

APPROXIMATE SOLUTION OF CAUCHY PROBLEM FOR ABSTRACT HYPERBOLIC EQUATION USING UNITARY GROUP APPROXIMATION

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The paper discusses Cauchy problem for an abstract hyperbolic equation in a Hilbert space. It is well-known that the solution to this problem is given in terms of sine and cosine operator functions, which gives a possibility to define unknown function and its derivative in terms of vectors using time-variable-dependent matrix-operator that generates a unitary group. The following work offers two-layer vector scheme for unitary groups using high-order rational approximation. The scheme allows us to find the values of the unknown function as well as the values of its derivative on each time layer. Stability of this scheme is studied. Namely, it is proved that the norm of the transition matrix-operator does not exceed 1. This guarantees stability of the scheme in any finite time interval. It is also proved that the constructed rational approximation, which in fact is an operator analog of scalar Pade approximation, yields fourth order accuracy for values of the function and its derivative for any time layer. We also consider a case of weak nonlinearity, in particular, a case in which a nonlinear operator satisfying Lipschitz condition is added to the linear main part.