paper proposes an approach that allows one in a constructive way effectively write out the solution to the Cauchy problem in quadratures for one class of strictly hyperbolic equations of high order.

ON MACHINE LEARNING AIDED NUMERICAL SOLUTIONS FOR MULTIDIMENSIONAL SYSTEMS

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A multidimensional analogue of a two-dimensional system of nonlinear partial differential equations [1] describing the vein formation in plant leaves is studied. The proposed abstract investigates decomposition schemes [2, 3] for multidimensional models using machine learning algorithms [4]. Computer realizations have been implemented for above-mentioned decomposition schemes. Results from computer experiments are compared with theoretical conclusions and relevant analyses are carried out.

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SECTION OF PROBABILITY THEORY AND MATHEMATICAL STATISTICS

Dedicated to the 60th anniversary of the Nadaraya-Watson formula

Chairs: Elizbar Nadaraya, Omar Purtukhia
Co-chair: Petre Babilua

THE "Z-CRITERIA" OF HYPOTHESIS TESTING FOR THE HAAR STATISTICAL STRUCTURE

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The following necessary and sufficient conditions for the existence of the "Z-criterion" for hypothesis testing have been established.

Theorem 1. Let

$$M_B = \bigoplus_{h\in H} M_B\left(\bar{\mu}_h\right)$$

be a Banach space of measures, $card\ H \le c$, let E is Radon separable complete metric space. In order for the Borel orthogonal Haar statistical structure $\left\{E, S_1, \bar{\mu}_h, h\in H\right\}$, $card\ H \le c$ to admit the Z-criterion for hypothesis testing in the theory of $\left(\mathrm{ZFC}\right)\&\left(\mathrm{MA}\right)$ it is necessary and sufficient that the correspondence $f \longleftrightarrow \ell_f$ defined by the equality

$$\int_E f\left(x\right)\bar{\mu}_h\left(dx\right) = \ell_f\left(\bar{\mu}_h\right), \quad \bar{\mu}_h \in M_B,\ f\in F\left(M_B\right)$$

was one-to-one, where ℓ_f is a linear continuous functional on M_B, $F\left(M_B\right)$ the set of real function f such that $\int_E f\left(x\right)\bar{\mu}_h\left(dx\right)$ is define for all $\bar{\mu}_h \in M_B$.

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ON THE LIMIT DISTRIBUTION OF INTEGRAL SQUARE DEVIATION BETWEEN PROJECTION TYPE ESTIMATORS OF DISTRIBUTION DENSITY IN $p \geq 2$ INDEPENDENT SAMPLES

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The limiting distribution of the statistic, which describes the mutual deviation of the projection type estimates from each other of distribution density in $p \geq 2$ independent samples is established. The goodness-of-fit test is constricted. Various examples are given.

THE LAW OF LARGE NUMBERS FOR WEAKLY CORRELATED RANDOM ELEMENTS IN HILBERT SPACES

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The first rigorous proof of the law of large numbers (LLN) belongs to Bernoulli. More than hundred years later, Bernoulli's theorem was generalized by Poisson, who was the first to use the term "law of large numbers". Significant contribution in this direction belongs to Chebyshev, who presented the problem in terms of random variables and gave a very simple and precise proof of the generalized LLN. The LLN was a popular area of research and has been intensively studied by many authors for the sequences of independent (or uncorrelated) random variables. The study of the LLN for dependent random variables started later. One of the first results in this direction was obtained in 1928 by Khinchin. we proved an analogue of the Khinchin's theorem for weakly correlated random elements with values in Hilbert spaces.

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THE "Z-CRITERIA" OF HYPOTHESIS TESTING FOR EXPONENTIAL STATISTICAL STRUCTURE

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The following necessary and sufficient conditions for the existence of the "Z-criterion" for hypothesis testing have been established.

Theorem 1. Let

$$M_H = \bigoplus_{h\in H} M_H\left(\bar{\mu}_h\right)$$

be a Hilbert space of measures, $card\ H \le c$, let E is Radon separable complete metric space. In order for the Borel orthogonal Haar statistical structure $\left\{E, S_1, \bar{\mu}_h, h\in H\right\}$, $card\ H \le c$ to admit the Z-criterion for hypothesis testing in the theory of $(\mathrm{ZFC})\&(\mathrm{MA})$ it is necessary and sufficient that the correspondence $f \longleftrightarrow \ell_f$ $\left(f \in F\left(M_H\right)\right)$ defined by the equality

$$\int_E f(x)\,\bar{\mu}_h(dx) = \left(\psi_f, \bar{\mu}_h\right), \quad \bar{\mu}_h \in M_H$$

was one-to-one, where $F\left(M_H\right)$ the set of real function f such that $\int_E f(x)\,\bar{\mu}_h(dx)$ is define for all $\bar{\mu}_h \in M_H$.

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NAVIGATING THE FRONTIER: AI MODEL RISK IN FINANCIAL INSTITUTIONS - EMBRACING OPPORTUNITIES AMIDST CHALLENGES

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Amidst the rapid advancements in Machine Learning and Artificial Intelligence (AI) models, financial institutions are progressively integrating these innovations into domains such

as portfolio optimization, robo-advisory, and trading. Eclipsing conventional statistical methodologies in data comprehension, these models offer distinct opportunities intertwined with inherent complexities. However, the management of these intricate models presents notable challenges. The lack of proper data management, complex mathematical models, shortage of highly skilled professionals, and insufficient model validation mechanisms are the most prominent challenges that most financial institutions will face. The existence of an independent model risk management unit necessitates the implementation of systematic backtesting, scenario analysis, and capital quantification. This paper highlights these challenges and provides recommendations for financial institutions to swiftly transform their AI models.

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ENHANCING TAIL RISK MEASUREMENT: A PRACTICAL APPROACH TO MANAGING MODEL RISK OF TAIL RISK

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The tail market risk measurement has become one of the most crucial assignments of financial institutions' risk management units. Same time arious papers show that tail risk measures are especially sensitive to model misidentification. In this paper we address this practical problem. We propose model risk robust approach for measuring tail risk based on superposed risk measures. Superposed risk measures consider novel approach to measure market and model risk in a consistent way. This paper has two major goals. First, we investigate several practical superposed market risk measures under the extreme value theory. Second, we demonstrate our results via the case study of DAX 30 index

The modern risk management requires monitoring tail events that are rare in

frequency, but they are associated with large losses. Stock crashes, unexpected news in capital markets, political instability, oil shocks can lead to extreme unexpected losses. Extreme value theory is a framework that enables statistical modeling of tail events [1]. The extreme value theory was first introduced by [2] and [3]. In this paper we focus on the parametric approach of the extreme value theory based on the generalized Pareto (GP) distribution. [4] understand under model the probability distribution. Following this approach, we consider a complete model set that includes all probability distributions modelling extreme tail risk of financial position. Our case study shows that the tail risk measures are highly sensitive on model misspecification. Further, it is surprisingly robust and easy to construct the market tail risk measures that capture model risk. Finally, we show that model risk of tail market risk measures can be effectively managed.

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COMPARISON OF CONSTRAINED BAYESIAN AND CLSSICAL METHODS OF TESTING STATISTICAL HYPOTHESES IN SEQUENTIAL EXPERIMENTS

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The article focuses on the discussion of basic approaches to hypotheses testing in sequential experiments, which are Wald and Berger sequential tests and the test based on Constrained Bayesian Method (CBM). The positive and negative aspects of these approaches are considered and demonstrated on the basis of computed examples.

THE MINIMAL REVERSE ENTROPY MARTINGALE MEASURE IN THE TRINOMIAL FINANCIAL MODEL

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We consider the financial scheme where the evolution of risky asset is described by the equation $S_n = S_{n-1}(1 + \rho_n)$, here ρ_n, $1 \le n \le N$, is the sequence of independent identically distributed random variables which takes three values with known probabilities. In this incomplete market model the probability measure Q equivalent to the reference measure P and minimizing reverse entropy (negentropy) is constructed.

ON AN APPLICATION OF A SEQUENCE WITH CHAIN DEPENDENCE

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On the probability space (Ω, F, P), a stationary, in the narrow sense, two-component sequence $\{\xi_i, Y_i\}_{i\ge1}$ is considered. $\{\xi_i\}_{i\ge1}$ $(\xi_i : \Omega \to \Xi)$ is a finite, homogeneous, regular Markov chain. $\{Y_i\}_{i\ge1}$ $(Y_i : \Omega \to R^1)$ is a sequence with a chain dependence, the members of which are observations of the random variable Y with an unknown density. An estimate of this density has been constructed. The accuracy of the constructed estimate has been established.

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MONTE CARLO METHODS FOR COMPARING TWO POPULATION MEANS

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Monte Carlo methods are widely used in modern scientific research and technology. Using Monte Carlo methods is the best way to deeply understand and study complex statistical methods. In this paper, we will use Monte Carlo methods to create sampling distributions, confidence intervals, and test hypotheses for comparing two population means.

CUNSTRUCTIVE STOCHASTIC INTEGRAL REPRESENTATION OF SOME PATH-DEPENDENT BROWNIAN FUNCTIONAL

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One of the important properties of Ito's stochastic integral is the following: Ito's stochastic integral as a process from a square integrable adapted process is a martingale with respect to the natural filtration of Brownian motion. On the other hand, according to the well-known Clark (1970) formula, the inverse statement (so-called stochastic integral representation theorem) is also true, where, by virtue of Ocone ([1]), the integrand of the Itô integral is the optional projection of the stochastic derivative of the functional. Further, Glonti and Purttukhia ([2]) generalized the Clark-Ocone formula in case, when the functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method of finding the integrand. Here we consider some path-dependent Brownian functionals and derive constructive formulas for the stochastic integral representation.

For any non-negative integer $n \geq 0$ we denote $F(2n+1) := \left(\int_0^T B_s ds \right)^{2n+1}$.

Theorem. For any non-negative integer $n \geq 0$ the Brownian functional $G(2n+1) := [F(2n+1)]^+$ admits the following stochastic integral representation

$$G(2n+1) = EG(2n+1) + (2n+1)\int_0^T (T-t) \sum_{r=0}^{2n} C_{2n}^r \sigma^r y^{2n-r} I_r(\sigma, y)|_{y=\int_0^t (T-s)dB_s} dB_t ,$$

where

$$I_{2r-1}(\sigma, y) := \frac{1}{\sqrt{2\pi}} \int_{-y/\sigma}^{+\infty} x^{2r-1} \exp\{-\frac{x^2}{2}\} dx = \varphi(\frac{y}{\sigma}) \sum_{k=0}^{r-1} \frac{(2r-2)!!}{(2k)!!} (\frac{y}{\sigma})^{2k} ,$$

$$I_{2r}(\sigma, y) := \frac{1}{\sqrt{2\pi}} \int_{-y/\sigma}^{+\infty} x^{2r} \exp\{-\frac{x^2}{2}\} dx = (2r-1)!! \Phi(\frac{y}{\sigma}) + \varphi(\frac{y}{\sigma}) \sum_{k=1}^{r} \frac{(2r-1)!!}{(2k-1)!!} (-\frac{y}{\sigma})^{2k-1} ,$$

$\sigma^2 = (T-t)^3/3$ and Φ (resp. φ) standard normal distribution function (resp. standard normal distribution density function).

Proposition. $\left(\int_0^T B_s ds\right)^+ = \sqrt{\frac{T^3}{6\pi}} + \int_0^T (T-t)\Phi\left(\int_0^t (T-s)dB_s\right)dB_t$.

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A SIZE STRUCTURED MODEL OF FISH BASED ON A STOCHASTIC GROWTH EQUATION

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A stochastic growth equation is given whose solution density satisfies size-structured population growth equation and boundary conditions.

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SECTION OF MECHANICS OF CONTINUA

Chairs: George Jaiani
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A NEW HAMILTONIAN SEMI-ANALYTICAL APPROACH TO VIBRATION ANALYSIS OF ELASTIC MULTI-LAYERED COMPOSITE PLATES

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This contribution addresses the development of a new semi-analytical Hamiltonian method for the vibration analysis of elastic multi-layered composite plates. By performing a Legendre Transform, the classical Lagrangian functional is recast into a Hamiltonian one, so that the formulation has now the displacements and their transverse and normal stresses dual variables as arguments.

A numerical model developed in the framework of mixed formalism usually uses displacements and stresses dual variables as nodal unknowns. This implies a doubling of the size of the discretized equations in comparison with classical Lagrangian approach. This further makes the resulting computational burden a crucial point. To address this issue, the approach adopted in the present paper employs a well-established procedure used in sub-structuring techniques. When assembling the dynamic stiffness's of two adjacent layers, the unknowns are split in two subsets: one contains the unknowns associated with the common interface and the other contains the remaining unknowns associated with each sub-layer interiors. This partitioning induces a procedure that merges the two adjacent layers into one unique layer by eliminating the interior degrees of freedom while enforcing the interface continuity constraints.

Another key issue in using the Hamiltonian formalism in vibration analysis is the transcendental nature of the resulting equations once the interfaces constraints and the unloaded surfaces conditions are imposed. Solving transcendental equations inevitably raises the question of using adequate algorithm to compute the unknown natural frequencies. To alleviate this bottleneck, a Wittrick-William algorithm is adopted here, that allows counting the number of eigenvalues in a given interval, thanks to the Sturm Sequences property and the Schur Complement technique. Aside from solving the above difficulties, this paper provides a rigorous and systematic approach to deal with boundary conditions (BCs) so that arbitrary ones can be coped with. Thus, the proposed method overcomes the inherent limitations of the traditional State Space Method when dealing with non-simply supported BCs such as clamped or free end conditions.

To further assess the robustness of the proposed method, we investigate the numerical accuracy of the proposed approach by studying the vibration analysis of elastic multi-layered composite plates. Numerical examples are presented to show the efficiency of the proposed procedure.

SOME BOUNDARY PROBLEMS FOR THE CIRCLE WITH QUADRUPLE POROSITY

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In this talk the linear theory of elasticity for quadruple porosity isotropic materials with macro-, meso-, micro-, and submicropores is considered [1]. A two-dimensional system of equations of plane deformation is written in the complex form and its general solution is represented by means of three analytic functions of a complex variable and three solutions of Helmholtz equations. Concrete problem is solved for the circle.

Acknowledgement. This work was supported by Shota Rustaveli National Science Foundation of Georgia under the project MR-23-346.

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CONSTRUCTION AND INVESTIGATION OF NONCLASSICAL ONE-DIMENSIONAL MODELS OF THERMOELASTIC BARS WITH THREE PHASE-LAGS

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In the present paper, the Roy Choudhuri dynamic three-dimensional model [1] of the nonclassical theory of thermoelasticity is considered for a bar with a variable rectangular cross-section with thickness or width which may vanish on one of the ends of the bar. The butt end of the bar with positive area is clamped and temperature vanishes on it, and along the remaining part of the boundary, the densities of surface force and heat flux along the outward normal vector are given. Applying variational formulation of the initial-boundary value problem corresponding to the three-dimensional model a hierarchy of dynamic one-dimensional models of bar is constructed. The initial-boundary value problems corresponding to the obtained models are investigated in suitable spaces of vector-valued distributions with values in weighted Sobolev spaces defined on a one-dimensional domain. Moreover, the pointwise with respect to the time variable convergence of the sequence of vector-functions of three space variables restored from the solutions of the reduced one-dimensional problems to the solution of the original three-dimensional problem is proved and under additional regularity conditions, the rate of convergence is estimated.

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BOUNDARY VALUE PROBLEMS OF THE COUPLED THEORY OF PLANE ELASTICITY FOR AN INFINITE ARE WITH A CIRCULAR HOLE WITH DOUBLE-POROSITY

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In the present talk the linear model of elastic double-porosity materials in which the coupled phenomenon of the concepts of Darcy's law and the volume fractions is considered [1]. For the plane deformation the corresponding system of differential equations is written in a complex form and its general solution is presented with the use of three analytic functions of a complex variable and three solutions of the Helmholtz equation. The boundary value problems are solved for the infinite plane with a circular hole analytically.

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SOME DYNAMICAL PROBLEMS FOR CUSPED PRISMATIC SHELLS WITH THE THICKNESS DEPENDING ON TIME

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Cusped Prismatic Shells with the thickness as follows

$$2h(x_1, x_2, t) = h_0 x_2^{\kappa_1} t^{\kappa_2},\ h_0, \kappa_1, \kappa_2 = const > 0$$

is considered. Within the framework of I. Vekua's hierarchical models the setting of initial conditions is analyzed. Correct IBVP is investigated.

ON WELL-POSEDNESS OF IBVPS FOR CUSPED PRISMATIC SHELLS WITH THIKNESS DEPENDING ON TIME

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The title problem leads to investigation of IBVPs for singular hyperbolic equations and systems. That is connected with non-classical setting of initial conditions.

We consider prismatic shells of the following thicknesses

1. $2h(x_1,x_2,t)=h_0x_2^{\kappa_1}t^{\kappa_2}$, $h_0,\kappa_1,\kappa_2=const>0$ at an initial moment $t=0$ we have membrane; that afterwards $-\ t>0$ becomes a shell with the cusped edge for $x_2=0$;
2. $2h(x_1,x_2,t)=h_0x_2^{\kappa_1}+t^{\kappa_2}$ at an initial moment $t=0$ we have a shall with cusped edge for $x_2=0$, that afterwards $-\ t>0$ becomes a shall with a blunt edge for $x_2=0$.

Within the framework of I. Vekua's hierarchical models a model equation is analysed.

SOME BOUNDARY VALUE PROBLEMS FOR MULTI-CONNECTED PLATES WITH VOIDS

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The report considers static problems of tension-compression of elastic plates with voids. The system of two-dimensional equations describing the equilibrium of such bodies is derived from the three-dimensional linear Cowin-Nunziato model [1]. Dimensionality reduction is implemented using the Vekua method, which is called the method of sequential differentiation [2]. The general solution of the system of equations under consideration is represented using any two harmonic functions and the solution of the Helmholtz equation [3]. Based on the constructed general solution, using the method of fundamental solutions, approximate solutions of boundary value problems for perforated plates are obtained.

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PROBLEM OF THE PLANE THEORY OF VISCOUS ELASTICITY FOR A TRIANGULAR ARE WITH A CIRCULAR HOLE

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The problem of the plane theory of viscous elasticity for a triangular plate with a circular hole based on the Kelvin-Voigt model is considered. It is assumed that rigid absolutely smooth stamps act on the sides of the triangle with normal forces with a given main vector (or known normal displacements of the border points), and the inner boundary is loaded with uniformly distributed normal forces.

Based on the methods of conformal mappings and the theory of analytic functions, the problem is reduced to the Riemann-Hilbert boundary value problem for a circular ring, based on the solution of the latter, the required complex potentials are effectively constructed (in analytical form).

SOME PLANE PROBLEMS FOR THE COUPLED LINEAR THEORY OF ELASTICITY FOR POROUS MATERIALS

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The equations of linear theory for the elasticity of simple porous bodies are based on four types of interrelated ratios [1, 2]. The general solution of two-dimensional system of equations of plane deformation is represented by means of three analytic functions of a complex variable and solutions of Helmholtz equations. Concrete problems are solved for the circle and circular ring.

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EXPLICIT SOLUTION OF BOUNDARY VALUE PROBLEMS FOR THE ELASTIC CIRCLE WITH TRIPLE POROSITY

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The system of equilibrium equations of the linear theory of elasticity for isotropic materials with triple porosity has the form

$$\mu\Delta\mathbf{u}(\mathbf{x})+(\lambda+\mu)\,grad\,div\,\mathbf{u}(\mathbf{x})-grad\,(\boldsymbol{\beta}\mathbf{p})=0,$$
$$a_1\Delta p_1+a_{12}(p_2-p_1)+a_{13}(p_3-p_1)=0,$$
$$a_2\Delta p_2+a_{21}(p_1-p_2)+a_{23}(p_3-p_2)=0,$$
$$a_3\Delta p_3+a_{31}(p_1-p_3)+a_{32}(p_2-p_3)=0,$$

where $\mathbf{u}(x)=(u_1,u_2)$ is the displacement vector; $\boldsymbol{\beta}=(\beta_1,\beta_2,\beta_3)$ is the constants vector, $\mathbf{p}=(p_1,p_2,p_3)$, p_1, p_2 and p_3 are the pressures in pores; $\boldsymbol{\beta}\mathbf{p}=\beta_j p_j$; λ and μ are the Lame constants; a_j, a_{ij} are constants, characterizing the porosity of the body, $i, j=1,2,3$.

Static two-dimensional boundary value problems for an elastic porous circle with voids were solved. Special representations of the general solution of a system of differential equations through elementary functions have been constructed, which allow the original system of equations to be reduced to equations of a simple structure and facilitate the solution of the original problems. Solutions to problems are presented explicitly, in the form of absolutely and uniformly convergent series.

THE CONDUCTING LIQUID FLOW BETWEEN POROUS WALLS WITH HEAT TRANSFER

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The flow of viscous electroconductive fluid between porous walls with heat transfer has been studied when in perpendicular to walls the external homogeneous magnetic field is applied. The fluid flow is caused by pulsation fall and pulsation motion of porous walls. The temperature change in the porous pipe walls and in the pipe is carried out with pulsation. In the heat transfer equation the caused due result of the friction the energy dissipation is considered, as well as joule heat.

The physical characteristics of fluid flow are found.

STUDIES OF ELECTRICAL CONDUCTION PROCESSES IN BLOOD VESSELS USING CABLE THEORY

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Electrical conduction processes in the vasculature have traditionally been described using the cable theory, i.e., locally induced signals (membrane potential V_m) decaying passively along the arteriolar wall. The decay is typically quantified using the length-constant, λ, derived from the cable theory. However, the applicability of the cable theory to blood vessels depends on assumptions that are not necessarily fulfilled in small arteries and arterioles. It is known that arterioles consist of at least two cell layers—an endothelial cell (EC), and one or more smooth muscle cell (SMC) layers that are coupled by myoendothelial gap junctions (MEGJ). This work discusses arterioles with two cell layers and studies the change in membrane potential in the EC and SMC layers. The corresponding stationary problems are posed and solved analytically by the method of separation of variables. Numerical modeling of membrane potential propagation is performed using MATLAB software. Membrane isopotential contours, and 2D and 3D graphs corresponding to the obtained numerical results are presented.

SECTION OF MATHEMATICAL MODELING AND NUMERICAL ANALYSIS

Chairs: Teimuraz Davitashvili, Jemal Rogava, Tamaz Vashakmadze
Co-chair: Archil Papukashvili

THE CLASS OF NON-INTEGER VERTICES FOR THE INITIAL RELAXATION POLYTOPE OF THE LINEAR ORDER PROBLEM

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The NP– hard linear order problem is solved as an integer linear programming problem. In this case, an important role is played by the study of the initial relaxation polytope for the polytope of the linear order problem. In this paper, we consider a non-integer point and we prove that this point is a non-integer vertex for the initial relaxation polytope of the linear order problem.

TO APPLICATION OF MULTIPOINT METHOD

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We report the application of multipoint method for numerical solution of two-dim boundary value problems for linear and nonlinear differential equations of elliptic type by using the continuous analogue of the alternating direction method. We also give the results of numerical realizations, which in one-dim case would-be compared with A. Tikhonov-A. Samarskii and E. Volkov methods.

MATHEMATICAL AND COMPUTER MODELS OF THE TRANSFORMATION OF THE PROTO-KARTVELIAN POPULATION INTO THE SVAN AND GEORGIAN-COLCHIAN POPULATIONS

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Earlier, we proposed a scenario for the transformation of the Proto-Kartvelian population into the Georgian, Megrelian, Laz and Svan populations [1].

In [1], using mathematical and computer modeling, the first stage (5000-2500 BC) of the dynamics of the Proto-Kartvelian population was studied. In particular cases, exact

analytical solutions are obtained, and in the general case of variable coefficients - numerical solutions.

In [2], the second stage was studied (2500-1000 BC). For the dynamics of Pelasgians emigrating to Europe, an exact analytical solution has been obtained.

A two-dimensional dynamic system describing the interaction of the Svan and Georgian-Colchian populations has been studied in particular cases. In some cases, exact analytical solutions have been found, and in some cases theorems on the coexistence of these two populations have been proved, when complete assimilation of the Svan population does not occur.

This paper considers computer modeling of a nonlinear dynamic system with variable coefficients, which describes at the second stage the interaction of the Svan and Georgian-Colchian populations. Exponential and qualitative functions are taken as variable coefficients. A numerous computer experiment was carried out.

In the case of positive demographic factors of both populations, it is shown that the Svan population from approximately (0.5-0.6) million, despite the growing demographic factor, due to assimilation decreased to (0.3-0.4) million. The Georgian-Colchian population increased from (2.4-2.5) million to (3-4) million.

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MODELING VARIATIONS OF THE BUBA AND TBILISA GLACIERS AGAINST THE BACKDROP OF REGIONAL CLIMATE CHANGE

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Due to global warming, the glaciers of the Caucasus (Georgia) have also undergone significant changes. In particular, some of them have disappeared, and the majority are degraded. During the retreat, the area of glaciers decreased, but at the same time the total number of glaciers increased [1]. In general, glaciers play an important role on both a global and regional scale, and their reduction or disappearance would cause significant damage to natural ecosystems and economies. Since glacier-climate interactions are complex, non-linear processes, we use mathematical modeling to predict the adaptation of Georgia's glaciers to ongoing climate change [2,3]. In this paper, for the first time, using mathematical modeling, the process of melting of Buba and Tbilisi glaciers is evaluated against the background of regional climate change. Some simulation results are presented and analyzed.

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CLIMATE CHANGE VIA SOLAR - TERRESTRIAL INTERACTIONS

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The Earth's climate is determined by complex interactions among the Sun, oceans, atmosphere, cryosphere, land surface and biosphere. The Sun is the principal driving force for the Earth's weather and climate. The influence of solar activity on the Earth's global surface is determined due to temperature variation, which in turn drives the instabilities and is expressed via turbulent effects. Standard approaches to identify such connections are often based on correlations between the appropriate time series. Here we present a novel method of Granger causality, which can infer/detect relationships between any two fields. We compare Solar activity – climate connections via magnetic turbulence identified by correlation and Granger causality at different timescales.

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REALISATION THE m -LAYER SEMI-DISCRETE SCHEMES FOR APPROXIMATION SOLUTION THE ABSTRACT EVOLUTIONARY PROBLEMS

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In the present work we consider the approximate solution of the abstract evolutionary problems. We study the solution of the m -layer semi-discrete schemes for evolutionary problem by perturbation algorithm. We have formulated the algorithm of an approximate solution of initial problem for the general case, namely, when for the approximation of the first derivative we use the m -layer scheme. Solutions of these schemes are used for constructing an approximate solution of the original problem with high order approximation.

EXACT TRAVELING WAVE SOLUTION OF THE (2+1) - GARDNER EQUATION

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Traveling wave special exact solutions of (2+1)D nonlinear Gardner partial differential equation is obtained. The result is found through hyperbolic secant function and has spatially isolated structural forms. Appropriate graphical illustration is given.

THE 2D NAVIER-STOKES EQUATIONS FOR THE INCOMPRESSIBLE FLUDS

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We study the steady incompressible Newtonian 2D fluid flow in the finite or infinite area [1]. The corresponding system of Navier-Stokes equations (NSE) with the suitable boundary conditions is considered. The solutions of this system are obtained by means of the conformal mapping method [2, 3].

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ON THE APPROXIMATE SOLUTION OF THE J. BALL'S BEAM EQUATION IN THE CASE OF TEMPERATURE DEPENDENCE OF EFFECTIVE VISCOSITY

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The presented article is a direct continuation of articles [1]-[3] that considered an initial-boundary value problem for the J. Ball integro-differential equation, which describes the dynamic state of a beam. The solution is approximated utilizing the Galerkin method, stable symmetrical difference scheme and the Jacobi iteration method. In the articles [1]-[2] the algorithm has been approved by tests. In the article [3]-[4] and this paper presents the approximate solution to one practical problem. Particularly, the results of numerical computations of the initial-boundary value problem for an iron beam. In the presented article the case where the effective viscosity depends on the temperature is discussed. The results of numerical calculations qualitatively satisfactorily describe the process under consideration.

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A NUMERICAL ALGORITHM AND THE TEST RESULTS FOR THE NONLINEAR INHOMOGENEOUS KIRCHHOFF STRING EQUATION

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An initial boundary value problem for an integro-differential nonlinear inhomogeneous equation describing the string oscillation is considered. Using the Galerkin method and the Crank-Nicolson type difference scheme the solution is discretized with respect to the spatial and time variables. Thus, the problem is reduced to a system of nonlinear algebraic equations which is solved by the Jacobi iteration method and Cardano formula. The algorithm has been approved by tests and the results of calculations are presented.

ON THE CONVERGENCE OF A THREE-LAYER SEMI-DISCRETE SCHEME FOR THE NONLINEAR DYNAMIC STRING EQUATION OF KIRCHHOFF-TYPE WITH TIME-DEPENDENT COEFFICIENTS

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In this work, we consider the Cauchy problem for the nonlinear dynamic string equation of Kirchhoff-type with time-varying coefficients. To find an approximate solution to this initial boundary value problem, a symmetric three-layer semi-discrete scheme is designed concerning the temporal variable, wherein the value of a nonlinear term is evaluated at the middle node point. This nuance is important because it allows us to obtain an approximate solution by inverting the linear operator at each time layer. This approach reduces the considered non-linear integral-differential equation to a system of second-order linear ordinary differential equations. Issues of convergence of the discussed scheme have been studied. For numerical implementation, a spatial high-order scheme is developed. We have conducted several numerical experiments using the proposed algorithm for various test problems to validate its performance. The resulting numerical results are analyzed and found to align with the theoretical findings.

ON THE SOLUTION OF DIRICHLET GENERALIZED AND CLASSICAL SPATIAL HARMONIC PROBLEMS BY THE MPS IN NEIGHBORHOOD OF THE CONSIDERED DOMAIN SURFACE

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The Dirichlet generalized and classical harmonic problems for the special type irregular 3-sided pyramidal domain is considered. Under a generalized problem is meant the problem when a boundary function has a finite number of first kind discontinuity curves. In the considered case, edges of the pyramid are in a role of the mentioned curves and height of the pyramid passes through the vertex of the acute angle of the base. In spite of difficulty of the problem domain, the algorithm for numerical solution of the boundary problem is constructed, which consists of the following main steps: a) application of the method of probabilistic solution (MPS), which in its turn is based on a computer modeling of the Wiener process; b) finding the intersection point of the path of Wiener process simulation and the pyramid surface; c) development of a code for the numerical realization and checking the accuracy of calculated results; d) calculating the meaning of a sought for function at any chosen points, which lie in the neighborhood of the domain surface. For illustration a numerical example is considered and results are presented.

CONTENT

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