Material Balance Laws in Linear Elasticity with Applications

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Abstract Material conservation laws and associated path-independent integrals play a prominent role in the assessment of defects in structures. Especially Rice's J-integral is widely used in fracture mechanics. For systems governed by a Lagrangian, the usual tool for the derivation of material conservation laws is the application of Noether's first theorem in combination with Bessel-Hagen's extension. The so-called Neutral-Action (NA) method is a different approach. Its advantage in comparison with the classical Noether's approach lies in the fact that it is applicable to field equations that are not necessarily the Euler-Lagrange equations of a variational principle, i.e., for systems not governed by a Lagrangian. After a short review of the NA method, a complete set of characteristics and the associated conserved currents are derived and interpreted in physical terms. As an example, path-independent integrals are evaluated around a crack tip and a defect-interaction problem is treated. Finally, the application of conservation laws in strength-of-materials theories is discussed.