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SEMI-DISCRETE SCHEME FOR ONE SYSTEM OF NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS WITH SOURCE TERMS

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Abstract. One system of nonlinear integro-differential equations with source terms is considered. The model is based on the well known Maxwell system. Semi-discrete difference scheme is studied.

Keywords and phrases: System of nonlinear integro-differential equations, semi-discrete difference scheme.

AMS subject classification: 45K05, 65M06.

One nonlinear integro-differential model arising on mathematical simulation of the process of penetration of a magnetic field into a substance [1] is considered. This model were introduced after reduction of nonlinear Maxwell's differential system [2] to the integro-differential form. One-dimensional simple analog with source terms has the following form:

$$
\begin{gathered}
\frac{\partial U}{\partial t}-\frac{\partial}{\partial x}\left\{\left(1+\int_0^t\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]d\tau\right)^p\frac{\partial U}{\partial x}\right\}+|U|^{q-2}U=0,\\
\frac{\partial V}{\partial t}-\frac{\partial}{\partial x}\left\{\left(1+\int_0^t\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]d\tau\right)^p\frac{\partial V}{\partial x}\right\}+|V|^{q-2}V=0,
\end{gathered}
\tag{1}
$$

where $0<p\le 1$, $q\ge 2$.

Many works are dedicated to the investigation and numerical resolution of the integro-differential (1) type models described in [1]. Especially, in [1], [3]-[9] solvability and uniqueness of the initial-boundary value problems for these type equations and systems are studied. Asymptotic behavior of solutions as $t\to\infty$ is investigated in many works also (see, for example, [7]-[18] and references there in). Numerical resolution by finite difference scheme is given in works [9], [13]-[15], [18]-[20] and in a number of other works as well.

The aim of this note is to construct and study semi-discrete scheme for the system (1).

In the $[0,1]\times[0,T)$, where T is positive number, let us consider the following initial-boundary value problem for system (1):

$$
\begin{gathered}
U(0,t)=U(1,t)=V(0,t)=V(1,t)=0,\\
U(x,0)=U_0(x),\quad V(x,0)=V_0(x),
\end{gathered}
\tag{2}
$$

where $U_0 = U_0(x)$ and $V_0 = V_0(x)$ are given functions.

On [0,1] let us introduce a net with mesh points denoted by $x_i = ih$, $i = 0, 1, \ldots, M$, with $h = 1/M$. The boundaries are specified by $i = 0$ and $i = M$. The semi-discrete approximation at (x_i, t) is designed by $u_i = u_i(t)$ and $v_i = v_i(t)$. The exact solution to the problem at (x_i, t) is denoted by $U_i = U_i(t)$ and $V_i = V_i(t)$. At points $i = 1, 2, \ldots, M-1$, the integro-differential equation will be replaced by approximation of the space derivatives by a forward and backward differences. We will use the following known notations:

$$r_{x,i}(t) = \frac{r_{i+1}(t) - r_i(t)}{h}, \quad r_{\bar{x},i}(t) = \frac{r_i(t) - r_{i-1}(t)}{h}.$$

Using usual methods of construction of discrete analogs (see, for example, [21]) let us construct the following semi-discrete scheme for problem (1),(2):

$$\begin{gathered}
\frac{du_i}{dt} - \left\{ \left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2 \right] d\tau \right)^p u_{\bar{x},i} \right\}_x + |u_i|^{q-2} u_i = 0, \\
\frac{dv_i}{dt} - \left\{ \left(1 + \int_0^t \left[(u_{\bar{x},i})^2 + (v_{\bar{x},i})^2 \right] d\tau \right)^p v_{\bar{x},i} \right\}_x + |v_i|^{q-2} v_i = 0, \\
i = 1, 2, \ldots, M-1,
\end{gathered} \tag{3}$$

$$u_0(t) = u_M(t) = v_0(t) = v_M(t) = 0, \tag{4}$$

$$u_i(0) = U_{0,i}, \quad v_i(0) = U_{0,i}, \quad i = 0, 1, \ldots, M. \tag{5}$$

It is not difficult to show validity of the following estimations for solution of (3)-(5) problem:

$$\|u(t)\|^2 + \int_0^t \|u_{\bar{x}}]|^2 d\tau \le C, \quad \|v(t)\|^2 + \int_0^t \|v_{\bar{x}}]|^2 d\tau \le C, \tag{6}$$

where, here and below, C denotes a positive constant which does not depend on h and the norms $\|\cdot\|$ and $\|\cdot]|$ are defined as follows:

$$\|r\| = \left(h \sum_{i=1}^{M-1} r_i^2 \right)^{1/2}, \qquad \|r]| = \left(h \sum_{i=1}^{M} r_i^2 \right)^{1/2}.$$

The a priori estimate (6) guarantee the stability of the scheme (3) and global solvability of the problem (3) - (5).

The following statement takes place.

Theorem. *If $0 < p \le 1$, $q \ge 2$ and the initial-boundary value problem (1),(2) has the sufficiently smooth solution $U = U(x,t)$, $V = V(x,t)$, then the semi-discrete scheme (3)-(5) converges and the following estimate is true*

$$\left\| u^j - U^j \right\| + \left\| v^j - V^j \right\| \le Ch.$$

Note that for solving the finite difference scheme corresponding to (3)-(5) we use an algorithm analogical to [19]. So, it is necessary to use Newton iterative process [22]. According to this method the great numbers of numerical experiments are carried out. These experiments agree with the theoretical result given in the Theorem.

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