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SOME UNSTEADY PROBLEMS OF CONDUCTING FLUID NEAR AN INFINITE FLAT PLATE WITH EXTERNAL MAGNETIC FIELD

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In this paper two cases of unsteady flow of viscous incompressible conductive fluid are considered. They arise with the motion of infinitely plate in the presence of transverse magnetic field.

1. In the first case we consider unsteady flow of viscous conductive fluid near infinitelly plane plate, when plate begins impulsively motion at $t = 0$ with a velocity $u_w(t)$ in the presence of non-homogeneous external magnetic field [1]-[4]:

$$\vec{B}^l \Big\{0,\ -\frac{B_0 y}{a},\ \frac{B_0 z}{a}\}. \tag{1}$$

In the following case we take into account induction of magnetic field in the fluid:

$$\vec{B}^i\{b(z,t),0,0\}.$$

The equations which describe the flow of electrically conductive fluid and induction of magnetic field in the fluid with boundary and initial conditions in non-dimensional quantities are [1-4]

$$\begin{gathered}\frac{\partial^2 v}{\partial z^2} - \frac{\partial v}{\partial t} + Ha\, z\, \frac{\partial b}{\partial z} = 0,\\ \frac{\partial^2 b}{\partial z^2} - \frac{\partial b}{\partial t} + Ha\, z\, \frac{\partial v}{\partial z} = 0,\end{gathered} \tag{2}$$

$$\begin{array}{lll} v(z,0) = 0, & v(0,t) = u_w(t), & v(\infty,t) = 0,\\ b(z,0) = 0, & b(0,t) = 0, & b(\infty,t) = 0,\end{array} \tag{3}$$

where $Ha = B_0 a\sqrt{\frac{\sigma}{\rho\nu}}$ is Hartmann number. As for the non-dimensional quantities, they are

$$v = \frac{\nu_m}{a}\,\overline{v},\quad b = \frac{1}{a}\sqrt{\frac{\rho\nu}{\sigma}}\,\overline{b},\quad z = a\overline{z},\quad t = \frac{a^2}{\nu}\,\overline{t},\quad u_w(t) = \frac{\nu_m}{a}\,\overline{u}_w(\overline{t}\,).$$

Here $v(z,t)$ is velocity of fluid, $b(z,t)$ is induction of magnetic field, $u_w(t)$ is the velocity of the plate, ν is the coefficient of kinematic viscosity, $\nu_m = \frac{1}{\sigma\mu_0}$ is magnetic kinematic viscosity. Assume $\nu = \nu_m$, $u_w(0) = 0$.

Problem (2), (3) can be solved by the Laplace transformation method and with successive approximations, when $Ha \ll 1$.

Then the velocity of fluid and the magnetic field in the first two approximations will be the following solutions:

$$v(z,t) \approx v_0(z,t) + Ha\, v_1(z,t) =$$
$$= \int_0^t u_w(\tau)\left[\frac{1}{2\sqrt{\pi(t-\tau)^3}} - \frac{Ha}{4\sqrt{\pi(t-\tau)}}\right] z e^{-\frac{z^2}{4(t-\tau)}}\, d\tau,$$
$$b(z,t) \approx b_0(z,t) + Ha\, b_1(z,t) = -\frac{Ha}{16}\int_0^t u_w(\tau)\frac{z^3}{\sqrt{\pi(t-\tau)}}\, e^{-\frac{z^2}{4(t-\tau)}}\, d\tau.$$

In particular case when plate starts motion with constant velocity ($u_w(t) = u_w = const$) we will have:

$$v(x,t) = u_w\Big[\mathrm{erfc}\frac{z}{2\sqrt{t}} - Ha\left(\frac{z}{2}\sqrt{\frac{t}{\pi}}\, e^{-\frac{z^2}{4t}} - \frac{z^2}{4}\,\mathrm{erfc}\frac{z}{2\sqrt{t}}\right)\Big],$$

$$b(z,t) = -\frac{Ha\, u_w z^2}{4}\,\mathrm{erfc}\frac{z}{2\sqrt{t}}.$$

2. In the second case we assume that the infinitely plane plate is porous. If $v_w(t)$ is the suction velocity and external magnetic field is constant, then equations of magnetic hydrodynamics and boundary and initial conditions in non-dimensional quantities are:

$$\begin{aligned}
&\frac{\partial^2 v}{\partial z^2} + R_0 v_w(t)\,\frac{\partial v}{\partial z} - \frac{\partial v}{\partial t} + Ha\,\frac{\partial b}{\partial z} = 0,\\
&\frac{\partial^2 b}{\partial z^2} + R_0 v_w(t)\,\frac{\partial b}{\partial z} - \frac{\partial b}{\partial t} + Ha\,\frac{\partial v}{\partial z} = 0,\\
&v(z,0) = u_w(0) = 0, \quad v(0,t) = u_w(t), \quad v(\infty,t) = 0,\\
&b(z,0) = 0, \quad b(0,t) = 0, \quad b(\infty,t) = 0.
\end{aligned} \tag{4}$$

Here $R_0 = \frac{v_0 a}{\nu}$ is Reynolds percolation number. If $v_w(t) = 1$, then Problem (4) can be solved by the Laplace transformation method and with successive approximations, when $R_0 \ll 1$.

Then the velocity of fluid and the magnetic field in the first two approximations will be the following solutions:

$$v(z,t) \approx v_0(z,t) + R_0 v_1(z,t) =$$

$$= \left(1 - \frac{R_0 z}{2}\right) \int_0^t u_w(\tau) L_0(z, t-\tau) d\tau \cdot \mathrm{ch}\frac{Ha\, z}{2} +$$

$$+ R_0 \frac{Ha\, z}{4} \int_0^t u_w(\tau) L_1(z, t-\tau) d\tau \cdot \mathrm{sh}\frac{Ha\, z}{2},$$

$$b(z,t) \approx b_0(z,t) + R_0 b_1(z,t) =$$

$$= \left(\frac{R_0 z}{2} - 1\right) \int_0^t u_w(\tau) L_0(z, t-\tau) d\tau \cdot \mathrm{sh}\frac{Ha\, z}{2} -$$

$$- R_0 \frac{Ha\, z}{4} \int_0^t u_w(\tau) L_1(z, t-\tau) d\tau \cdot \mathrm{ch}\frac{Ha\, z}{2}.$$

In case when plate starts motion with constant velocity ($u_w = const$) we will have:

$$v(x,t) \approx \frac{u_w}{2} \left[\left(1 - \frac{R_0 z}{2}\right) f_+(z,t) \mathrm{ch}\frac{Ha\, z}{2} + R_0 \frac{z}{2\sqrt{2}} f_-(z,t) \mathrm{sh}\frac{Ha\, z}{2} \right],$$

$$b(x,t) \approx -\frac{u_w}{2} \left[\left(1 - \frac{R_0 z}{2}\right) f_+(z,t) \mathrm{sh}\frac{Ha\, z}{2} + R_0 \frac{z}{2\sqrt{2}} f_-(z,t) \mathrm{ch}\frac{Ha\, z}{2} \right],$$

where

$$f_\pm(z,t) = e^{-\frac{Ha}{2} z} \mathrm{erfc}\left(\frac{z}{2\sqrt{t}} - \frac{Ha\sqrt{t}}{2}\right) \pm e^{\frac{Ha}{2} z} \mathrm{erfc}\left(\frac{z}{2\sqrt{t}} + \frac{Ha\sqrt{t}}{2}\right),$$

$$L_0(z,t) = \frac{1}{2\pi i} \int_{p-i\infty}^{p+i\infty} e^{-\sqrt{\frac{Ha^2}{4} + s}\, z} e^{st} dt,$$

$$L_1(z,t) = \frac{1}{2\pi i} \int_{p-i\infty}^{p+i\infty} \frac{e^{-\sqrt{\frac{Ha^2}{4} + s}\, z} e^{st}}{\sqrt{\frac{Ha^2}{4} + s}} dt.$$

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