Multilinear Integral Operators in Weighted Function Spaces

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Recently, much attention has been paid to the study of the boundedness of various types of operators between weighted L^p-spaces playing an important role in analysis, in particular, in harmonic analysis and its applications in partial differential equations (PDE). One of our aims is to discuss the results of the last decade regarding mapping properties of multilinear integral operators in weighted Lebesgue spaces. We are focused on multi(sub)linear maximal and fractional integral operators. Multisublinear maximal operators appeared naturally in connection with multilinear Calderón-Zygmund theory. It was used to obtain a precise control on multilinear singular integral operators of Calderón-Zygmund type and to build a theory of weights adapted to the multilinear setting. The study of mapping properties of multilinear fractional integral operators were initiated by Grafakos (1992), Kenig and Stein (1999), Grafakos and Kalton (2001).

The second part of our presentation is dedicated to the boundedness problem of general multisublinear operators generated by quasi-concave functions between weighted Banach function lattices. These operators, in particular, generalize the Hardy-Littlewood and fractional maximal functions playing an important role in harmonic analysis. We show that under some general geometrical assumptions on Banach function lattices two-weight weak type and also strong type estimates for these operators are true. To derive the main results the strong type estimate for a variant of multilinear averaging operators is characterized. As special cases boundedness results for fractional maximal operators in concrete function spaces will be presented.