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BOUNDARY VALUE PROBLEMS FOR AN INFINITE STRIP WITH VOIDS

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Abstract. The object of the present paper is to construct explicit solutions of BVPs for an isotropic elastic infinite strip with voids. General representations of a regular solution of a system of equations for a homogeneous isotropic medium with voids are constructed by means of the elementary (harmonic, bi-harmonic and meta-harmonic) functions. Using the Fourier method, the basic BVPs are solved effectively (in quadratures) for the infinite strip.

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1. Introduction

Elastic materials with porosity are very important and have applications in many fields of engineering, such as the petroleum industry, material science and biology.

The theory of porous materials with voids is used for investigating various types of geological and biological materials for which classical theory of elasticity is not adequate. This theory studies the behavior of elastic porous materials like the rock, the bone and the manufactured porous materials. The voids are assumed to contain nothing of mechanical or energetic significance.

Recently the linear theory of elasticity for materials with voids has been expanding and developing in different directions. For example, the non-linear version of elastic materials with voids was proposed by Nunziato and Cowin [1] and the linear version was developed by Cowin and Nunziato [2] to study mathematically the mechanical behavior of porous solids. Ieşan in [3] established a variational theory for thermoelastic materials with voids. In [4, 5] Ciarletta and Scalia studied a linear theory of thermoelasticity for materials with voids and established uniqueness and reciprocal theorems. In [6] Ieşan and Quintanilla have developed the theory of Nunziato and Cowin for thermoelastic deformable materials with double porosity structure by using the mechanics of materials with voids.

Many problem have been considered for elastic materials with voids by many authors (some of those articles can be seen, for instance, in [7-23] and the references cited therein).

In the present paper we consider the elastic infinite strip with voids. General representations of a regular solution of a system of equations for a homogeneous isotropic medium with voids are constructed by means of the elementary (harmonic, bi-harmonic and meta-harmonic) functions. Using the Fourier method, the basic BVPs are solved effectively (in quadratures) for the infinite strip.

2. Basic equations and boundary value problems

Let D denote an infinite strip with voids $0 < x_2 < H, \quad -\infty < x_1 < +\infty$. Let $\mathbf{x} := (x_1, x_2) \in D$. $\partial_{\mathbf{x}} := \left(\dfrac{\partial}{\partial x_1}, \dfrac{\partial}{\partial x_2}\right)$. The boundaries of D are $x_2 = 0$ and $x_2 = H$.

We say that a body is subject to a plane deformation if the component u_3 of the displacements vector $u(u_1, u_2, u_3)$ vanish and the other components are functions of the variables only x_1, x_2.Then the basic system of linearized equations of motion in the theory of elasticity for homogeneous and isotropic materials with voids structure can be written as

$$\begin{cases} \mu\Delta\mathbf{u} + (\lambda+\mu)\mathrm{graddiv}\mathbf{u} + \beta\mathrm{grad}\varphi = 0, \\ (\alpha\Delta - \varsigma)\varphi - \beta\,\mathrm{div}\mathbf{u} = 0, \end{cases} \tag{1}$$

where $\mathbf{u} = (u_1, u_2)^\top$ is the displacement vector in a solid, φ is the changes of volume fractions from the reference configuration. λ, μ, β, α, ς, are constitutive coefficients, Δ is the 2D Laplace operator. Throughout this paper the superscript $^\top$ denotes transposition.

For the equation (1) the basic BVPs for an infinite strip are formulated as follow: Find a regular function $\mathbf{U}(x)$, satisfying in D the system (1), when on the boundary of the domain D one of the the following conditions are given:

Problem 1.

$$\mathbf{u}^+ = \mathbf{f}^+(x_1), \quad \varphi = f_3^+, \quad x_2 = 0, \quad \mathbf{u}^- = \mathbf{F}^-(x_1), \quad \varphi = F_3^-, \quad x_2 = H,$$

Problem 2.

$$[\mathbf{TU}]^+ = \mathbf{f}^+(x_1), \quad \varphi = f_3^+, \quad x_2 = 0, \quad [\mathbf{TU}]^- = \mathbf{F}^-(x_1), \quad \varphi = F_3^- \quad x_2 = H,$$

where $\mathbf{T}(\partial\mathbf{x},\mathbf{n})\mathbf{u}$ is the following vector

$$\mathbf{T}(\partial\mathbf{x},\mathbf{n})\mathbf{u} := \begin{pmatrix} \mu\dfrac{\partial}{\partial x_2} & \mu\dfrac{\partial}{\partial x_1} \\ \lambda\dfrac{\partial}{\partial x_1} & \mu_0\dfrac{\partial}{\partial x_2} \end{pmatrix} \mathbf{u} + \beta\mathbf{n}\varphi,$$

$$\mu_0 = \lambda + 2\mu, \quad \mathbf{n} = (0, 1).$$

The vectors functions $\mathbf{f}(f_1, f_2)$, $\mathbf{F}(F_1, F_2)$ and the functions f_3, F_3 are given functions on the boundary D, satisfying certain smoothness conditions and also the conditions at infinity.

3. A representation of regular solutions

The following theorems hold:

Theorem 1. *If $\boldsymbol{U} := (\boldsymbol{u}, \varphi)$ is a regular solution of the homogeneous system (1) then $\boldsymbol{u}$, and φ satisfy the equations*

$$\begin{cases} \Delta\Delta(\Delta - s_1^2)\mathbf{u} = 0, \\ \Delta(\Delta - s_1^2)\varphi = 0, \end{cases} \tag{2}$$

where

$$s_1^2 = \frac{\mu_0 \varsigma - \beta^2}{\mu_0 \alpha}.$$

Proof. From (1) it follows that

$$\mathrm{div}\mathbf{u} = \frac{1}{\beta}[\alpha\Delta - \zeta]\varphi. \tag{3}$$

Applying the operator div to equation $(1)_1$ and taking into account (3), we obtain

$$\Delta(\Delta - s_1^2)\varphi = 0 \tag{4}$$

Further, applying the operator $\Delta(\Delta - s_1^2)$ to equation $(1)_1$, and using the latter relation we obtain

$$\Delta\Delta(\Delta - s_1^2)\mathbf{u} = 0,$$

which completes the proof.

Theorem 2. *The regular solution $\boldsymbol{U} = (\boldsymbol{u}, \varphi)$ of system (1) admits in the domain of regularity a representation*

$$\begin{aligned} &\mathbf{u} = \mathbf{\Psi} - \mathrm{grad}\left[a_0 h_0 + \frac{\beta h_1}{\mu_0 s_1^2}\right], \\ &\varphi = h + h_1, \quad \mathrm{div}\mathbf{u} = -\frac{\varsigma}{\beta}h - \frac{\beta}{\mu_0}h_1, \end{aligned} \tag{5}$$

where

$$a_0 = \frac{\beta^2 - (\lambda + \mu)\varsigma}{\mu\beta}, \quad \mu_0 = \lambda + 2\mu, \quad \Delta h = 0, \quad (\Delta - s_1^2)h_1 = 0,$$

the functions $\mathbf{\Psi}$ and h_0 are chosen so that

$$\Delta h_0 = h, \quad \mathrm{div}\mathbf{\Psi} = mh, \quad m = \frac{\beta^2 - \mu_0\varsigma}{\mu\beta}.$$

Proof. Since φ is the solution of equation (4), we can write [26]

$$\varphi = h + h_1, \tag{6}$$

where

$$\Delta h = 0, \quad (\Delta - s_1^2)h_1 = 0.$$

Further, substituting (6) into (3), we conclude

$$\mathrm{div}\mathbf{u} = -\frac{\varsigma}{\beta}h - \frac{\beta}{\mu_0}h_1.$$

Let us assume that the functions h and h_1 are known, when $\mathbf{x} \in D$, then from (1), after obvious transformations, for $\mathbf{u}$ we get the following nonhomogeneous equation

$$\Delta\mathbf{u} = -\mathrm{grad}\left[a_0 h + \frac{\beta}{\mu}h_1\right], \tag{7}$$

The general solution of equation (7) has the form

$$\mathbf{u} = \mathbf{\Psi} + \mathbf{u}_0,$$

where $\mathbf{u}_0$ is a particular solution of equation (7)

$$\mathbf{u}_0 = -\mathrm{grad}\left[a_0 h_0 + \frac{\beta}{\mu_0 s_1^2}h_1\right], \tag{8}$$

The functions $\mathbf{\Psi}$ and h_0 satisfy the following conditions

$$\Delta\mathbf{\Psi} = 0, \quad \Delta h_0 = h, \quad \mathrm{div}\mathbf{\Psi} = mh.$$

Thus, we have obtained the general solution of system (1) in the form (5).

Here and in what follows we assume that the prescribed on the boundaries ($x_2 = 0$ and $x_2 = H$.) functions be representable by the Fourier integrals (see [27])

$$\widehat{\mathbf{F}}(x_1) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\mathbf{F}(\xi)\exp(-ix_1\xi)d\xi$$

and the inversion formula

$$\mathbf{F}(\xi) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\widehat{\mathbf{F}}(x_1)\exp(ix_1\xi)dx_1$$

is valid.

4. Solution of Problem 1 for an infinite strip

In this section, to illustrate the suggested method, we construct an explicit solution of Problem 1 for an infinite strip with voids in details. Quite similarly, we can construct the solution of Problem 2.

We are looking for a solution of the system (1), under BCs of Problem 1, in the form (5), where the functions h, $h_1(\mathbf{x})$, and $\mathbf{\Psi}$ are sought in the form

$$h=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\left[\eta(\xi)e^{-x_2|\xi|}+\nu(\xi)e^{(x_2-H)|\xi|)}\right]e^{ix_1\xi}d\xi,$$

$$\mathbf{\Psi}=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\left[\boldsymbol{\eta}^{(\mathbf{4})}(\xi)e^{-x_2|\xi|}+\boldsymbol{\nu}^{(\mathbf{4})}e^{(x_2-H)|\xi|}\right]exp(ix_1\xi)d\xi, \tag{9}$$

$$h_1=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\left[\eta_1(\xi)e^{-x_2r_1}+\nu_1(\xi)e^{(x_2-H)r_1}\right]e^{ix_1\xi}]d\xi,$$

$$r_1^2=\xi^2+s_1^2,\quad \boldsymbol{\eta}^{(\mathbf{4})}=(\eta_1^{(4)},\eta_2^{(4)}),\quad \boldsymbol{\nu}^{(\mathbf{4})}=(\nu_1^{(4)},\nu_2^{(4)}),$$

where $\boldsymbol{\eta}^{(4)}$, $\boldsymbol{\nu}^{(\mathbf{4})}$, η_k, η, ν and ν_k are absolutely integrable unknown vector functions;

Owing to the fact that $\Delta h_0=h$, the function h_0 can be represented in the following form

$$h_0=\frac{-x_2}{2\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\eta(\xi)e^{-x_2|\xi|}e^{ix_1\xi}\frac{d\xi}{|\xi|}+\frac{x_2-H}{2\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\nu(\xi)e^{(x_2-H)|\xi|}e^{ix_1\xi}\frac{d\xi}{|\xi|}, \tag{10}$$

We introduce the following functions

$$[\mathrm{div}\mathbf{u}]^+=F_4^+,\quad [\mathrm{div}\mathbf{u}]^-=F_4^-.$$

Let us substitute expression (9) into (5), pass to the limit as $x_2\to 0$ and $x_2\to H$, and taking into account boundary conditions, for determining the unknown values, from (5), we obtain the following system of equations

$$\begin{aligned}
&[\mathbf{\Psi}]^+=\mathbf{f}^++\left[\mathrm{grad}\left[a_0h_0+\frac{\beta h_1}{\mu_0 s_1^2}\right]\right]^+,\\
&h^++h_1^+=f_3^+,\quad -\frac{\varsigma}{\beta}h^+-\frac{\beta}{\mu_0}h_1^+=F_4^+,\\
&[\mathbf{\Psi}]^-=\mathbf{F}^-+\left[\mathrm{grad}\left[a_0h_0+\frac{\beta h_1}{\mu_0 s_1^2}\right]\right]^-,\\
&h^-+h_1^-=F_3^-,\quad -\frac{\varsigma}{\beta}h^--\frac{\beta}{\mu_0}h_1^-=F_4^-,
\end{aligned} \tag{11}$$

By solving $h^\pm$, $h_1^\pm$ and $\Psi^\pm$ from Eq. (11) we obtain:

$$h^+ = -\frac{\beta}{\alpha s_1^2}\left[\frac{\beta}{\mu_0} f_3^+ + F_4^+\right] = G^+, \quad h_1^+ = \frac{\beta}{\alpha s_1^2}\left[\frac{\varsigma}{\beta} f_3^+ + F_4^+\right] = G_1^+,$$

$$[\boldsymbol{\Psi}]^+ = \mathbf{f}^+ + \left[\operatorname{grad}\left[a_0 h_0 + \frac{\beta h_1}{\mu_0 s_1^2}\right]\right]^+ = \mathbf{G}_2^+,$$

$$h^- = -\frac{\beta}{\alpha s_1^2}\left[\frac{\beta}{\mu_0} F_3^- + F_4^-\right] = G^-, \quad h_1^- = \frac{\beta}{\alpha s_1^2}\left[\frac{\varsigma}{\beta} F_3^- + F_4^-\right] = G_1^-, \tag{12}$$

$$[\boldsymbol{\Psi}]^- = \mathbf{F}^- + \left[\operatorname{grad}\left[a_0 h_0 + \frac{\beta h_1}{\mu_0 s_1^2}\right]\right]^- = \mathbf{G}_2^-,$$

On the other hand, from (9) and (12) it is evident that

$$\eta(\xi) + \nu(\xi)e^{-H|\xi|} = \widehat{G^+}, \qquad \eta(\xi)e^{-H|\xi|} + \nu(\xi) = \widehat{G^-},$$

$$\eta_1(\xi) + \nu_1(\xi)e^{-Hr_1} = \widehat{G_1^+}, \qquad \eta_1(\xi)e^{-Hr_1} + \nu_1(\xi) = \widehat{G_1^-}, \tag{13}$$

$$\boldsymbol{\eta}^{(4)}(\xi) + \boldsymbol{\nu}^{(4)}e^{-H|\xi|} = \widehat{\mathbf{G}_2^+}, \quad \boldsymbol{\eta}^{(4)}(\xi)e^{-H|\xi|} + \boldsymbol{\nu}^{(4)} = \widehat{\mathbf{G}_2^-},$$

where $\widehat{G_k^+}$, $\widehat{G_k^-}$ ... are Fourier transform of the functions G_k^+, G_k^- .... respectively. After some transformation, from (13) we find

$$\eta = \frac{\widehat{G^+} - e^{-H|\xi|}\widehat{G^-}}{1 - e^{-2H|\xi|}}, \quad \nu = \frac{\widehat{G^-} - e^{-H|\xi|}\widehat{G^+}}{1 - e^{-2H|\xi|}},$$

$$\eta_1 = \frac{\widehat{G_1^+} - e^{-Hr_1}\widehat{G_1^-}}{1 - e^{-2Hr_1}}, \quad \nu_1 = \frac{\widehat{G_1^-} - e^{-Hr_1}\widehat{G_1^+}}{1 - e^{-2Hr_1}}, \tag{14}$$

$$\boldsymbol{\eta}^{(4)} = \frac{\widehat{\mathbf{G}_2^+} - e^{-H|\xi|}\widehat{\mathbf{G}_2^-}}{1 - e^{-2H|\xi|}}, \quad \boldsymbol{\nu}^{(4)} = \frac{\widehat{\mathbf{G}_2^-} - e^{-H|\xi|}\widehat{\mathbf{G}_2^+}}{1 - e^{-2H|\xi|}},$$

The obtained values (14) are substituted into (9), which yields

$$h(\mathbf{x}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\widehat{G^+}\sinh(H - x_2)|\xi| + \widehat{G^-}\sinh x_2|\xi|}{\sinh H|\xi|}\right]e^{ix_1\xi}d\xi,$$

$$h_1 = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\widehat{G_1^+}\sinh(H - x_2)r_1 + \widehat{G_1^-}\sinh x_2 r_1}{\sinh Hr_1}\right]e^{ix_1\xi}d\xi, \tag{15}$$

$$\boldsymbol{\Psi}(\mathbf{x}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\widehat{\mathbf{G}_2^+}\sinh(H - x_2)|\xi| + \widehat{\mathbf{G}_2^-}\sinh x_2|\xi|}{\sinh H|\xi|}\right]e^{ix_1\xi}d\xi,$$

Substituting (15) into (5), we get the solution $\mathbf{U} = (\mathbf{u}, \phi)$ of Problem 1 in quadratures.

5. Solution of Problem 2 for an infinite strip

Following the procedure, quite similarly as above, we can construct a solution of Problem 2 for an elastic strip with voids.

We are looking for a solution of the system (1), in the form (5), where the functions h, h_1 and $\boldsymbol{\Psi}$ are sought in the form (9).

Keeping in mind that

$$[TU]_1 = \mu\left[\frac{\partial\psi_1}{\partial x_2} + \frac{\partial\psi_2}{\partial x_1} - 2\frac{\partial^2}{\partial x_1\partial x_2}\left(a_0h_0 + \frac{\beta h_1}{\mu_0 s_1^2}\right)\right].$$

$$[TU]_2 = \left[\beta - \frac{\lambda\zeta}{\beta}\right]h + 2\mu\frac{\partial\psi_2}{\partial x_2} + \frac{2\beta\mu}{\mu_0}\left[1 - \frac{1}{s_1^2}\frac{\partial^2}{\partial x_2^2}\right]h_1 - 2\mu a_0\frac{\partial^2 h_0}{\partial x_2^2}, \tag{16}$$

and using the boundary conditions, we obtain:

when $x_2 = 0$

$$[TU]_1 = \mu\left[-|\xi|\eta_1^{(4)} + |\xi|\nu_1^{(4)}e^{-H|\xi|} + i\xi(\eta_2^{(4)} + \nu_2^{(4)}e^{-H|\xi|})\right]$$

$$-\mu a_0 i\xi\left[\frac{-\eta}{|\xi|} + \nu\left(\frac{1}{|\xi|} - H\right)e^{-H|\xi|}\right] - 2\mu i\xi\frac{\beta r_1}{\mu_0 s_1^2}\left(-\eta_1 + \nu_1 e^{-Hr_1}\right) = \hat{f_1^+},$$

$$[TU]_2 = \left[\beta - \frac{\lambda\zeta}{\beta}\right][\eta + \nu e^{-H|\xi|}] + 2\mu|\xi|[-\eta_4^{(4)} + \nu_2^{(4)}e^{-H|\xi|}] \tag{17}$$

$$-2\mu|\xi|^2\frac{\beta}{\mu_0 s_1^2}\left(\eta_1 + \nu_1 e^{-Hr_1}\right) - 2\mu a_0\left[\eta + \nu\left(1 - \frac{H|\xi|}{2}\right)e^{-H|\xi|}\right] = \hat{f_2^+},$$

$$h^+ + h_1^+ = f_3^+, \quad -\frac{\varsigma}{\beta}h^+ - \frac{\beta}{\mu_0}h_1^+ = F_4^+,$$

when $x_2 = H$

$$[TU]_1 = \mu\left[-|\xi|\eta_1^{(4)}e^{-H|\xi|} + |\xi|\nu_1^{(4)} + i\xi(\eta_2^{(4)}e^{-H|\xi|} + \nu_2^{(4)})\right]$$

$$-\mu a_0 i\xi\left[\left(H - \frac{1}{|\xi|}\right)\eta e^{-H|\xi|} + \frac{\nu}{|\xi|}\right] - 2\mu i\xi\frac{\beta r_1}{\mu_0 s_1^2}\left(-\eta_1 e^{-Hr_1} + \nu_1\right) = \hat{F_1}^-,$$

$$[TU]_2 = \left[\beta - \frac{\lambda\zeta}{\beta}\right][\eta e^{-H|\xi|} + \nu] + 2\mu|\xi|[-\eta_2^{(4)}e^{-H|\xi|} + \nu_2^{(4)}] \tag{18}$$

$$-2\mu|\xi|^2\frac{\beta}{\mu_0 s_1^2}\left(\eta_1 e^{-Hr_1} + \nu_1\right) - 2\mu a_0\left[\nu + \left(1 - \frac{H|\xi|}{2}\right)\eta e^{-H|\xi|}\right] = \hat{F_2}^-,$$

$$h^- + h_1^- = F_3^-, \quad -\frac{\varsigma}{\beta}h^- - \frac{\beta}{\mu_0}h_1^- = F_4^-,$$

From (17, 18), by the same calculations, as above, we derive

$$h^+ = -\frac{\beta}{\alpha s_1^2}\left[\frac{\beta}{\mu_0} f_3^+ + F_4^+\right] = G^+, \quad h_1^+ = \frac{\beta}{\alpha s_1^2}\left[\frac{\varsigma}{\beta} f_3^+ + F_4^+\right] = G_1^+,$$

$$h^- = -\frac{\beta}{\alpha s_1^2}\left[\frac{\beta}{\mu_0} F_3^- + F_4^-\right] = G^-, \quad h_1^- = \frac{\beta}{\alpha s_1^2}\left[\frac{\varsigma}{\beta} F_3^- + F_4^-\right] = G_1^-,$$

On the other hand, for determining the unknown coefficients, we obtain from (9) the equations

$$\begin{aligned} &\eta(\xi) + \nu(\xi) e^{-H|\xi|} = \widehat{G^+}, \qquad \eta(\xi) e^{-H|\xi|} + \nu(\xi) = \widehat{G^-}, \\ &\eta_1(\xi) + \nu_1(\xi) e^{-Hr_1} = \widehat{G_1^+}, \qquad \eta_1(\xi) e^{-Hr_1} + \nu_1(\xi) = \widehat{G_1^-}, \end{aligned} \tag{19}$$

After some transformation, from (19) we find

$$\begin{aligned} \eta &= \frac{\widehat{G^+} - e^{-H|\xi|}\widehat{G^-}}{1 - e^{-2H|\xi|}}, \quad \nu = \frac{\widehat{G^-} - e^{-H|\xi|}\widehat{G^+}}{1 - e^{-2H|\xi|}}, \\ \eta_1 &= \frac{\widehat{G_1^+} - e^{-Hr_1}\widehat{G_1^-}}{1 - e^{-2Hr_1}}, \quad \nu_1 = \frac{\widehat{G_1^-} - e^{-Hr_1}\widehat{G_1^+}}{1 - e^{-2Hr_1}}, \end{aligned} \tag{20}$$

substituting (20) into(9), we get

$$\begin{aligned} h(\mathbf{x}) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left[\frac{\widehat{G^+}\sinh(H - x_2)|\xi| + \widehat{G^-}\sinh x_2|\xi|}{\sinh H|\xi|}\right] e^{ix_1\xi} d\xi, \\ h_1 &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left[\frac{\widehat{G_1^+}\sinh(H - x_2)r_1 + \widehat{G_1^-}\sinh x_2 r_1}{\sinh Hr_1}\right] e^{ix_1\xi} d\xi, \end{aligned} \tag{21}$$

Let us find the expression for Ψ_k. To this end from (9), (17), and (18), using some algebraic manipulations, we get

$$\begin{aligned} &\left(-\eta_1^{(4)} + \nu_1^{(4)} e^{-H|\xi|}\right)|\xi| + i\xi\left(\eta_2^{(4)} + \nu_2^{(4)} e^{-H|\xi|}\right) \\ &= a_0 i\xi\left[\frac{-\eta}{|\xi|} + \nu\left(\frac{1}{|\xi|} - H\right)e^{-H|\xi|}\right] + 2i\xi\frac{\beta r_1}{\mu_0 s_1^2}\left(-\eta_1 + \nu_1 e^{-Hr_1}\right) + \frac{\hat{f}_1}{\mu} = \widehat{G_2}, \\ &2|\xi|\left(-\eta_2^{(4)} + \nu_2^{(4)} e^{-H|\xi|}\right) = 2|\xi|^2\frac{\beta}{\mu_0 s_1^2}\left(\eta_1 + \nu_1 e^{-Hr_1}\right) \\ &+2a_0\left[\eta + \nu\left(1 - \frac{H|\xi|}{2}\right)e^{-H|\xi|}\right] + \frac{1}{\mu}\hat{f}_2 - \frac{1}{\mu}\left[\beta - \frac{\lambda\zeta}{\beta}\right]\left(\eta + \nu e^{-H|\xi|}\right) = \widehat{G_3} \end{aligned} \tag{22}$$

$$
\begin{gathered}
\left(-\eta_1^{(4)} e^{-H|\xi|}+\nu_1^{(4)}\right)|\xi|+i\xi(\eta_2^{(4)} e^{-H|\xi|}+\nu_2^{(4)}) \\
=a_0 i\xi\left[\left(H-\frac{1}{|\xi|}\right)\eta e^{-H|\xi|}+\frac{\nu}{|\xi|}\right]+2i\xi\frac{\beta r_1}{\mu_0 s_1^2}\left(-\eta_1 e^{-Hr_1}+\nu_1\right)+\frac{1}{\mu}\hat{F}_1=\widehat{G_4}, \\
2|\xi|\left(-\eta_2^{(4)} e^{-H|\xi|}+\nu_2^{(4)}\right)=2|\xi|^2\frac{\beta}{\mu_0 s_1^2}\left(\eta_1 e^{-Hr_1}+\nu_1\right) \\
-2a_0\left[\nu+\left(1-\frac{H|\xi|}{2}\right)\eta e^{-H|\xi|}\right]+\frac{1}{\mu}\hat{F}_2-\frac{1}{\mu}\left[\beta-\frac{\lambda\zeta}{\beta}\right][\eta e^{-H|\xi|}+\nu]=\widehat{G_5}, \quad (23)
\end{gathered}
$$

After certain calculations, it follows from Eqs. (22) and (23) that

$$
\begin{aligned}
\eta_2^{(4)} &= \frac{\widehat{G_3}-e^{-H|\xi|}\widehat{G_3}}{2|\xi|(-1+e^{-2H|\xi|})}, \quad \nu_2^{(4)}=\frac{-\widehat{G_5}+e^{-H|\xi|}\widehat{G_3}}{2|\xi|(-1+e^{-2H|\xi|})}, \\
\eta_1^{(4)} &= \frac{\widehat{G_2}'-e^{-H|\xi|}\widehat{G_4}'}{|\xi|(-1+e^{-2H|\xi|})}, \quad \nu_1^{(4)}=\frac{-\widehat{G_4}'+e^{-H|\xi|}\widehat{G_2}'}{|\xi|(-1+e^{-2H|\xi|})},
\end{aligned} \qquad (24)
$$

where

$$
\widehat{G_2}'=\widehat{G_2}-i\xi(\eta_2^{(4)}+\nu_2^{(4)}e^{-H|\xi|}), \quad \widehat{G_4}'=\widehat{G_4}-i\xi(\eta_2^{(4)}e^{-H|\xi|}+\nu_2^{(4)}).
$$

We assume that $\widehat{f(0)}=\widehat{F(0)}$. This condition means that the principal vector of external stresses is equal to zero.

Substituting (24) into (9), we get

$$
\begin{aligned}
\Psi_1(\mathbf{x}) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\widehat{G_2'}\cosh(H-x_2)|\xi|-\widehat{G_4'}\cosh x_2|\xi|}{-|\xi|\sinh H|\xi|}\right]e^{ix_1\xi}d\xi, \\
\Psi_2(\mathbf{x}) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\widehat{G_3}\cosh(H-x_2)|\xi|-\widehat{G_5}\cosh x_2|\xi|}{-2|\xi|\sinh H|\xi|}\right]e^{ix_1\xi}d\xi,
\end{aligned} \qquad (25)
$$

For the existence of a solution of Problem 2 it is necessary that the principal vector and the principal moment of external stresses acting on the boundaries of the domain D be equal to zero. As is known from the general theory these, conditions are sufficient as well.

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