HIGHER GRADIENT THEORIES: APPLICATIONS TO THE SIZE EFFECT IN SUBMICROSTRUCTURES INCL. EXPERIMENTAL ANALYSIS OF ELASTIC MATERIAL PARAMETERS

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Experiments on micro-specimens have shown that the deformation behavior of materials can be size dependent. The size dependence for some materials is, for example, reflected in a stiffer elastic response on the sub-microscale. A quantitative understanding of the size effect is important during the design phase of micro- and nanosize systems which, for example, is accompanied by finite element (FE) simulations. In our research project higher-order theories of elasticity will be presented and used for the description of the bending behavior of micro-beams and micro-plates. These theories include additional material parameters in order to describe the size effect. Thus they go beyond the limits of the classical Boltzmann continuum. Special attention is paid to such non-local theories as the strain gradient- and the couple stress theory of linear elasticity. The latter is based on the ideas of E. and F. Cosserat (1909) by introducing an independent degree of freedom intrinsic to the material, named rotation.

We interpret the material points of a body as rigid particles. As a consequence we require only one material length scale parameter for connecting the rotation vector to the mathematical rotation of the displacement gradients. The objectives of the research project are to determine the material length scale parameter by analyzing experimental data obtained from force-displacement measurements on extremely small cantilever beams or plates with analytical as well as numerical analysis tools based on higher gradient theory. In particular, deflection measurements were performed and force data was recorded for submicron beams made of silicon and silicon nitride and for micro plates made of the polymer BCB. Simple beam bending and plate bending experiments were performed with decreasing thicknesses of the beams and plates. This is in line with the corresponding “method of size effect” from Lakes (1995). Bending rigidities and effective strains will be measureable in our labs with the help of atomic force microscopy and Raman spectroscopy, respectively.

The analytical solutions of Euler-Bernoulli beam theory or Kirchhoff plate theory incorporating the terms of higher gradient theory are presented. In contrast to existing work on the implementation of the superior strain gradient theory in terms of a finite element solution the crucial differential equation used here is consistently based on the balance of linear momentum and on the uncoupled balance of angular momentum. The resulting analytical formulae, the obtained data from the numerical modeling and the obtained data from the experiments could be used for evaluation of higher gradient coefficients. The developed variational formulation of the couple stress theory is implemented into an open-source finite element code, FEniCS©, using equidistantly distributed tetrahedral and continuous Lagrange elements with a polynomial degree of two in observance to the rank of the resulting elliptical partial differential equation. This novel open-source FE-software provides a collection of open-source packages for automated, efficient solutions of various differential equations.

The experiments were performed by using a highly sensitive atomic force microscope (MV-1000 from Nanonics Imaging Ltd.). In addition, Raman spectroscopy was used for recording strains (Renishaw© RS). Raman spectroscopy takes advantage of the morphic effect in opaque materials: The spectroscopic detection of a modification in the characteristic phonon frequencies can be attributed to mechanical strains. These strain data could be used to fit the solutions of strain gradient theories.

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