SOME PLANE STRAIN PROBLEMS FOR ELASTIC MATERIALS WITH DOUBLE VOIDS

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Abstract

In this paper deals with the basic boundary value problems of the plane theory of elasticity for a circular ring with double voids. general solution of the governing system of equations of the plane strain is represented by means of two analytic functions of the complex variable and two solutions of Helmholtz equations. Using the obtained solutions, the problems of the plane theory of elasticity for a circular ring are solved analytically.

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1 Introduction

Nunziato and Cowin [1, 2] have established a theory for the behavior of single porous deformable materials in which the skeletal or matrix materials are elastic and the interstices are voids (vacuous pores). Recently, Ieşan and Quintanilla [3] have developed the theory of Nunziato and Cowin [2] for thermoelastic deformable materials with double porosity structure by using the mechanics of materials with voids. In addition, in these models the dependent variables are the displacement vector, the volume fractions of pores and fissures and the variation of temperature. Such materials include, in particular, rocks and soils, granulated and some other manufactured porous materials. The basic BVPs of this theory are studied in [4–7].

Furthermore, plane waves, uniqueness theorems and existence of eigenfrequencies in the theory of rigid bodies with double voids are investigated by Svanadze [5]. The existence of classical solutions in the external BVPs of steady vibrations of this theory is established by the same author in [8].

The problems of porous elasticity for materials with voids were considered in [9-19].

The present paper deals with plane strain problem for material with double voids. The boundary value problem is solved for a circular ring.

2 Basic equations for materials with double voids

Let $x = (x_1; x_2; x_3)$ be a point of the Euclidean three dimensional space R^3. We assume that the subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate.

The governing equations of the theory of isotropic and homogeneous elastic materials with double voids can be expressed in the following form [3]:

• Equations of equilibrium

$$\begin{array}{l} t_{ji,j} + \rho_0 f_i = 0, \;\; i,j = 1,2,3, \\ \sigma_{j,j} + \xi + \rho_0 g = 0, \\ \tau_{j,j} + \zeta + \rho_0 l = 0, \end{array} \tag{1}$$

where t_{ij} is the symmetric stress tensor, f_i is the body force per unit mass, ρ_0 is the mass density, σ_i and τ_i are the equilibrated stress vectors, ξ and ζ are the intrinsic equilibrated body forces, g is the extrinsic equilibrated body force per unit mass associated to macro pores, l is the extrinsic equilibrated body force per unit mass associated to fissures.

• Constitutive equations

$$\begin{array}{l} t_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi, \\ \sigma_i = \alpha\varphi_{,i}, + b_1\psi_{,i}, \\ \tau_i = b_1\varphi_{,i}, + \gamma\psi_{,i}, \\ \xi = -be_{kk} - \alpha_1\varphi - \alpha_3\psi, \\ \zeta = -de_{kk} - \alpha_3\varphi - \alpha_2\psi, \end{array} \tag{2}$$

where λ and μ are the Lamé constants, α, b, d, b_1, α_1, α_2 and α_3 are the constants characterizing the body porosity, δ_{ij} is the Kronecker delta, φ is a changes of volume fraction corresponding to pores, ψ is a a changes of volume fraction corresponding to fissures, e_{ij} is the strain tensor and

$$e_{ij} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{3}$$

where u_i, $i = 1,2,3$ are the components of the displacement vector.

The constitutive equations also meet some other conditions, following from physical considerations

$$\begin{array}{l}\mu>0,\ \ 3\lambda+2\mu>0,\ \ \alpha_2>0,\ \ \alpha_1\alpha_2-\alpha_3^2>0,\ \ \alpha>0,\\(3\lambda+2\mu)(\alpha_1\alpha_2-\alpha_3^2)>3(\alpha_1 d^2+\alpha_2 b^2-2\alpha_3 bd),\ \ \alpha\gamma>b_1^2.\end{array}\tag{4}$$

Substituting (2) and (3) into (1) we obtain equations with respect to the components of the displacement and the functions φ and ψ

$$\begin{array}{l}\mu\tilde{\Delta}u_i+(\lambda+\mu)\partial_i\Theta+b\partial_i\varphi+d\partial_i\psi=0,\ \ j=1,2,3\\(\alpha\tilde{\Delta}-\alpha_1)\varphi+(b_1\tilde{\Delta}-\alpha_3)\psi-b\Theta=0,\\(b_1\tilde{\Delta}-\alpha_3)\varphi+(\gamma\tilde{\Delta}-\alpha_2)\psi-d\Theta=0,\end{array}\tag{5}$$

where $\partial_i\equiv\frac{\partial}{\partial x_i}$, $\Theta=\partial_k u_k$, $\Delta\equiv\partial_{11}+\partial_{22}+\partial_{33}$ is the three-dimensional Laplace operator, $f_i=g=l=0$.

3 Basic (governing) equations of the plane deformation

In the case of plane deformation $u_3=0$ while the functions u_1, u_2, φ and ψ do not depend on the coordinate x_3 [20].

As it follows from formulas (2) and (3), in the case of plane strain

$$t_{k3}=t_{3k}=0,\ \ \sigma_3=0,\ \ \tau_3=0,\ \ k=1,2.$$

In this case from system (5) we obtain the following system of governing equations of statics with respect to the functions u_1, u_2 and φ, ψ

$$\begin{array}{l}\mu\Delta u_k+(\lambda+\mu)\partial_k\theta+b\partial_k\varphi+d\partial_k\psi=0,\ \ k=1,2\\(\alpha\Delta-\alpha_1)\varphi+(b_1\Delta-\alpha_3)\psi-b\theta=0,\\(b_1\Delta-\alpha_3)\varphi+(\gamma\Delta-\alpha_2)\psi-d\theta=0,\end{array}\tag{6}$$

Note that $\Delta\equiv\partial_{11}+\partial_{22}$ is the two-dimensional Laplace operator.

On the plane Ox_1x_2, we introduce the complex variable $z=x_1+ix_2=re^{i\vartheta}$, $(i^2=-1)$ and the operators $\partial_z=0.5(\partial_1-i\partial_2)$, $\partial_{\bar{z}}=0.5(\partial_1+i\partial_2)$, $\bar{z}=x_1-ix_2$, and $\Delta=4\partial_z\partial_{\bar{z}}$.

We can rewrite system (6) in the complex form

$$\begin{array}{l}2\mu\partial_{\bar{z}}\partial_z u_++(\lambda+\mu)\partial_{\bar{z}}\theta+b\partial_{\bar{z}}\varphi+d\partial_{\bar{z}}\psi=0,\\(\alpha\Delta-\alpha_1)\varphi+(b_1\Delta-\alpha_3)\psi-b\theta=0,\\(b_1\Delta-\alpha_3)\varphi+(\gamma\Delta-\alpha_2)\psi-d\theta=0.\end{array}\tag{7}$$

The general solution of the system (7) is represented as follows:

$$2\mu u_+=\varkappa f(z)-z\overline{f'(z)}-\overline{g(z)}-p_1\partial_{\bar{z}}\chi_1(z,\bar{z})-p_2\partial_{\bar{z}}\chi_2(z,\bar{z}),$$

$$\varphi = l_{11}\chi_1(z,\bar z) + l_{12}\chi_2(z,\bar z) - E_1(f'(z) + \overline{f'(z)}), \tag{8}$$
$$\psi = l_{21}\chi_1(z,\bar z) + l_{22}\chi_2(z,\bar z) - E_2(f'(z) + \overline{f'(z)}).$$

where $f(z)$ and $g(z)$ is an arbitrary analytic function of z, $\chi_1(z,\bar z)$ and $\chi_2(z,\bar z)$ are a general solutions of the Helmholtz equations

$$\Delta\chi(z,\bar z) - \kappa_1\chi(z,\bar z) = 0,$$

$$\Delta\chi(z,\bar z) - \kappa_2\chi(z,\bar z) = 0.$$

where κ_α are eigenvalues and $(l_{11},\ l_{21})$, $(l_{12},\ l_{22})$ are eigenvectors of the matrix C

$$C = \begin{pmatrix} \alpha & b_1 \\ b_1 & \gamma \end{pmatrix}^{-1} \cdot \begin{pmatrix} \alpha_1 - \frac{b^2}{\lambda+2\mu} & \alpha_3 - \frac{bd}{\lambda+2\mu} \\ \alpha_3 - \frac{bd}{\lambda+2\mu} & \alpha_2 - \frac{d^2}{\lambda+2\mu} \end{pmatrix},$$

and

$$\varkappa = \frac{\lambda + 3\mu + 2\mu(bS_1 + dS_2)}{\lambda + \mu - 2\mu(bS_1 + dS_2)},$$

$$p_1 = \frac{4\mu(bl_{11} + dl_{21})}{\kappa_1(\lambda + 2\mu)},$$

$$p_2 = \frac{4\mu(bl_{12} + dl_{22})}{\kappa_2(\lambda + 2\mu)},$$

$$E_1 = R \cdot S_1, \quad E_2 = R \cdot S_2,$$

$$R = \frac{2(\lambda + 2\mu)}{\lambda + \mu - 2\mu(b_1S_1 + b_2S_2)},$$

$$S_1 = \frac{b\alpha_2 - d\alpha_3}{2((\alpha_1\alpha_2 - \alpha_3^2)(\lambda + 2\mu) - \alpha_1 d^2 - \alpha_2 b^2 + 2\alpha_3 bd)},$$

$$S_2 = \frac{d\alpha_1 - b\alpha_3}{2((\alpha_1\alpha_2 - \alpha_3^2)(\lambda + 2\mu) - \alpha_1 d^2 - \alpha_2 b^2 + 2\alpha_3 bd)}.$$

4 The boundary value problem for a circular ring

In this section, we consider a boundary value problem for a concentric circular ring with radius R_1 and R_2 (Fig. 1). The origin of coordinates is at the center of the circle.

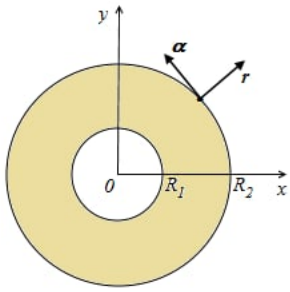

Figure 1: The circular ring.

We consider the following problem:

$$u_+ = \begin{cases} A', & r = R_1, \\ A'', & r = R_2, \end{cases} \tag{9}$$

$$\varphi = \begin{cases} B', & r = R_1, \\ B'', & r = R_2, \end{cases} \tag{10}$$

$$\psi = \begin{cases} C', & r = R_1, \\ C'', & r = R_2, \end{cases} \tag{11}$$

where A', A'', B' B'', C' and C'' are sufficiently smooth functions.

The analytic functions $f(z)$, $g(z)$ and the metaharmonic functions $\chi_1(z,\overline{z})$, $\chi_2(z,\overline{z})$ are represented as the following series [21]

$$\begin{gathered} f(z) = T_1 z \ln z + T_2 \ln z + \sum_{-\infty}^{\infty} a_n z^n, \\ g(z) = T_3 \ln z + \sum_{-\infty}^{\infty} b_n z^n, \end{gathered} \tag{12}$$

$$\begin{aligned} \chi_1(z,\overline{z}) &= \sum_{-\infty}^{+\infty} \left(\alpha_n' I_n(\sqrt{\kappa_1} r) + \alpha_n'' K_n(\sqrt{\kappa_1} r)\right) e^{in\alpha}, \\ \chi_2(z,\overline{z}) &= \sum_{-\infty}^{+\infty} \left(\beta_n' I_n(\sqrt{\kappa_2} r) + \beta_n'' K_n(\sqrt{\kappa_2} r)\right) e^{in\alpha}, \end{aligned} \tag{13}$$

where $I_n(\cdot)$ and $K_n(\cdot)$ are the modified Bessel functions of the first and second kind of n-th order.

Expand the function A', A'', B', B'', C' and C'', given on $r = R_1$ and $z = R_2$, in a complex Fourier series

$$\begin{gathered} A' = \sum_{-\infty}^{+\infty} A'_n e^{in\alpha}, \quad A'' = \sum_{-\infty}^{+\infty} A''_n e^{in\alpha}, \\ B' = \sum_{-\infty}^{+\infty} B'_n e^{in\alpha}, \quad B'' = \sum_{-\infty}^{+\infty} B''_n e^{in\alpha}, \\ C' = \sum_{-\infty}^{+\infty} C'_n e^{in\alpha}, \quad C'' = \sum_{-\infty}^{+\infty} C''_n e^{in\alpha}. \end{gathered} \tag{14}$$

Substituting (12), (13) in (8), taking into account the boundary conditions (9-11), (14) and assuming that the series converge on the circumference $r = R_1$ and $r = R_2$, one finds

$$\begin{gathered} \varkappa\left(T_1 R_1(\ln R_1 + i\alpha)e^{i\alpha} + T_2(\ln R_1 + i\alpha) + \sum_{-\infty}^{\infty} R_1^n a_n e^{in\alpha}\right) \\ -(\ln R_1 - i\alpha + 1)R_1 T_1 e^{i\alpha} - T_2 e^{2i\alpha} - \sum_{-\infty}^{\infty} n R_1^n \bar{a}_n e^{-i(n-2)\alpha} \\ -\frac{p_1\sqrt{\kappa_1}}{2}\sum_{-\infty}^{\infty}\left(I_{n+1}(\sqrt{\kappa_1}R_1)\alpha'_n - K_{n+1}(\sqrt{\kappa_1}R_1)\alpha''_n\right)e^{i(n+1)\alpha} \\ -\frac{p_2\sqrt{\kappa_2}}{2}\sum_{-\infty}^{\infty}\left(I_{n+1}(\sqrt{\kappa_2}R_1)\beta'_n - K_{n+1}(\sqrt{\kappa_2}R_1)\beta''_n\right)e^{i(n+1)\alpha} \\ -T_3(\ln R_1 - i\alpha) - \sum_{-\infty}^{\infty} R_1^n \bar{b}_n e^{-in\alpha} = \sum_{-\infty}^{\infty} A'_n e^{in\alpha}, \end{gathered} \tag{15}$$

$$\begin{gathered} \varkappa\left(T_1 R_2(\ln R_2 + i\alpha)e^{i\alpha} + T_2(\ln R_2 + i\alpha) + \sum_{-\infty}^{\infty} R_2^n a_n e^{in\alpha}\right) \\ -(\ln R_2 - i\alpha + 1)R_2 T_1 e^{i\alpha} - T_2 e^{2i\alpha} - \sum_{-\infty}^{\infty} n R_2^n \bar{a}_n e^{-i(n-2)\alpha} \\ -\frac{p_1\sqrt{\kappa_1}}{2}\sum_{-\infty}^{\infty}\left(I_{n+1}(\sqrt{\kappa_1}R_2)\alpha'_n - K_{n+1}(\sqrt{\kappa_1}R_2)\alpha''_n\right)e^{i(n+1)\alpha} \\ -\frac{p_2\sqrt{\kappa_2}}{2}\sum_{-\infty}^{\infty}\left(I_{n+1}(\sqrt{\kappa_2}R_2)\beta'_n - K_{n+1}(\sqrt{\kappa_2}R_2)\beta''_n\right)e^{i(n+1)\alpha} \\ -T_3(\ln R_2 - i\alpha) - \sum_{-\infty}^{\infty} R_2^n \bar{b}_n e^{-in\alpha} = \sum_{-\infty}^{\infty} A''_n e^{in\alpha}, \end{gathered} \tag{16}$$

$$
\begin{aligned}
&l_{11}\sum_{-\infty}^{+\infty}\left(\alpha_n' I_n(\sqrt{\kappa_1}R_1)+\alpha_n'' K_n(\sqrt{\kappa_1}R_1)\right)e^{in\alpha}+\frac{T_2}{R_1}\left(e^{i\alpha}+e^{-i\alpha}\right)\\
&+l_{12}\sum_{-\infty}^{+\infty}\left(\beta_n' I_n(\sqrt{\kappa_2}R_1)+\beta_n'' K_n(\sqrt{\kappa_2}R_1)\right)e^{in\alpha}+2T_1\left(\ln R_1+1\right)\\
&\quad -E_1\sum_{-\infty}^{\infty} nR_1^{n-1}\left(a_n e^{i(n-1)\alpha}+\bar{a}_n e^{-i(n-1)\alpha}\right)=\sum_{-\infty}^{\infty}B_n' e^{in\alpha},
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
&l_{11}\sum_{-\infty}^{+\infty}\left(\alpha_n' I_n(\sqrt{\kappa_1}R_2)+\alpha_n'' K_n(\sqrt{\kappa_1}R_2)\right)e^{in\alpha}+\frac{T_2}{R_2}\left(e^{i\alpha}+e^{-i\alpha}\right)\\
&+l_{12}\sum_{-\infty}^{+\infty}\left(\beta_n' I_n(\sqrt{\kappa_2}R_2)+\beta_n'' K_n(\sqrt{\kappa_2}R_2)\right)e^{in\alpha}+2T_1\left(\ln R_2+1\right)\\
&\quad -E_1\sum_{-\infty}^{\infty} nR_2^{n-1}\left(a_n e^{i(n-1)\alpha}+\bar{a}_n e^{-i(n-1)\alpha}\right)=\sum_{-\infty}^{\infty}B_n'' e^{in\alpha},
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
&l_{21}\sum_{-\infty}^{+\infty}\left(\alpha_n' I_n(\sqrt{\kappa_1}R_1)+\alpha_n'' K_n(\sqrt{\kappa_1}R_1)\right)e^{in\alpha}+\frac{T_2}{R_1}\left(e^{i\alpha}+e^{-i\alpha}\right)\\
&+l_{22}\sum_{-\infty}^{+\infty}\left(\beta_n' I_n(\sqrt{\kappa_2}R_1)+\beta_n'' K_n(\sqrt{\kappa_2}R_1)\right)e^{in\alpha}+2T_1\left(\ln R_1+1\right)\\
&\quad -E_2\sum_{-\infty}^{\infty} nR_1^{n-1}\left(a_n e^{i(n-1)\alpha}+\bar{a}_n e^{-i(n-1)\alpha}\right)=\sum_{-\infty}^{\infty}C_n' e^{in\alpha},
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
&l_{21}\sum_{-\infty}^{+\infty}\left(\alpha_n' I_n(\sqrt{\kappa_1}R_2)+\alpha_n'' K_n(\sqrt{\kappa_1}R_2)\right)e^{in\alpha}+\frac{T_2}{R_2}\left(e^{i\alpha}+e^{-i\alpha}\right)\\
&+l_{22}\sum_{-\infty}^{+\infty}\left(\beta_n' I_n(\sqrt{\kappa_2}R_2)+\beta_n'' K_n(\sqrt{\kappa_2}R_2)\right)e^{in\alpha}+2T_1\left(\ln R_2+1\right)\\
&\quad -E_2\sum_{-\infty}^{\infty} nR_2^{n-1}\left(a_n e^{i(n-1)\alpha}+\bar{a}_n e^{-i(n-1)\alpha}\right)=\sum_{-\infty}^{\infty}C_n'' e^{in\alpha}.
\end{aligned} \tag{20}
$$

We use the condition of single-valuedness of the displacements which in the present case are expressed as

$$T_1=0,\quad T_2=0,\quad T_3=0.$$

Comparing in (15)–(20) the coefficients of $e^{in\alpha}$ we have

$$
\begin{aligned}
&\varkappa R_1^n a_n+(n-2)R_1^{-n+2}\bar{a}_{-n+2}-R_1^{-n}\bar{b}_{-n}-\frac{p_1\sqrt{\kappa_1}}{2}I_n(\sqrt{\kappa_1}R_1)\alpha_{n-1}'\\
&+\frac{p_1\sqrt{\kappa_1}}{2}K_n(\sqrt{\kappa_1}R_1)\alpha_{n-1}''-\frac{p_2\sqrt{\kappa_2}}{2}I_n(\sqrt{\kappa_2}R_1)\beta_{n-1}'\\
&+\frac{p_2\sqrt{\kappa_2}}{2}K_n(\sqrt{\kappa_2}R_1)\beta_{n-1}''=A_n',
\end{aligned} \tag{21}
$$

$$\begin{aligned}&\varkappa R_2^n a_n+(n-2)R_2^{-n+2}\bar{a}_{-n+2}-R_2^{-n}\bar{b}_{-n}-\frac{p_1\sqrt{\kappa_1}}{2}I_n(\sqrt{\kappa_1}R_2)\alpha'_{n-1}\\&+\frac{p_1\sqrt{\kappa_1}}{2}K_n(\sqrt{\kappa_1}R_2)\alpha''_{n-1}-\frac{p_2\sqrt{\kappa_2}}{2}I_n(\sqrt{\kappa_2}R_2)\beta'_{n-1}\\&+\frac{p_2\sqrt{\kappa_2}}{2}K_n(\sqrt{\kappa_2}R_2)\beta''_{n-1}=A''_n,\end{aligned}\tag{22}$$

$$\begin{aligned}&l_{11}I_n(\sqrt{\kappa_1}R_1)\alpha'_n+l_{11}K_n(\sqrt{\kappa_1}R_1)\alpha''_n+l_{12}I_n(\sqrt{\kappa_2}R_1)\beta'_n+\\&l_{12}K_n(\sqrt{\kappa_2}R_1)\beta''_n-(n+1)R_1^nE_1a_{n+1}+(n-1)R_1^{-n}E_1\bar{a}_{1-n}=B'_n,\end{aligned}\tag{23}$$

$$\begin{aligned}&l_{11}I_n(\sqrt{\kappa_1}R_2)\alpha'_n+l_{11}K_n(\sqrt{\kappa_1}R_2)\alpha''_n+l_{12}I_n(\sqrt{\kappa_2}R_2)\beta'_n+\\&l_{12}K_n(\sqrt{\kappa_2}R_2)\beta''_n-(n+1)R_2^nE_1a_{n+1}+(n-1)R_2^{-n}E_1\bar{a}_{1-n}=B''_n,\end{aligned}\tag{24}$$

$$\begin{aligned}&l_{21}I_n(\sqrt{\kappa_1}R_1)\alpha'_n+l_{21}K_n(\sqrt{\kappa_1}R_1)\alpha''_n+l_{22}I_n(\sqrt{\kappa_2}R_1)\beta'_n+\\&l_{22}K_n(\sqrt{\kappa_2}R_1)\beta''_n-(n+1)R_1^nE_1a_{n+1}+(n-1)R_1^{-n}E_1\bar{a}_{1-n}=C'_n,\end{aligned}\tag{25}$$

$$\begin{aligned}&l_{21}I_n(\sqrt{\kappa_1}R_2)\alpha'_n+l_{21}K_n(\sqrt{\kappa_1}R_2)\alpha''_n+l_{22}I_n(\sqrt{\kappa_2}R_2)\beta'_n+\\&l_{22}K_n(\sqrt{\kappa_2}R_2)\beta''_n-(n+1)R_2^nE_1a_{n+1}+(n-1)R_2^{-n}E_1\bar{a}_{1-n}=C''_n,\end{aligned}\tag{26}$$

All coefficients in series (12)-(13) are found by solving (21)-(26). It is easy to prove the absolute and uniform convergence of the series obtained in the the circle (including the contours) when the functions set on the boundaries have sufficient smoothness.

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