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MATHEMATICAL AND COMPUTER MODELS OF THE TRANSFORMATION OF THE PROTO-KARTVELIAN POPULATION INTO THE SVAN AND GEORGIAN-COLCHIAN POPULATIONS

Temur Chilachava Gia Kvashilava George Pochkhua

Abstract. Earlier, we proposed a scenario for the transformation of the Proto-Kartvelian population into the Georgian, Megrelian, Laz and Svan populations.
Using mathematical and computer modeling, the first stage (5000–2500 BC) of the dynamics of the Proto-Kartvelian population was studied. In particular cases, exact analytical solutions are obtained, and in the general case of variable coefficients - numerical solutions.
The second stage was studied (2500–1000 BC). For the dynamics of Pelasgians emigrating to Europe, an exact analytical solution has been obtained.
A two-dimensional dynamic system describing the interaction of the Svan and Georgian-Colchian populations has been studied in particular cases. In some cases, exact analytical solutions have been found, and in some cases theorems on the coexistence of these two populations have been proved, when complete assimilation of the Svan population does not occur.
This paper considers computer modeling of a nonlinear dynamic system with variable coefficients, which describes at the second stage the interaction of the Svan and Georgian-Colchian populations. Exponential and qualitative functions are taken as variable coefficients. Numerous computer experiments were carried out.
In the case of positive demographic factors of both populations, it is shown that the Svan population from approximately 0.5–0.6 million, despite the growing demographic factor, due to assimilation decreased to 0.3–0.4 million. The Georgian-Colchian population increased from 2.4–2.5 million to 3–4 million.

Keywords and phrases: Transformation of the Proto-Kartvelian, Svan and Georgian-Colchian populations, mathematical and computer models.

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1 Introduction. Previously, we studied the first period of Proto-Kartvelian population dynamics through mathematical and computer modeling [1]. The second period of transformation of the Proto-Kartvelian population was also discussed, when the impact of Indo-European and Semitic tribes on the Proto-Kartvelian population began. As a result of their influence, Pelasgian, which went to Europe, as well as Svan and Georgian-Colchian (Georgian-Zan) stood out from the Proto-Kartvelian population.

2 Mathematical and computer modeling of the second period. A two-dimensional nonlinear dynamical system describes the interaction and mutual influence of Svan and Georgian-Colchian populations:

$$\begin{cases} \frac{du(t)}{dt} = \alpha_2(t)u(t) - \beta_1(t)u^2(t) + \beta_2(t)u(t)v(t) - p_1(t)u(t) \\ \frac{dv(t)}{dt} = \alpha_3(t)v(t) - \beta_3(t)v^2(t) - \beta_4(t)u(t)v(t), \end{cases} \quad (1)$$

$$u(t_1) = u_1,\ v(t_1) = v_1, \quad (2)$$

where $t \in (t_1; t_2),\ u(t),\ v(t) \in C^1[t_1, t_2],\ \beta_i(t) \geqslant 0,\ i \in \overline{1-4},\ p_1(t) > 0,\ \beta_i(t),\ p_1(t) \in C[t_1, t_2],\ \alpha_2(t),\ \alpha_3(t) \in C[t_1, t_2], t_1 = 2500$ BC, $t_2 = 1000$ BC; $u(t)$ is the number of Georgian-Colchian population at time t; $v(t)$ is the number of Svan population at time t; $\alpha_2(t), \alpha_3(t)$ are demographic factors (natural reproduction-mortality rate) respectively for the Georgian-Colchian and Svan populations; $\beta_1(t), \beta_3(t)$ are self-limiting factors respectively of Georgian-Colchian and Svan populations; $\beta_2(t), \beta_4(t)$ are coefficients of assimilation of Svan population; $p_1(t) > 0$ is factor of unnatural reduction of Georgian-Colchian population due to forced hostilities with neighboring people.

Qualitative analysis of the system of equations (1), considering the adequacy and non-triviality of the mathematical model leads to limiting the variable coefficients of the dynamic system:

$$\begin{cases} \alpha_3(t) > 0 \\ \beta_1(t) \geqslant 0 \\ \beta_3(t) \geqslant 0 \\ \beta_2(t) > 0 \\ \beta_4(t) > 0 \\ p_1(t) > 0 \end{cases}, \quad (3)$$

where the coefficient $\alpha_2(t)$ can be sign variable or equal to zero.

The second period is described by two different mathematical models:

a. The unknown function that determines the number of Pelasgian population is described by a Bernoulli equation with variable coefficients taking into account the term representing the assimilation process. The analytical solution of the Cauchy problem is obtained in quadratures [1].

b. A mathematical model is used to describe the process of interaction and mutual influence of populations speaking Svan and Georgian-Colchian languages, which is given by a nonlinear dynamic system with nonlinear members of self-limitation and takes into account the unnatural reduction of the Georgian-Colchian population as a result of hostilities with neighboring peoples. For a dynamic system without a self-limiting nonlinear member, in the case of certain dependence between variable coefficients, the first integral was found, by means of which the Bernoulli equation with variable coefficients for one of the unknown functions was obtained [2].

For the general mathematical model, in four cases of certain interdependencies between constant coefficients, taking into account Bendixon's principle, theorems about the change of sign of the divergence of an unknown vector function and the existence of closed trajectories in some singly connected domain in a single connected area containing a point (the starting point of the trajectory) located on a semi-line have been proved [2].

This paper presents the description of the interaction of the Svan and Georgian-Colchian populations of the second period computer modeling of a nonlinear dynamic system (1), (2) with general variable coefficients. Exponential, qualitative and trigonometric functions are taken as variable coefficients. Numerous computer experiments have been conducted.

Computer modeling of the first period showed us that the Proto-Kartvelian population increased from 1 million to about 3.5 million. We assume that about 0.5 million Pelasgians went to southern Europe. The remaining 3 million Proto-Kartvelian population was divided into two parts and we assume that at the beginning of the second period, the number of Georgian-Colchian and Svan populations were $u(0) = 2.4 \cdot 10^6$ and $v(0) = 0.6 \cdot 10^6$, respectively. The interval of consideration of the second period is $[0, T]$, where $T = 1500$ years.

According to the historical materials and some logic, we have selected the values of the unknown functions which are placed in the following intervals: $3 \cdot 10^6 < u(1500) < 4 \cdot 10^6$, $0.25 \cdot 10^6 < v(1500) < 0.4 \cdot 10^6$.

176,096 cases were obtained for the exponential functions

$$\begin{cases} \frac{du(t)}{dt} = \alpha_{20} e^{\alpha_{21}\frac{t}{T}} u(t) - \beta_{10} e^{\beta_{11}\frac{t}{T}} u^2(t) + \beta_{20} e^{\beta_{21}\frac{t}{T}} u(t)v(t) - p_{10} e^{p_{11}\frac{t}{T}} u(t) \\ \frac{dv(t)}{dt} = \alpha_{30} e^{\alpha_{31}\frac{t}{T}} v(t) - \beta_{30} e^{\beta_{31}\frac{t}{T}} v^2(t) - \beta_{40} e^{\beta_{41}\frac{t}{T}} u(t)v(t), \end{cases}$$

439,056 cases were considered for the qualitative functions

$$\begin{cases} \frac{du(t)}{dt} = \alpha_{20} \left(\frac{t+1}{T}\right) u(t) - \beta_{10} \left(\frac{t+1}{T}\right) u^2(t) + \beta_{20} \left(\frac{t+1}{T}\right) u(t)v(t) - p_{10} \left(\frac{t+1}{T}\right) u(t) \\ \frac{dv(t)}{dt} = \alpha_{30} \left(\frac{t+1}{T}\right) v(t) - \beta_{30} \left(\frac{t+1}{T}\right) v^2(t) - \beta_{40} \left(\frac{t+1}{T}\right) u(t)v(t), \end{cases}$$

354,294 cases were obtained for trigonometric functions

$$\begin{cases} \frac{du(t)}{dt} = \alpha_{20} \sin\left(\frac{6(t+1)}{T}\right) u(t) - \beta_{10} \sin\left(\frac{1.1t}{T} + \frac{\pi}{3}\right) u^2(t) \\ +\beta_{20} \sin\left(\pi\left(\frac{t}{2T} + \frac{1}{6}\right)\right) u(t)v(t) - p_{10} \sin\left(\pi\left(\frac{t}{2T} + \frac{1}{4}\right)\right) u(t) \\ \frac{dv(t)}{dt} = \alpha_{30} \sin\left(\frac{3(t+1)}{T}\right) v(t) - \beta_{30} \sin\left(\frac{2.3t}{T} + \frac{\pi}{4}\right) v^2(t) - \beta_{40} \sin\left(\pi\left(\frac{t}{2T} + \frac{1}{3}\right)\right) u(t)v(t), \end{cases}$$

Numerous computer experiments allow us to make the following conclusion.

3 Conclusion. In the case of a positive demographic factor of the Svan population, and a variable demographic factor of the Georgian-Colchian population, it is accepted for the exponential, qualitative and trigonometric functions that the Svan population decreased from approximately 0.6 million, despite increasing demographic factor, as a result of assimilation to 0.25-0.4 million, and the Georgian-Colchian population increased from 2.4 million to 3-4 million.

Thus, computer modeling in the case of variable coefficients, as well as mathematical analysis in the case of constant coefficients, showed us that there is no complete assimilation of the Svan population by the Georgian-Colchian population, and these two populations coexist in one geographical region.

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Author(s) address(es):

Temur Chilachava, Gia Kvashilava, George Pochkhua
Sokhumi State University
Politkovskaya str. 61, 0186 Tbilisi, Georgia
E-mail: temo_chilachava@yahoo.com, gia.kvashilava@tsu.ge, g.pochkhua@sou.edu.ge