NONLOCAL PROBLEM WITH DISCONTINOUS BONDING CONDITIONS FOR LINEAR PARABOLIC EQUATIONS OF MIXED TYPE

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*An existence and uniqueness of solution of a nonlocal problem are considered for linear parabolic equation of mixed type, in the right -parabolic portion the integrally condition connects of the breaking line and the right border of a domain. To prove of the uniqueness, we use the principle of extremes. The existence is proved by equivalent reduction to an existence of a solution of a singular integral equations system of normal type with zero index.

Let $\Omega = \Omega^+ \cup \Omega^- \cup S_0$ - limited domain of the plane (x,t), where $\Omega^+ = \{(x,t): 0 < x < 1, \quad 0 < t \leq 1\}$, $\Omega^- = \{(x,t): -1 < x < 1, 0 < t \leq 1\}$, $S_0 = \{(x,t):\ x = 0\ ,\ 0 < t < 1\}$.

In the domain Ω we consider the linear equation

$$U_{xx} + +c(x,t)U - sgnx \cdot U_t = f(x,t) \qquad \text{(I)}$$

where $f(x,t)\epsilon C^{(1,h)}(\Omega)$, $c(x,t)\epsilon C^{(0,h)}(\overline{\Omega})$ и $c(x,t) \leq 0\ in\ \Omega$.

Use the following notation:

$\Gamma_0 = \{(x,t): 0 \leq x \leq 1,\ \ t = 0\}$, $\Gamma_1 = \{(x,t): -1 \leq x \leq 0,\ \ t = 1\}$, $S_1^+ = \{(x,t): x = 1,\ \ 0 < t < 1\}$, $S_1^- = \{(x,t): x = -1,\ \ 0 < t < 1\}$. Let $x = \gamma_i(t), i = 1, \dots n$ – given functions from $C^1[0,1]$, wherein they do not alter its signs. $-1 \leq \gamma_i(t) \leq 0, i = 1, \dots n$

Problem A. Find a function $U(x,t)$ with the following properties:

1) $U(x,t) \in C(\overline{\Omega}/S_0) \cap C^1(\Omega \cup S_1^+ \cup S_1^- / S_0)$;
2) $U(x,t)$ is a regular solution of the equation (I) at Ω/S_0;
3) satisfies the conditions

$$U/_{\Gamma_0} = g_1(x),\ \ U/_{\Gamma_1} = g_2(x) \qquad (2)$$

$$\int_0^1 U(x,t)dx = \mu_1(t),\ U/_{S_1^-} = \mu_2(t),\ \ 0 < t < 1 \qquad (3)$$

$$U(-1,t) + \sum_{i=1}^{n} b_i(t) U(\gamma_i(t),t) = \mu_2(t),\ \ 0 \leq t \leq 1, \qquad (4)$$

4) and conditions of bonding in the following forms

$$U(+0,t) = U(-0,t) + \alpha_1(t),$$

$$U_x(+0,t) = \alpha_2(t) U_x(-0,t) + \alpha_3(t),$$

where $\alpha_1(t)\epsilon C[0,1] \cap C^1(0,1)$, $\alpha_2(t)$, $\alpha_3(t)\epsilon C[0,1]$, $\alpha_2(t) > 0$,
$b_i(t), i = 1, \dots n\ , f(x,t), g_i(x), \mu_i(t)\ \ i = 1,2$ – given sufficiently smooth functions, wherein

$$\mu_1(0) = \int_0^1 g_1(x)dx,\ \ \mu_2(1) = g_2(-1) + \sum_{i=1}^{n} b_i(1) g_2(\gamma_i(1))$$

Theorem. Let $\sum_{i=1}^{n} |b_i(t)| \leq 1$. The problem has no more than one solution.

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