On a singular integral equation of the heat conductivity theory in infinite degenerating domain

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In the domain $G_\omega = \{(x;\ t) : t > 0,\ 0 < x < t^\omega,\ 1/2 < \omega < +\infty\}$ the following family of boundary value problems is considered:

$$\frac{\partial u(x,t)}{\partial t} - a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0,\ \{x,t\} \in G_\omega, \tag{1}$$

$$u(x,t)_{|\,x=0} = v_0(t), \tag{2}$$

$$u(x,t)_{|\,x=t^\omega} = v_1(t), \tag{3}$$

where $a = const > 0$, $v_0(t)$ and $v_1(t)$ are given. Solving problem (1) — (3) is reducing to studying the following integral equation [1], [2]

$$\varphi(t) - \int_0^t K(t,\,\tau)\,\varphi(\tau)\,d\tau = f(t) \equiv F[v_0,\,v_1],\ t > 0.$$

Property of the kernel:

$$\lim_{t\to+0} \int_0^t K(t,\,\tau)\,d\tau = 1.$$

The main result of the paper:

Theorem. Homogeneous boundary value problem (1)–(3) ($v_0(t) \equiv 0,\ v_1(t) \equiv 0$) has exactly $N_1 + N_2 + 1$ nontrivial solutions (up to a constant factor).

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References

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