ON INVESTIGATION OF SOME NONCLASSICAL MODELS FOR THERMOELASTIC BODIES

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The present paper is devoted to investigation of initial-boundary value problems corresponding to nonlcassical dynamical linear three-dimensional models of thermoelastic bodies in corresponding spaces of vector-valued distributions. We consider Lord-Shulman, Green-Lindsay and Chandrasekharaiah-Tzou linear three-dimensional models for anisotropic inhomogeneous thermoelastic bodies, which were constructed to eliminate shortcomings of the classical thermoelasticity, such as infinite velocity of thermoelastic disturbances that is inconsistent with the real physical properties of elastic bodies, unsatisfactory thermoelastic response of a solid to short laser pulses, and poor description of thermoelastic behavior at low temperatures.

In Lord-Shulman [9] model instead of the classical Fourier law of heat conduction Maxwell-Cattaneo law was used, which is a generalization of Fourier law and depend on one relaxation time parameter. Hence, the equation corresponding to the temperature field involve second order derivatives of temperature and divergence of displacement vector-function with respect to the time variable. The problem of propagation of a thermoelastic wave was studied and domain of influence result for Lord-Shulman model in spaces of classical smooth enough functions was obtained in [8], and problems of steady oscillations and pseudo-oscillations were investigated in [1] applying methods of the theory of integral equations.

Different approach was used by A. Green and K. Lindsay [6] to obtain nonclassical model for thermoelastic bodies, which is characterized by a system of partial differential equations where, in comparison to the classical linear system of thermoelasticity, the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times. For Green-Lindsay nonclassical model the problem of propagation of a thermoelastic wave was studied and domain of influence result was obtained in [2] in classical spaces of twice continuously differentiable functions. In the case of infinite and semi-infinite bodies initial-boundary value problems corresponding to Green-Lindsay model were investigated in [5, 7]. Applying method of potential and theory of integral equations the problems of stable and pseudo oscillations for Green-Lindsay nonclassical model were studied in [1].

Further, Tzou [12] proposed a dual-phase-lag heat conduction model, where the phase-lag corresponding to temperature gradient is caused by microstructural interactions such as phonon scattering or phonon-electron interactions, while the second phase-lag is interpreted as the relaxation time due to fast-transient effects of thermal inertia. Applying Tzou's model Chandrasekharaiah [3] constructed nonclassical model for thermoelastic bodies, where the classical Fourier's law of heat conduction was replaced with its generalization proposed by Tzou. In this model the equation describing the temperature field involves the third order derivative with respect to the time variable of the temperature and divergence of the third order derivative with respect to the time variable of the displacement. Note that the Chandrasekharaiah-Tzou model is an extension of the Lord-Shulman nonclassical model for thermoelastic bodies. Spatial behavior of solutions of the dual-phase-lag heat conduction equation and problems of stability of dual-phase-lag heat conduction models have been investigated and particular one-dimensional initial-boundary value problems have been analysed in the Chandrasekharaiah-Tzou theory [4,10,11].

We consider variation formulations of initial-boundary value problems in differential form corresponding to Lord-Shulman, Green-Lindsay and Chandrasekharaiah-Tzou dynamical noncla-

ssical three-dimensional models and show their equivalence in the spaces of smooth enough functions. On basis of variation formulation we define spaces of vector-valued distributions in which the initial-boundary value problems corresponding to Lord-Shulman, Green-Lindsay and Chandrasekharaiah-Tzou models are well-posed, and applying suitable a priori estimates we prove the existence and uniqueness of solutions of the three-dimensional initial-boundary value problems. In addition, we obtain energetic identities, which permits one to prove continuous dependence of solutions on initial and boundary conditions and densities of body forces and heat sources.

REFERENCES

1. T.V. Burchuladze, T.G. Gegelia, Development of the method of potential in the theory of elasticity, Mecniereba, Tbilisi, 1985, (Russian).
2. B. Carbonaro, J. Ignaczak, Some theorems in temperature-rate dependent thermoelasticity for unbounded domains, J. Thermal Stresses, **10** (1987), 193-220.
3. D.S. Chandrasekharaiah, Hyperbolic thermoelasticity: a review of recent literature, Appl. Mech. Review, **51** (1998), 705-729.
4. D.S. Chandrasekharaiah, One-dimensional wave propagation in the linear theory of thermoelasticity without energy dissipation, J. Thermal Stresses, **19** (1996), 695-710.
5. G. Fichera, A boundary-value problem connected with response of semi-space to a short laser pulse, Rend. Mat. Acc. Lincei, **9**, 8 (1997), 197-228.
6. A.E. Green, K.A. Lindsay, Thermoelasticity, J. Elasticity, **2** (1972), 1-7.
7. R.B. Hetnarski, J. Ignaczak, Generalized thermoelasticity response of semi-space to a short laser pulse, J. Thermal Stresses, **17** (1994), 377-396.
8. J. Ignaczak, B. Carbonaro, R. Russo, Domain of influence theorem in thermoelasticity with one relaxation time, J. Thermal Stresses, **9** (1986), 79-91.
9. H.W. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, J. Mech. Phys. Solids, **15** (1967), 229-309.
10. R. Quintanilla, A condition on the delay parameters in the one-dimensional dual-phase-lag thermoelastic theory, J. Thermal Stresses, **28** (2005), 43-57.
11. R. Quintanilla, R. Racke, Qualitative aspects in dual-phase-lag heat conduction, Proc. Royal Society A, **463** (2007), 659-674.
12. D.Y. Tzou, A unified approach for heat conduction from macro to micro-scales, J. Heat Transfer, **117** (1995), 8-16.