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# DYNAMICS AND RENORMDYNAMICS

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**Abstract**. Concise introduction in QCD Renormdynamics and Hamiltonization methods of the dynamical systems with some application.

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**1. Renormdynamics.** Quantum field theory (QFT) and Fractal calculus provide Universal language of fundamental physics (see e.g. [6] ). In QFT existence of a given theory means, that we can control its behavior at some scales by renormalization theory [2]. If the theory exists, then we want to solve it, which means to determine what happens on other scales. This is the problem (and content) of Renormdynamics. The result of the Renormdynamics, the solution of its discrete or continual motion equations, is the effective QFT on a given scale (different from the initial one). Perturbation theory series have the following qualitative form

$$f(x)=\sum_{n\geq 0}P(n)n!x^n=P(\delta)\Gamma(1+\delta)\frac{1}{1-x},\ \delta=x\frac{d}{dx}, \tag{1}$$

So, we reduce previous series to the standard geometric progression series. This series is convergent for $|x|<1$ or for $|x|_p=p^{-k}<1,\ x=p^k a/b,\ k\geq 1,\ p=2,3,5,...,29,...,137,...$ With an appropriate nomalization of the expansion parameter, the coefficients of the series are rational numbers and if experimental data indicates for some prime value for $x$, e.g. in QED, $x=\alpha=e^2/(4\pi)=1/137.036...$, then we can take corresponding prime number and consider p-adic convergence of the series. In the Yukawa theory of strong interactions (see e.g. [1] ), we take $x=\alpha_{\pi N}=13$. So, the series is convergent. If the limit is a rational number, we consider it as an observable value of the corresponding physical quantity. In $MSSM$ (see [4] ) coupling constants unify at $\alpha_u^{-1}=26.3\pm 1.9\pm 1$. So, $23.4<\alpha_u^{-1}<29.2$.
Question: how many primes are in this interval? (24, 25, 26, 27, 28, 29) Only one!
Proposal: take the value $\alpha_u^{-1}=29.0...$ which will be two orders of magnitude more precise prediction and find the consequences for the $SM$ scale observables.

**2. Renormdynamics (RD) of QCD.** The RD equation for the coupling constant

$$\dot{a}=\beta(a)=\beta_2a^2+\beta_3a^3+\beta_4a^4+\beta_5a^5+..., \tag{2}$$

can be reparametrized,

$$a(t)=f(A(t))=A+f_2A^2+...+f_nA^n+...=\sum_{n\geq 1}f_nA^n, \tag{3}$$

$$\dot{A}=b_1A+b_2A^2+...=\sum_{n\geq 1}b_nA^n,\ \dot{a}=\sum_n\beta_na^n=\dot{A}f'(A),$$

$$\sum_n \beta_n(\sum_m f_m A^m)^n = \sum_n b_n A^n \sum_m m f_m A^{m-1}$$

$$b_1 = \beta_1,\ b_2 = \beta_2 + f_2\beta_1 - 2f_2 b_1 = \beta_2 - f_2\beta_1,$$

$$b_3 = \beta_3 + 2f_2\beta_2 + f_3\beta_1 - 2f_2 b_2 - 3f_3 b_1 = \beta_3 + 2(f_2^2 - f_3)\beta_1,$$

$$b_4 = \beta_4 + 3f_2\beta_3 + f_2^2\beta_2 + 2f_3\beta_2 - 3f_4 b_1 - 3f_3 b_2 - 2f_2 b_3, ... \tag{4}$$

so, by reparametrization, beyond the critical dimension ($\beta_1 \neq 0$) we can change any coefficient but $\beta_1$. We can fix any higher coefficient with zero value, $b_n = 0,\ n \geq 2$, if we take $f_2 = \beta_2/\beta_1,\ f_3 = \beta_3/2\beta_1 + f_2^2,$ ... In the critical dimension of the space-time, $\beta_1 = 0$ and we can change by reparametrization any coefficient but $\beta_2$ and $\beta_3$. From the relations (4), in the critical dimension ($\beta_1 = 0$), we find the minimal form of the RD equation $\dot{A} = \beta_2 A^2 + \beta_3 A^3$, with the solution as the following implicit function,

$$u^{\beta_3/\beta_2} e^{-u} = c e^{\beta_2 t},\ u = \frac{1}{A} + \frac{\beta_3}{\beta_2}. \tag{5}$$

Then, as in the noncritical case, explicit solution for $a$ will be given by reparametrization representation (3) [7]. If we know somehow the coefficients $\beta_n$, e.g. for first several exact and for others asymptotic values (see e.g. [5] ) then we can construct reparametrization function (3) and find the dynamics of the running coupling constant. This is similar to the action-angular canonical transformation of the analytic mechanics (see e.g. [3] ). Statement: The reparametrization series for $a$ is p-adically convergent, when $\beta_n$ and $A$ are rational numbers.

It was noted [8] that parton densities given by the solution of the Altarelli-Parisi equation with the constituent-valence quark initial condition at a valence quark scale $m$ and $\alpha_s(m^2) = 2$, gives the experimental values for the moments of parton densities. We have seen, that for $\pi\rho N$ model $\alpha_{\pi\rho N} = 3$, and for $\pi N$ model $\alpha_{\pi N} = 13$. It is nice that $\alpha_s^2 + \alpha_{\pi\rho N}^2 = \alpha_{\pi N}$; to $\alpha_s = 2$ corresponds $g = \sqrt{4\pi\alpha_s} = 5.013 = 5+$.

**3. Hamiltonization of dynamical systems.** Let us consider the following system of the ordinary differential equations [9]

$$\dot{x}_n = v_n(x),\ 1 \leq n \leq N, \tag{6}$$

Lagrangian,

$$L = (\dot{x}_n - v_n(x))\psi_n \tag{7}$$

and the corresponding motion equations

$$\dot{x}_n = v_n(x), \dot{\psi}_n = -\frac{\partial v_m}{\partial x_n}\psi_m. \tag{8}$$

The system (8) extends the general system (6) by linear equation for the $\psi$. The extended system can be put in the Hamiltonian form [12].

In the Faddeev-Jackiw formalism [11] for the Hamiltonian treatment of systems defined by first-order Lagrangians,

$$L = f_n(x)\dot{x}_n - H(x), \tag{9}$$

motion equations

$$f_{mn}\dot{x}_n = \frac{\partial H}{\partial x_m}, \tag{10}$$

for the regular structure function $f_{mn}$, can be put in the explicit Hamiltonian form

$$\dot{x}_n = f^{-1}_{nm}\frac{\partial H}{\partial x_m} = \{x_n, x_m\}\frac{\partial H}{\partial x_m} = \{x_n, H\}, \tag{11}$$

where the fundamental Poisson (Dirac) bracket is

$$\{x_n, x_m\} = f^{-1}_{nm},\ f_{mn} = \partial_m f_n - \partial_n f_m. \tag{12}$$

The system (8) is an important example of the first order regular Hamiltonian systems. Indeed, in the new variables, $y^1_n = x_n, y^2_n = \psi_n$, Lagrangian (7) takes the following first order form

$$\begin{aligned} L &= (\dot{x}_n - v_n(x))\psi_n \Rightarrow \frac{1}{2}(\dot{x}_n\psi_n - \dot{\psi}_n x_n) - v_n(x)\psi_n \\ &= \frac{1}{2}y^a_n\varepsilon^{ab}\dot{y}^b_n - H(y) = f^a_n(y)\dot{y}^a_n - H(y), \\ f^a_n &= \frac{1}{2}y^b_n\varepsilon^{ba},\ H = v_n(y^1)y^2_n,\ f^{ab}_{nm} = \frac{\partial f^b_m}{\partial y^a_n} - \frac{\partial f^a_n}{\partial y^b_m} = \varepsilon^{ab}\delta_{nm}; \end{aligned} \tag{13}$$

corresponding motion equations and the fundamental Poisson brackets are

$$\dot{y}^a_n = \varepsilon_{ab}\delta_{nm}\frac{\partial H}{\partial y^b_m} = \{y^a_n, H\},\ \{y^a_n, y^b_m\} = \varepsilon_{ab}\delta_{nm}. \tag{14}$$

Nabu mechanics (NM) [14,16] is a proper generalization of the HM, which makes the difference between dynamical systems with different numbers of integrals of motion explicit (see, e.g. [13] ). In Nambu formulation, the Poisson bracket is replaced by the Nambu bracket with $n+1, n \geq 1$, slots.

The quasi-classical description of the motion of a relativistic point particle with spin in accelerators and storage rings includes the equations of orbit motion

$$\begin{aligned} &\dot{x}_n = f_n(x),\ f_n(x) = \varepsilon_{nm}\partial_m H,\ \varepsilon_{n,n+3} = 1,\ n = 1,2,3;\ n,m = 1,2,...,6; \\ &H = e\Phi + c\sqrt{\wp^2 + m^2c^2},\ x_n = q_n,\ x_{n+3} = p_n,\ \wp_n = p_n - \frac{e}{c}A_n \end{aligned} \tag{15}$$

and Thomas-BMT equations [15,10] of classical spin motion

$$\begin{aligned} &\dot{s}_n = \varepsilon_{nmk}\Omega_m s_k = \{H_1, H_2, s_n\},\ H_1 = \Omega\cdot s,\ H_2 = s^2, \\ &\{A, B, C\} = \varepsilon_{nmk}\partial_n A\partial_m B\partial_k C, \\ &\Omega_n = \frac{-e}{m\gamma c}((1+k\gamma)B_n - k\frac{(B\cdot\wp)\wp_n}{m^2c^2(1+\gamma)} + \frac{1+k(1+\gamma)}{mc(1+\gamma)}\varepsilon_{nmk}E_m\wp_k), \end{aligned} \tag{16}$$

where, parameters $e$ and $m$ are the charge and the rest mass of the particle, $c$ is the velocity of light, $k = (g-2)/2$ quantifies the anomalous spin $g$ factor, $\gamma$ is the Lorentz factor, $p_n$ are components of the kinetic momentum vector, $E_n$ and $B_n$ are the electric and magnetic fields. We put the spin motion equations in the Nambu-Poisson form. The general method of Hamiltonization of the dynamical systems are also used in the spinning particle case. We conceder unified configuration space $q = (x, p, s),\ x_n = q_n,\ p_n = q_{n+3},\ s_n = q_{n+6},\ n = 1, 2, 3$; extended phase space, $(q_n, \psi_n)$; Hamiltonian and motion equations

$$H = H(q, \psi) = v_n \psi_n,\ n = 1, 2, ...9; \qquad \dot{q}_n = v_n(q),\ \dot{\psi}_n = -\frac{\partial v_m}{\partial q_n}\psi_m, \tag{17}$$

where $v_n$ depends on external fields as control parameters which can be determined according to the optimal control criterion.

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