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GEOMETRIC FIGURES WHICH APPEAR AFTER VV-CUTTING IN THE RADIAL CROSS SECTION OF THE GML BODIES *

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Abstract. In the previous works, we were able to calculate all possible, and various options that appear after VV, VS or SS cuts of the GML bodies with the help of the so-called straight chordal knives [1]. Then we did not specify how many and what traces of flat figures appear on the radial cross section of the GML_m^n body, depending on: m-number of polygon vertices, n-number of twist and a parameter showing which vertices (sides) are connected by this knife!? In this article, a regularity is given with the help of which it is possible to calculate the number and nature of flat figures appearing after an arbitrary VV in arbitrary regular m-gon.It should be obligatorily noted that at present this regularity has been discovered and tested on many examples of parameters, but by this time there is no complete mathematical proof. Therefore, we call these regularity - hypothetical regularity.

Keywords and phrases: Analytic representation, Möbius-Listing's bodies.

AMS subject classification (2010): 53A05, 51B10, 51E12.

1 Introduction. In this article, we use all the traditional definitions and notation introduced in previous works [1], so we will not repeat them, but add only a few new parameters that turned out to be decisive for these results.

• 1: V_1V_i - is the "trace of corresponding knife" or diagonal connecting the first numbered vertex and the i-numbered vertex of the regular m-polygon in the radial cross section of the GML body. It is known from previous works that it suffices to consider $i = 3, ..., [m/2]$ if m is an odd number and $i = 3, ..., [m/2]$ if m is an even number, where $[m/2]$-is an integer part of a fraction.

• 2: $\kappa \equiv i - 1$- is the parameter describing the given "trace of corresponding knife". Always $\kappa > 1$ (if $\kappa = 1$- then we have the side of the polygon).

• 3: $j \equiv gcd(m, n)$- is a parameter characterizing this GML_m^n body, $j = 0$-means $n \equiv 0$ body without twisting or classical toroidal body.

General Remarks: In what follows, a polygon with three vertices is called an 3-gon, a polygons with 16 vertices is called a 16-gon etc.. Further, it should be noted that the methodology uses planar sections, whereas Generalized Möbius-Listing bodies are three

*The author expresses his deep gratitude to Johan Gielis for the constant and very attentive discussion of these issues and the constant belief that I will find a regularity, as well as for valuable advice. The author is very grateful to Levan Roinishvili, who is his former student, for creating a convenient and easy-to-use computer program, with the help of which examples of figures appearing after such cuts were built.

dimensional. This means that the resulting ν-gons with the same color and shape form a single body after cutting.

We have identified four cases in which different regularities of appearance of different flat geometric figures in the radial cross section of the GML^n_m bodies are obtained.

• Case T.If $j = 0$ or $j = km$, $k \in Z$ and for arbitrary m and $\kappa \leq [m/2]$,one $\kappa + 1$-gon and one $[m - (\kappa - 1)]$-gon appear after such VV cutting.

First Case.The diagonal contains the center of symmetry of the polygon. Then $m = 2k$ and$\kappa = k$ - in this case for arbitrary $j = 1, .., m$ and the center of symmetry of the polygon is located on this diagonal.

• Case 1A. If $j = m = 2k$ or $j = 0$, then two different, but identical plane $k + 1$- gons appear after such VV cutting, but the Möbius phenomenon never occurs.

• Case 1B. If $j = k$ and $m/j = 2$, then two identical plane $k + 1$-gons appear after such VV cutting and the Möbius phenomenon always occurs.

• Case 1C. If $j < k$, and m/j is an even number, then m/j pieces identical $j + 2$- gons appear after such VV cutting and the Möbius phenomenon always occurs.

• Case 1D. If $j = 2\beta$ is an even number and m/j - is an odd number, then two different groups appear after VV cutting, each of which consists of m/j pieces of $\beta + 2$-gons!

		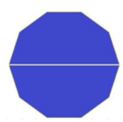	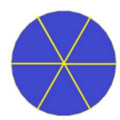	
Case T.$m = 10, k = 4, j = 0$ or 10 one 5-gonsand one 7-gon	Case 1A.$m = 10, k = 5, j = 0$ or 10 (2 different 6-gons)	Case 1B. $m = 10, k = 5$ (2 different 6-gons) **Möbius ph.**	Case 1C. $m = 30$, $k = 15, j = 5$ (6 similar 7-gons) **Möbius ph.**	Case 1D. $m = 18, k = 9, j = 6$ (two groups of 3 similar 5-gons)

Table 1: Examples for the cases T and 1 with different parameter values.

Second Case. For arbitrary m when $\kappa \leq j$, then (m/j)- similar $(\kappa + 1)$-gons and one piece of $[m - (\kappa - 1) \cdot (m/j)]$-gon appears after such VV-cutting!

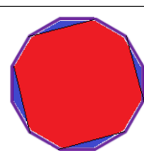	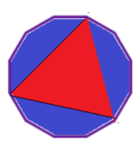	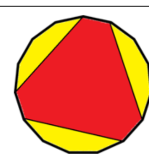	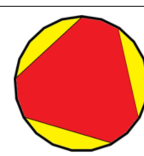
$m = 12, \kappa = 2 < j = 3$ 4 similar 3-gons and one 8-gon	$m = 12, \kappa = 4 \leq j = 4$ 3 similar 5-gons and one 3-gon	$m = 15, \kappa = 4 \leq j = 5$ 3 similar 5-gons and one 6-gon	$m = 21, \kappa = 5 \leq j = 7$ 3 similar 6-gons and one 9-gon

Table 2: Examples for the second case with different parameter values.

Third case For arbitrary m when $\kappa > j$: This turned out to be the most difficult case to study, which has many branches and shows a strong connection with the structure of numbers and geometric shapes.

• Case 3.I. **This subcase is considered separately, since for any values?? of m and n (even when these numbers are coprime) it is realized!** For arbitrary m and$\kappa = 2, ..., m/2$ when $j = 1$, then two different groups, each of which consists of [m/j] pieces 3$-$ gons and $\kappa - 2$ different groups, each of which consists of m/j similar 4-gons and one m/j-gon appears after such VV cutting!

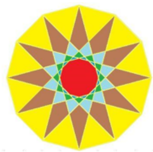	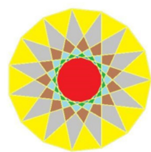	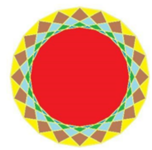	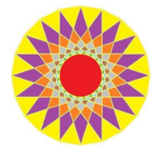
$m = 12, \kappa = 5 > j =$ 12-gr. (12 similar 3-gons), 3 -gr. (12 similar 4-gons) and one 12-gon	m=15, $\kappa = 6 > j = 12$-gr. (15 similar 3-gons), 4-gr. (15 similar 4-gons) and one 15-gon	$m = 20, \kappa = 5 > j =$ 12-gr. (20 similar 3-gons), 3 gr. (20 similar 4-gons) and one 20-gon	$m = 20, \kappa = 8 > j =$ 12-gr. (20 similar 3-gons), 6 gr.(20 similar 4-gons) and one 20-gon

Table 3: Examples for the third case 3.1. with different parameter values.

• Case 3.GA. For arbitrary m and $\kappa = j\beta < m/2$,where $\beta \in Z$ - integer and $\beta > 1$, then one group consisting of m/j -pieces 3- gons, $\beta - 2$ different groups, each of which consists of m/j - pieces 4-gons, one group consisting of m/j- pieces $j + 2$- gons and one piece of m/j -gon appears after such VV cutting!

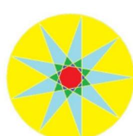	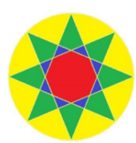	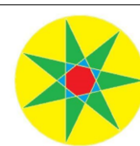
m=36, $\kappa = 16 > j = 4, \beta = 4$, 1-gr. (9 similar 3-gons), 2 gr. (9 similar 4-gon), 1-gr. (9 similar 6-gons) and one 8-gon	m=40, $\kappa = 15 > j = 5 , \beta = 3$, 1-gr. (8 similar 3-gons), 1-gr. (8 similar 4-gon), 1-gr. (8 similar 7-gons) and one 8-gon	m=42, $\kappa = 18 > j = 6 , \beta = 3$, 1-gr. (7 similar 3-gons), 1-gr. (7 similar 4-gon), 1-gr. (7 similar 8-gons) and one 7-gon

Table 4: Examples for the case GA with different parameter values.

• Case 3.GB. For arbitrary m and $\kappa = j\beta + l < m/2$, $\beta > 1$, and $\beta \in Z$, where $l = 1, 2, ..., \beta - 1$, then one group consisting of m/j- pieces 3- gons, $\beta - 2$ different groups , each of which consists of m/j pieces of 4-gons, one group consisting of m/j -pieces $[j - (l - 4)]$- gons, one group consisting of m/j- pieces $l + 2$- gons and one piece of m/j -gon appears after such VV cutting!

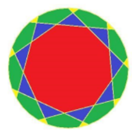		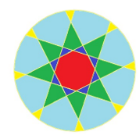	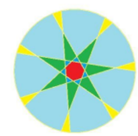
m=36, $\kappa = 9 > j = 4, \beta = 2, l = 1$ 1-gr. (9 similar 3-gons), 1-gr. (9 similar 7-gons), 1-gr. (9 similar 3-gons) and one 9-gon	m=36, $\kappa = 17 > j = 4$ $\beta = 4, l = 1$ 1-gr. (9 similar 3-gons), 2- gr. (9 similar 4-gon), 1-gr. (9 similar 7-gons), 1-gr. (9 similar 3-gons) and one 9-gon	m=40, $\kappa = 16 > j = 5$ $\beta = 3, l = 1$ 1-group (8 similar 3-gons), 1-group (8 similar 4-gon), 1-group (8 similar 8-gons), 1-group (8 similar 3-gons) and one 8-gon	m=42, $\kappa = 19 > j = 6$ $\beta = 3, l = 1$ 1-group (7 similar 3-gons), 1- group (7 similar 4-gon), 1-group (7 similar 9-gons), 1-group (7 similar 3-gons) and one 7-gon

Table 5: Examples for the subcase GB with different parameter values.

• Case 3. GC. For arbitrary m and $j < m/2$, when $\kappa = j\beta + l$, $\beta = 1$, $l = 1, 2, ..., j-1$ then: one group consisting of m/j pieces $[j - (l-3)]$- gons and one group consisting of m/j pieces $l+2$-gons and one piece of m/j -gon appears after such VV cutting!

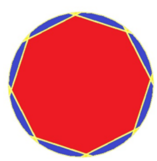	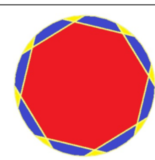	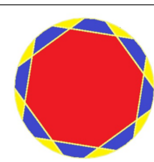	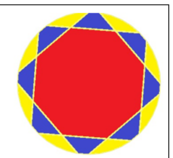
m=40, $\kappa = 6 > j = 5$, l=1 1-gr. (8 similar7-gons), 1-gr. (8 similar 3-gons) and one 8-gon	m=40, $\kappa = 7 > j = 5$, l=2 1-gr.(8 similar 6-gons), 1-gr.(8 similar 4-gons), and one 8-gon	m=40, $\kappa = 8 > j = 5$, l=3 1-gr.(8 similar 5-gons), 1-gr.(8 similar 5 gons), and one 8-gon	m=40, $\kappa = 9 > j = 5$, l=4 1-gr.(8 similar 4-gons), 1-gr.(8 similar 6-gons), and one 8-gon

Table 6: Examples for the case GC with different parameter values.

Final Remark. It should be obligatorily noted that at present this regularity has been discovered and tested on many examples of parameters, but by this time there is no complete mathematical proof. Therefore, we call this regularity - hypothetical regularity. I also want to note that the situation is almost repeating itself, when in 2014 Johan Gielis and I found a general regularity about the number of the GML_m^n - cutting bodies in different ways, and only in 2019 we were able to fully prove it [1]!

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