

# Conjugate differential forms and applications

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The theory of differential forms can be considered a useful tool in several framework of mathematical analysis. A  $k$ -form  $u$  is represented in the natural coordinate system  $(x_1, \dots, x_n)$  as  $u = \frac{1}{k!} u_{i_1 \dots i_k} dx_{i_1} \dots dx_{i_k}$ , where  $u_{i_1 \dots i_k}$  are the components of a  $k$ -covector, i.e. the components of a skew-symmetric covariant tensor. We denote by  $*$ ,  $d$  and  $\delta$  the adjoint, the differential and the co-differential operators, respectively. We say that a  $k$ -form  $u$  and a  $(k+2)$ -form  $v$  are conjugate if  $du = \delta v$ ,  $\delta u = 0$  and  $dv = 0$ . This concept is an extension of the classical definition of conjugate harmonic functions and the main results in the theory of holomorphic functions of one complex variable can be generalized. Some conditions for the existence of conjugate forms in a multiply connected domain of  $\mathbb{R}^n$  can be found as well.

By means of the notion of conjugate differential forms, a new concept of conjugate Laplace series in higher dimensions was given in [3]. Several results concerning such series have been obtained, like criteria for different kind of summability ([2, 4]) and a Brother Riesz Theorem ([3]).

By considering the direct sum of  $k$ -form spaces,  $0 \leq k \leq n$ , it is possible to define non homogeneous differential forms,  $U = \sum_{k=0}^n u_k$ ,  $u_k$  being a  $k$ -form. The operators  $*$ ,  $d$  and  $\delta$  can be extended to these objects in a natural way. Moreover,  $U$  is said to be self-conjugate if  $dU = \delta U$  (see [5]). We consider some BVPs involving non homogeneous differential forms and we find necessary and sufficient conditions for their resolubility. The notion of self-conjugate differential forms extends the one of conjugate differential forms and some other real generalizations of Cauchy Riemann equations. In particular we shall consider here the Cimmino system. By means of results for self-conjugate differential forms, we obtain necessary and sufficient conditions also for the resolubility of the Dirichlet problem for the Cimmino system (see [1]).

## References

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- [5] A. Cialdea, *On the Theory of Self-Conjugate Differential Forms*, Atti Sem. Mat. Fis. Univ. Modena, XLVI (Suppl.) (1998), 595-620.