About one method of solution to the Cauchy problem for a singular poly-parabolic equation

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The importance of the study of high-order equations of the form $L^m u = 0,\ m = 1, 2, \ldots,$ pointed out A.V.Bitsadze [1], where L - linear differential operator of the second order, and $L^m = L^{m-1}L$ - m -th composition of this operator.

By R^n we denote the n - dimensional Euclidean space, $R^n_+ = \{x \in R^n : x_k > 0,\ k = \overline{1, n}\}$. In the given work, we consider an analogue of the Cauchy problem of determination in a domain $\Omega = \{(x, t) : x \in R^n_+,\ t > 0\}$ of a classical solution of the equation

$$L^m_\gamma(u) \equiv \left(\frac{\partial}{\partial t} - \Delta_B\right)^m u(x, t) = 0,\ \ (x, t) \in \Omega$$

satisfying conditions $\partial^k u/\partial t^k\Big|_{t=0} = \varphi_k(x),\ x \in R^n_+,\ \partial^{2k+1}u/\partial x_j^{2k+1}\Big|_{x_j=0} = 0,\quad t > 0,\ j = \overline{1, n},\ k = \overline{0, m-1}$, where $\Delta_B = \sum\limits_{k=1}^{n} B^{x_k}_{\gamma_k},\ B^{x_k}_{\gamma_k} = \partial^2/\partial x_k^2 + [(2\gamma_k + 1)/x_k]\partial/\partial x_k$ is the Bessel operator on the variable $x_k,\ \gamma_k \in R,\ \gamma_k > -1/2,\ k = \overline{1, n},\ m \in N,\ \varphi_k(x)$ are given smooth functions.

In order to solve the formulated problem, we use multidimensional generalized Erdélyi-Kober operator [2]. For this operator the following is true:

Theorem. Let $\alpha_k > 0,\ \eta_k \geq -1/2,\ k = \overline{1, n},\ m \in N,\ f(x, y) \in C^{2m,lm}_{x,y}(\Omega^n \times \Omega^s)$, $\lim\limits_{x_k \to 0} x_k^{2\eta_k+1}[\partial/\partial x_k]\left[B^{x_k}_{\eta_k}\right]^j f(x, y) = 0$, $j = \overline{0, m-1}$, $k = \overline{1,\, n}$. Then

$$\left[L_y \pm \sum_{k=1}^{n}\left(B^{x_k}_{\eta_k+\alpha_k} + \lambda_k^2\right)\right]^m J^{(x)}_\lambda\left(\begin{array}{c}\alpha\\ \eta\end{array}\right) f(x, y) = J^{(x)}_\lambda\left(\begin{array}{c}\alpha\\ \eta\end{array}\right)\left[L_y \pm \sum_{k=1}^{n} B^{x_k}_{\eta_k}\right]^m f(x, y)$$

here L_y is a linear differential operator of l order on a variable $y \in R^s$ and independent on $x \in R^n$, $\Omega^n = \prod\limits_{k=1}^{n}(0,\, b_k)$ be the Cartesian product, $b_k > 0, k = \overline{1, n}$.

Applying this theorem, the explicit formula of a solution to the Cauchy problem is constructed.

References

1. Bitsadze, A.V. *Equations of mixed type.* Izd. Akad. Nauk SSSR, Moscow, 1959.
2. Karimov Sh.T. *Multidimensional generalized Erdélyi-Kober operator and its application to solving Cauchy problems for differential equations with singular coefficients.* // Fract. Calc. Appl. Anal., Vol. 18, No 4 (2015), pp. 845-861.