# HIERARCHICAL MODELING OF THERMOELASTIC SHELLS WITHIN THE FRAMEWORK OF NONCLASSICAL THEORIES OF THERMOELASTICITY

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Construction and implementation of algorithms of numerical solution of initial-boundary value problems corresponding to nonclassical Lord-Shulman, Green-Lindsay and Chandrasekharaiah-Tzou three-dimensional models for thermoelastic bodies are rather complicated tasks. Therefore, it is important to construct approximate two-dimensional models for three-dimensional problems taking into account geometric shape of thermoelastic body. Within the framework of Lord-Shulman, Green-Lindsay and Chandrasekharaiah-Tzou nonclassical theories of thermoelasticity we consider general thermoelastic shell with variable thickness in curvilinear coordinates, which may vanish on a part of the lateral surface, and construct its two-dimensional hierarchical models applying spectral method, which is a generalization of the dimensional reduction method suggested by I. Vekua [7, 8] in the theory of elasticity for plates with variable thickness and shells. To construct two-dimensional models of plate I. Vekua considered differential formulation of the three-dimensional initial-boundary value problem and approximating components of the displacement vector-function by partial sums of orthogonal Fourier-Legendre series with respect to the variable of plate thickness a hierarchy of initial-boundary value problems defined on two-dimensional space domain was obtained. Note that the classical Kirchhoff-Love and Reissner-Mindlin models can be incorporated into the hierarchy obtained by I. Vekua so that it can be considered as an extension of the frequently used engineering plate models. Static two-dimensional models constructed by I. Vekua for thin shallow shells first were investigated in Sobolev spaces in [4] and for homogenous isotropic plate the rate of approximation of the exact solution of the three-dimensional problem by the vector-functions of three space variables restored from the solutions of the reduced two-dimensional problems in the spaces of classical smooth functions was estimated in [5]. Later on, various two-dimensional and one-dimensional models were constructed and investigated for problems of the theory of elasticity and mathematical physics applying I. Vekua's reduction method and its generalizations (see [1-3, 6] and references given therein).

To obtain hierarchies of two-dimensional models of thermoelastic shell we construct sequences of subspaces of the spaces corresponding to the original three-dimensional initial-boundary value problems, which consist of vector-functions whose components are polynomials with respect to the variable of shell thickness. Note that the constructed subspaces are weighted Sobolev spaces of function defined in two-dimensional Lipschitz domain, when the thickness of the shell vanishes on a part of the boundary. Projecting the original three-dimensional problems on these subspaces we construct hierarchies of dynamical two-dimensional models, when surface force and heat flux densities or values of displacement and temperature are given along the face surfaces and along a part of the lateral surface, and the shell is clamped and the temperature vanishes along the remaining part of the boundary. We prove the existence and uniqueness of solutions of the obtained two-dimensional initial-boundary value problems in suitable spaces of vector-valued distributions. We also obtain energetic identity, which permits one to show continuous dependence of solutions on given functions. We investigate relationship between the hierarchies of dynamical two-dimensional models of thermoelastic shells obtained from Lord-Shulman, Green-Lindsay and Chandrasekharaiah-Tzou models and original three-dimensional initial-boundary value problems and prove that the sequences of vector-functions of three space variables constructed by means of the solutions of the reduced problems converges in corresponding Sobolev spaces pointwise with respect to the time variable to the exact solutions of the original three-dimensional initial-boundary value problems and under additional regularity conditions we obtain estimates of the rates of convergence. Note that the constructed algorithms of approximation can be used not only for simplification of algorithms of numerical solution of three-dimensional problems, but also the first approximations of the constructed hierarchies of two-dimensional initial-boundary value problems can be considered as independent nonclassical models for thermoelastic shells and can be used for mathematical modeling of engineering structures.

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