The boundary conditions of Bitsadze - Samarskiy for elliptic-parabolic potential.

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In the paper A. Bitsadze - A. Samarskii [1] for the first time established the correct solvability of the new class of non-local problems for the Laplace equation which relates the value of the solution on parts of the border with its value in the inside domain. In view of theoretical and applied importance of this task Bitsadze - Samarskii received rapid development.

In [2]-[3] found the boundary conditions of Newton and heat potential for the Laplace equation and the heat equation. This paper shows that regular elliptic- parabolic potential satisfies Bitsadze - Samarskii's condition.

Let be $\Omega \in R^n$-a finite domain with smooth boundary ∂D and $D = \Omega \times [-a, b]$, $a > 0, b > 0$ is cylindrical domain.

We define in D elliptic-parabolic potential as follows:

$$u = l_B^{-1} f = \begin{cases} (\varepsilon_{n+1}^{-} * f)(x,t) + \int_\Omega G(x,t,\xi,0)\nu(\xi)d\xi, & t < 0 \\ (\varepsilon_{n+1}^{+} * f)(x,t) + (\varepsilon_{n+1}^{+} * \tau)(x,t) & t > 0 \end{cases}$$

where ε_{n+1}^{-} is fundamental solution and $G(x,t,\varepsilon,\eta)$ is Green's function of Holmgren problem for $n+1$ dimensional Laplace equation, $\varepsilon_{n+1}^{+}(x,t)$ is fundamental solution for $n+1$ dimensional heat equation. $\nu(x) = \frac{\partial u}{\partial t}(x,0)$, when $\tau(x) = u(x,0)$, and $*$ is the convolution operation. Thus unknown functions is determined by the smoothness

$u = l_B^{-1} f \in {}^{2+\alpha}(\overline{D}^{+}) \cap {}^{2+\alpha}(\overline{D}^{-})$, $D^{+} = D \cup t > 0$, $D^{-} = D \cap t < 0$.

In this paper found that the integral operator $u = l_B^{-1} f$ satisfies the boundary conditions of Bitsadze - Samarskii.

References

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