# HEAT TRANSFER WITH THE FLOW OF CONDUCTING FLUID IN CIRCULAR PIPES WITH FINITE CONDUCTIVITY UNDER UNIFORM TRANSVERSE MAGNETIC FIELDS

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**Abstract**

The flow of conducting, viscous fluids in circular pipes under transverse magnetic field is studied theoretically. The correlation of Hartman's figure, Poisale's figure, Reynold's figure and conductivity of walls are considered.

*Keywords and phrases*: Heat transfer, circular, Flow, pipes, magnetic, fields.

*AMS subject classification (2010)*: 0000, 00000.

## 1 Introduction

Hartmann[3] carried out the pioneer work in the study of steady magneto-hydrodynamic channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall. The magnetohydrodynamic interaction under constant uniform pressure gradient is clearly demonstrated. Later Chang and Lundgren [7] solved the same problem with the channel wall of different conductivity. It is important to the basic understanding of magneto-hydrodynamics to extend this problem to include the effect of transient pressure gradient. This paper carries out such work.

It should be pointed out that the ordinary hydrodynamic channel flow under time-dependent pressure gradient is not a vailable in the literature except for Uchida's treatment of pipe flow under oscillatory pressure gradient. His work is related to a limiting case of the present work if the fluid is assumed to be electrically nonconductive, or if there is no applied magnetic field.

The problem is simplified by assuming fully developed laminar flow and a perfectly con-ducting channel wall, which leads to linear partial differential equations. Thus the equations are amenable to the methods of the placeLaplace transformation.

The basic differential equations, similar to the early work of Chang and Yen [8], are given and the initial boundary conditions are specified. For the case of a pressure gradient which is zero for time less than zero and periodic for time greater than zero, the solution can be conveniently decomposed into two parts, (a) quasi-stedy stage, and (b) initial stage. Next, the pressure gradients of step function of time and delta function of time are considered. For the step-function case, in the limit as time is allowed to go to infinity, it is shown that Hartmann's work of Chang and Lundgren [7] are confirmed. In the cases of step and pulse pressure gradients, many new and interesting features are given in contrast to the case of an electrically nonconducting fluid.

## 2 Basic part

Exploration of flows of electrically conducting fluid with two approaches are considered either in noninductive and inductive ways. Magnetohydrodynamics main equations in noninductive ($R_m << 1$) approach will be done as follows [4-7] :

$$\left\{ \begin{array}{c} \frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\nabla\right)\vec{V} = \frac{1}{\rho} grad p + \nu\Delta\vec{V} - \frac{\sigma}{\rho}\left(H \times \left(\vec{V} \times \vec{H}\right)\right), \\ div\vec{V} = 0, \quad div\vec{H} = 0, \quad rot\vec{H}, \\ \rho C_V \left(\frac{\partial T}{\partial t} + \left(\vec{V}\nabla\right) T\right) = \mathrm{K}\Delta T + \Phi + \sigma\left(\vec{V} \times \vec{H}\right)^2. \end{array} \right. \tag{1}$$

where $\sigma\left(\vec{V} \times \vec{H}\right)^2$ is a **Jole** heat, and $\Phi$ is a dissipation function as a result of friction and will be gauged as follows:

$$\Phi = 2\eta\left\{\frac{1}{2}\left[\left(\frac{\partial V_x}{\partial y}+\frac{\partial V_y}{\partial x}\right)^2+\left(\frac{\partial V_y}{\partial z}+\frac{\partial V_z}{\partial y}\right)^2+\left(\frac{\partial V_z}{\partial x}+\frac{\partial V_x}{\partial z}\right)^2\right]\right.$$
$$\left.+\left(\frac{\partial V_x}{\partial x}\right)^2+\left(\frac{\partial V_y}{\partial y}\right)^2+\left(\frac{\partial V_z}{\partial z}\right)^2\right\} \tag{2}$$

Let us consider flow of viscous incompressible weakly conducting fluid taking into account heat transfers under effects of external homogenous magnetis field ($H_0$) in square pipe. Here we may suppose that the conditions are created when the tension of electric field is equal to zero (E = 0). Induced magnetic field inside fluid is less in contrast with external magnetic field and it is ignored.

It is well known that the fluid speed has only one constituent: $\vec{V} = V_z(x,t)$ directed along axis OZ, and temperature T is considered to be the function of axis x and t (T=T (x,t)).

Taking into consideration of above mentioned the system of magneto-hydrodynamic equations in non-dimensional values is as follows[1,2,8]:

$$\begin{cases} \dfrac{\partial U}{\partial \tau}-\dfrac{\partial^2 U}{\partial R^2}-\dfrac{1}{R}\dfrac{\partial U}{\partial R}+M^2U=f(\tau), \\ P_r\dfrac{\partial \theta}{\partial \tau}-\dfrac{\partial^2 \theta}{\partial R^2}-\dfrac{1}{R}\dfrac{\partial \theta}{\partial R}=\left(\dfrac{\partial U}{\partial R}\right)^2+M^2U^2. \end{cases} \tag{3}$$

where $U = \frac{V}{V_0}$, $R = \frac{r}{a}$, $t = \frac{a^2\tau}{\nu}$, $\theta = \frac{k}{\eta V_0^2}$, $f(\tau) = -\frac{a^2}{\nu V_0 \rho}\frac{\partial P}{\partial z}$ - non-dimensional values, and $V_0$ and $a$–typical speed and length, correspondingly, $M = H_0 a\sqrt{\frac{\sigma}{\eta}}$ - **Hartmann's** number, $R_m = \frac{V_0 a}{\nu_m}$ - **Raynold's** magnetic number, $P_r = \frac{\eta C_V}{k}$ - **Prandtl's** number, $\alpha = \frac{\omega a^2}{\nu}$- simulation criterion, established by pulsating flow. $\rho$, $\omega$, $\nu$, $\eta$, $C_V$, $k$, $\sigma$, $\nu_m$ - density, frequency, kinematic viscosity, dynamic viscosity, heat capacity , heat conduction, electrical conduction and fluid magnetic viscosity coefficient, correspondingly.

Extreme conditions generally are as follows:

$$\begin{cases} U(R,0)=0, \quad U(1,\tau)=\varphi_1(\tau), \quad \theta_{1,2}(R,0)=q_{1,2}(R), \\ \theta(1,\tau)=\theta_1(1,\tau)+\theta_2(1,\tau)=q_1^{(1)}(\tau)+q_2^{(2)}(\tau)=q(\tau), \end{cases} \tag{4}$$

where $\theta_1(R,\tau)$ – temperature while in equation of heat-transfer is taken into account only viscous heat, and $\theta_2(R,\tau)$ - temperature while in equation of heat-transfer is taken into account only **Jole** heat.

It is well known that pulsating flow of fluid is caused only by pulsating drop of pressure ($f(\tau) = Ae^{i\alpha\tau}$), the pipe is not wheeled and change of temperature on the surface of the pipe is not equal to zero ($\varphi_1(\tau) = 0$, $\theta_{1,2}(R, 0) = 0$, $\theta_{1,2}(1, \tau) = B_{1,2}e^{2i\alpha\tau}$). In equation of heat-transfer is taken into account either viscous heat -$\left(\frac{\partial U}{\partial R}\right)^2$, or **Jole** heat (MU)$^2$.
Let us search solution of for the task (3) – (4) in following view [3]:

$$\begin{cases} U(R,\tau) = \varphi(R)e^{i\alpha\tau}, \\ \theta_1(R,\tau) = \psi(R)e^{2i\alpha\tau}, \\ \theta_2(R,\tau) = \psi_2(R)e^{2i\alpha\tau}. \end{cases} \tag{5}$$

Finally, for speed and heat transfer we will get :

$$U(R,\tau) = \frac{A}{M^2 + i\alpha}\left(1 - \frac{I_0\left(\sqrt{M^2 + i\alpha}R\right)}{I_0\left(\sqrt{M^2 + i\alpha}\right)}e^{i\alpha\tau}\right),$$

$$\theta_1(R,\tau) = \frac{B_1 I_0\left(\sqrt{2i\alpha P_r}R\right)}{I_0\left(\sqrt{2i\alpha P_r}\right)}e^{2i\alpha\tau} + \frac{A^2 e^{2i\alpha\tau}}{\sqrt{2i\alpha\tau P_r}\left(M^2 + i\alpha\right) I_0^2\left(\sqrt{M^2 + i\alpha}\right)}$$

$$\times\left\{\left[K_0\left(\sqrt{2i\alpha P_r}R\right)\right.\right.$$

$$\left.\times\int\frac{I_1^2\left(\sqrt{M^2 + i\alpha}R\right) I_0\left(\sqrt{2i\alpha P_r}R\right) dR}{I_0\left(\sqrt{2i\alpha P_r}R\right) K_1\left(\sqrt{2i\alpha P_r}R\right) + I_1\left(\sqrt{2i\alpha P_r}R\right) K_0\left(\sqrt{2i\alpha P_r}R\right)}\right]\Bigg]$$

$$-\left[I_0\left(\sqrt{2i\alpha P_r}R\right)\right.$$

$$\left.\left.\times\int\frac{I_1^2\left(\sqrt{M^2 + i\alpha}R\right) K_0\left(\sqrt{2i\alpha P_r}R\right) dR}{I_0\left(\sqrt{2i\alpha P_r}R\right) K_1\left(\sqrt{2i\alpha P_r}R\right) + I_1\left(\sqrt{2i\alpha P_r}R\right) K_0\left(\sqrt{2i\alpha P_r}R\right)}\right]\right\},$$

$$\theta_2(R,\tau) = \frac{B_2 I_0\left(\sqrt{2i\alpha P_r}R\right)}{I_0\left(\sqrt{2i\alpha P_r}\right)}e^{2i\alpha\tau} + \frac{A^2 M^2 e^{2i\alpha\tau}}{\sqrt{2i\alpha\tau P_r}\left(M^2 + i\alpha\right) I_0^2\left(\sqrt{M^2 + i\alpha}\right)}$$

$$\times\left\{\left[K_0\left(\sqrt{2i\alpha P_r}R\right)\right.\right.$$

$$\left.\times\int\frac{\left[I_0\left(\sqrt{M^2 + i\alpha}\right) - I_0\left(\sqrt{M^2 + i\alpha}R\right)\right]^2 I_0\left(\sqrt{2i\alpha P_r}R\right) dR}{I_0\left(\sqrt{2i\alpha P_r}R\right) K_1\left(\sqrt{2i\alpha P_r}R\right) + I_1\left(\sqrt{2i\alpha P_r}R\right) K_0\left(\sqrt{2i\alpha P_r}R\right)}\right]$$

$$-\left[I_0\left(\sqrt{2i\alpha P_r}R\right)\right.$$

$$\times \int \frac{\left[I_0\left(\sqrt{M^2+i\alpha}\right)-I_0\left(\sqrt{M^2+i\alpha}R\right)\right]^2 K_0\left(\sqrt{2i\alpha P_r}R\right)dR}{I_0\left(\sqrt{2i\alpha P_r}R\right)K_1\left(\sqrt{2i\alpha P_r}R\right)+I_1\left(\sqrt{2i\alpha P_r}R\right)K_0\left(\sqrt{2i\alpha P_r}R\right)}\Bigg]\Bigg]\Bigg\},$$

$$\theta(R,\tau)=\theta_1(R,\tau)+\theta_2(R,\tau),$$

where $\mathrm{I}_0$, $\mathrm{K}_0$ *and* $\mathrm{I}_1$, $\mathrm{K}_1$ are correspondingly the functions of zero and first order of **Bessel** and **Macdonald's.** $\left(I_0'=I_1', K_0'=-K_1\right)$.

Viscosity strength on the wall and fluid consumption through pipe profile are calculated as follows :

$$F=-\frac{A\eta}{\sqrt{M^2+i\alpha}}\cdot\frac{I_1\left(\sqrt{M^2+i\alpha}R\right)e^{i\alpha\tau}}{I_0\left(\sqrt{M^2+i\alpha}\right)},$$

$$Q=\frac{\pi A}{M^2+i\alpha}\left(1-\frac{\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\sqrt{M^2+i\alpha}\right)^{2k}}{(K+1)!\Pi(K)}}{\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\sqrt{M^2+i\alpha}\right)}{K!\Pi(K)}}\right)e^{i\alpha\tau},$$

where $\Pi(K)$ **Gauss** function $\left(\Pi(K)=\int_0^\infty e^{-x}x^k dx\right)$.

## 3 Conclusion

Calculations show that pulsating flow of weakly conducting viscous incompressible fluid in square pipe in presence of external homogenous magnetic field is hampered and maximum speed transfers from axis of pipe towards walls. The most intensive effect of retard is observed while the walls of channel are ideally conducting. At minor **Hartman's** figures viscous dissipation plays more important role than **Jole** heat. The fluid temperature in square pipe under pulsation drop of pressure is reduced with the length of **Hartman's** number and reduction of **Prandtl** number. This result corresponds with statement on retarded effect of magnetic field on fluid flow.

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