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THE METHODS OF HUMIDITY FIELDS COUNTINGS IN MESOMETEOROLOGICAL MODELS

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Abstract

Micro and macro physical methods of water phase transformation in mesoscale boundary layer of atmosphere is considered. The fog- and cloud formation process is simulated by one of their methods.

Let us consider the two-dimensional (x-z plane) nonstationary problem about a mesoscale boundary layer of atmospher (MBLA) over an thermal and humidity nonhomogeneous underlying surface taking into account humidity processes. We do not give in detail the simplification, which is necessary to obtain the basic equations. That is we have the equations of dynamics, heat, water-vapour and liquid-water mixing ratio.

Therefore initial basic equations have the following form [1,2]:

$$\frac{du}{dt} = -\frac{\partial \pi}{\partial x} + \Delta' u, \qquad (1)$$

$$\frac{\partial \pi}{\partial z} = \lambda \vartheta, \qquad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \qquad (3)$$

$$\frac{d\vartheta}{dt} + Sw = \frac{L}{c_p}\Phi + \Delta' \vartheta, \qquad (4)$$

$$\frac{dq}{dt} + \gamma_q w = -\Phi + \Delta' q, \qquad (5)$$

$$\frac{dv}{dt} = \Phi + \Delta' v, \qquad (6)$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z},$$

$$\Delta' = \mu\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial z^2}$$

where u, w are the horizontal and vertical components of air velocity, respectively, π , ϑ, q - the deviations of pressure analogy, potential temperature and water-vapour mixing ratio from their undisturbed fields, respectively, v - the liquid-water mixing ratio, λ, S- the parameters of atmospheric flotation and stratification, respectively, γ_q - the vertical gradient of undisturbed water-vapour mixing ratio, Φ- the rate of water-vapour condensation, L - the latent heat of condensation, c_p - the specific heat of dry air at constant pressure, μ, ν - the horizontal and vertical coefficients of turbulence, respectively.

General characteristic boundary and initial conditions have such view:

$$
\begin{array}{ll}
z = 0 & u = 0, \quad w = 0, \quad \vartheta = F1(x,t), \quad q = F2(x,t), \quad v = 0, \\
z = Z & u = 0, \quad \pi = 0, \quad \vartheta = 0, \quad \dfrac{\partial q}{\partial z} = 0, \quad \dfrac{\partial v}{\partial z} = 0, \\
x = 0,\ X & \dfrac{\partial u}{\partial x} = 0, \quad \dfrac{\partial \vartheta}{\partial x} = 0, \quad \dfrac{\partial q}{\partial x} = 0, \quad \dfrac{\partial v}{\partial x} = 0, \\
t = 0 & u = 0, \quad \vartheta = 0, \quad q = 0, \quad v = 0,
\end{array}
\qquad (7)
$$

where X, Z are the horizontal and vertical boundaries of MBLA, F1, and F2 - the given function from meteorological experiments.

Humidity processes are one of the essential factors, which are responsible for fog- and cloud formation, heat stream at water phase transformation, radiate and absorb of the radiation. While solving the humidity fields forecast problem we encounter the some difficulties, one of which is taking into account the condensation velocityΦ, included in the heat conductivity, water-vapour and liquid-water equations (4) - (6).

Actually, in [1] Φ has such view with exactly turbulence terms:

$$
\Phi = k\,\frac{c_p}{L}\,(\gamma_a - \gamma_b)\, w \qquad\qquad k = \begin{cases} 1 & at \quad q \ge q_s \\ 0 & at \quad q < q_s \end{cases} \qquad (8)
$$

where γ_a , γ_b -dryadiabatic and moistadiabatic temperature lapse rate, respectively, q, q_s - water-vapour mixing ratio and saturation water-vapour mixing ratio, respectively.

On the boundary of the saturation region (cloud) Φ has discontinuity, as out of cloud Φ = 0, and, consequently, while solving these equations an instability takes place in the numerical calculation.

And therefore in papers, where it is used directly method of Φ (8) estimate, the numerical simulation is possible for only short time interval because of unstable account.

Besides under the directly solution of the initial equation system (1) - (6) it is necessary the use highly complicated special monotonous finite-difference schemes.

The first step in order to remove these difficulties was done by Shvetch and his collaborators [3]. They worked out the forecast method, established on analysis of the equation for a moisture deficit and a dew point, in order to investigation of a fog- and cloud formation process.

The next step in this direction were the investigation, pushed off the ideas of Matveev [4] , which assumes a quite carry along clouds drops by turbulent air particles. Matveev excluded the condensation terms from (4) - (6) and introduces the new variables the total water content and the equivalent potential temperature, which are invariant relatively a phase transformation of humidity. He solved these equations relatively them and thus described a cloud evolution.

Pastushkov [5], in distinction of these authors, chooses the other method to get out of an embarrassing situation: he avoids the directly taking into account of condensation velocity by use virtual temperature instead of potential temperature.

The directly taking into account of condensation velocity was avoided by Dimnikov [6] using the new function, which is combination of the temperature, the water-vapour mixing ratio. This method is effective for problems of background weather forecast and common circulation.

Naturally, the previously mentioned models do not investigate cloud microphysics process of precipitation.

Now we shall consider models of cloud precipitation, which problem is solved only by equations of thermohydrodynamics in.

In papers of Srivastava, Orville and their collaborators [7, 8] there are given equations for rainwater content, cloud water content and total water content. It is formulated very complicated mechanism of self-transformation these water fraction, introduced elements of microphysics (the splash of drops, processes of condensation and coagulation).

Analogical manner was applied by Takeda [9] in order to investigation of cloud microphysics. He unlike Orville and Srivastava the drop spectra divided on 7 fraction (drop radius):1, 5, 20, 100, 200, 1000, 3000 μ . It is wrote the individual differential equation for each drop radius. It is take into account such microphysical process, as self-transformation of different drop fraction, condensation, evaporation, coagulation and drop capture. Naturally, mass of liquid water is constant at these transformations.

Of course, the investigation of cloud processes from the point of view of detailed mechanism of microphysics has more universal, temporary character. In this case we have the system of thermohydrodynamic equations (1) - (6) and the equation of cloud drop distribution function [10, 11].

In initial stage of cloud development condensation process prevails over coagulation and because it is firstly solved condensation equation and then it is solved coagulation equation. Thus the kinetics equation of cloud drop size distribution function (f) has such view at only condensation drop growth:

$$\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}+\frac{\partial}{\partial r}(\dot{r}f)+(w-c_s r^2)\frac{\partial f}{\partial z}=\nu\frac{\partial^2 f}{\partial z^2}+\mu\frac{\partial^2 f}{\partial x^2}\,; \qquad (9)$$

The velocity of drop growth

$$\dot{r}=\frac{dr}{dt}=\frac{D\Delta\rho_a}{\rho\eta r}$$

And velocity of condensation

$$\Phi=4\pi\rho\int_0^\infty \dot{r}r^2\, f(r,x,z,t)\,dr; \qquad (10)$$

where D, Δ are coefficients of water vapour diffusion and saturation, respectively, $\rho\,(\rho_a)$ - the water (air) density, η - correction on temperature difference of air and drop, c_s - Stoks parameter.

Let us remark, that in the case microphysics manner Φ (10) is estimated obviously, that is we elude those difficulties, which are at pure thermohydrodynamical modelling.

We have initial condition for (9):

at $t=t_c$ $\qquad f(r)=N_m\,\delta(r)$

This is the condensation nuclear activation, that is, when we first have achievement saturation condition (t_c) on some height, it is given certain number of condensation nuclear.

The boundary condition for (9) has such view:

On G (G is cloud boundary) f (r) = 0.

In a cloud there is activated the coagulation process on the late stage. The equation of coagulation has such view:

$$\frac{\partial f}{\partial t}+div\,(\vec{u}\,f)=\tfrac{1}{2}\int_0^{v_k}P(v'_k,v_k-v'_k)\,f(v_k-v'_k)\,f(v'_k)\,dv'_k-\int_0^{\infty}P(v_k,v'_k)\,f(v_k)\,f(v'_k)\,dv'_k;$$

where $\vec{u}$ is the drop velocity, $P\,(v_k\,,v'_k)$ - the probability of collision of drop between volumes v_k and v'_k .

The detail taking into account of cloud microphysics connects as with new physical difficulties, so with mathematical ones – now we have a system of integro-differential equations.

In relation to our work we use method of taking into account water phase transformation, described in [12], in our meso-meteorological models. It relates to previously mentioned direction [4], but it allows accounting more exactly fields of temperature and other meteo-elements in a cloud. According to this method, Φ was excluded from (4) -(6) and (5) - (6) and we have:

$$\frac{dA}{dt}+Sw=\Delta' A,$$

$$\frac{dB}{dt}+\gamma_q w=\Delta' B,$$

where

$$A=\begin{cases}\vartheta\\ \vartheta-L/c_p\end{cases}\qquad\qquad B=\begin{cases}q & at\quad q\ge q_s\\ q_s & at\quad q<q_s\end{cases}$$

We remark, not going into details of this method [12], that ϑ , q , v determine themselves by means of , already known, A and B.

As example let us describe a fog- and cloud formation on underground of local air circulation over thermal “island” at heating by daily march of temperature:

$$F(x,t)=\begin{cases}0 & 0\le x\le 32km,\quad 48km<x\le 80km\,,\\ 5\sin\omega\,t & 32km\le x\le 48km\,,\end{cases}$$

Where ω is an angular velocity of the Earth rotation.

The parameter of stratification S = 0.005 grad/m, relative humidity - 0.98.

Physical constants and parameters are given [13]. The model was integrated numerically by means of the finite-difference scheme with the first order accurate in time and with the second order accurate in space.

It was simulated numerically very interesting local air circulation regime, when have simultaneously stratus cloud and radiation fog (on fig. 1 are given isoline of liquid-water mixing ratio) at that moment (t = 15 hour), when a cloud is dissipating and a fog is intensifying. At this moment the cloud maximal liquid-water mixing ratio $v_{\max}$ = 0.7 g/kg and the fog v_{max} = 0.9 g/kg; the level of cloud v_{max} is 1800 m.

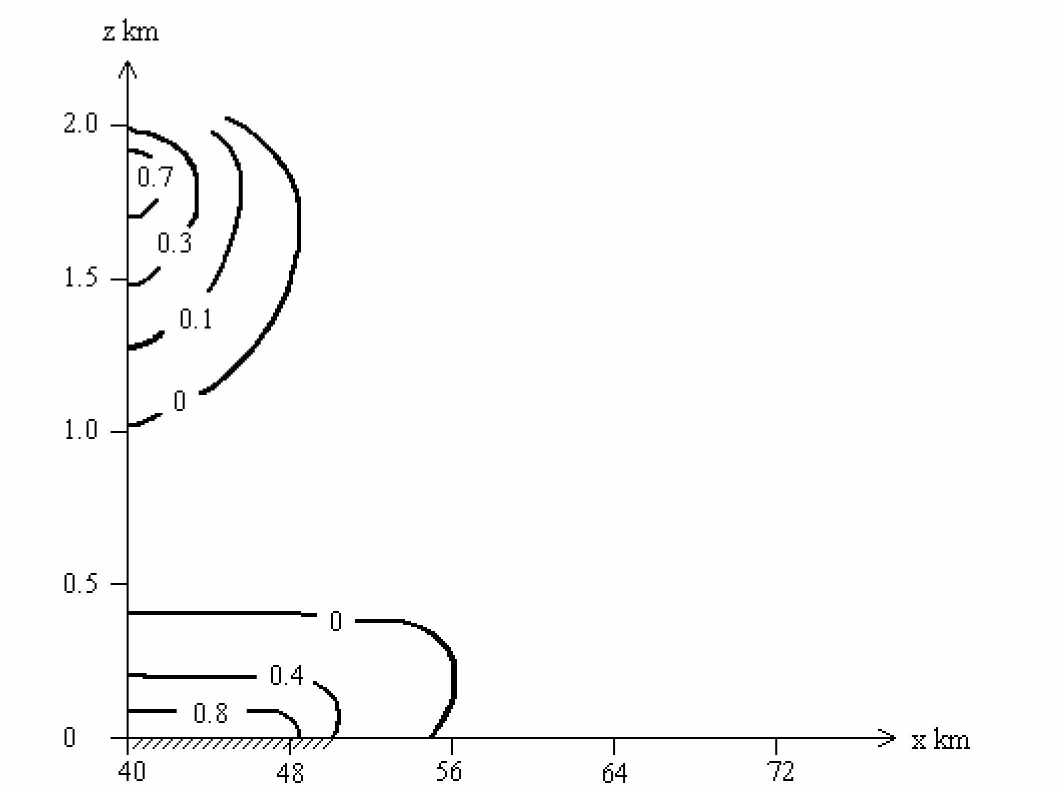

Fig. 1

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