SHALVA PKHAKADZE – A BRIEF OVERVIEW OF HIS LIFE

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Abstract

The paper is dedicated to the 100$^{\text{th}}$ anniversary of the founder of the school of classical mathematical logic in Georgia, honored scientist of Georgia, Shalva Pkhakadze and it is mainly based on Khimuri Rukhaia's, Otar Chankvetadze's and Konstantine Pkhakadze's earlier publications, which are titled as "Shalva Pkhakadze" (1999, Tbilisi State University, Vekua Institute of Applied Mathematics, 1-50) and "A short overview of Shalva Pkhakadze's Scientific and Pedagogical Activities" (2014, Book of Abstracts of V annual international conference of the Georgian Mathematical Union, 31-38). Thus, the paper is a short overview of Shalva Pkhakadze's personal and scientific life, in the final part of which we present his general semantic program, in other words, a natural semantic program, created with the aim of the foundation of mathematics. Also, in the paper, we give a new theoretical view on the origin of different natural languages. It must be underlined here that this new view together with natural specifics of the Georgian language is mainly based on Shalva Pkhakadze's notation theory and natural semantic program.

Brief Biographical Notes – Shalva Pkhakadze, Founder of the School of Classical Mathematical Logics in Georgia, Honored Scientist of Georgia

1. Shalva Pkhakadze was born on April 7, 1919, in the Zeda Sakara village of the Zestafoni region of the Georgian Democratic Republic.

2. In 1941 he graduated with honors from the Faculty of Physics and Mathematics of Tbilisi State University (TSU).
3. From 1942 to 1952, he had been teaching at various schools in the Zestafoni district.
4. In 1952 he was invited to Razmadze Institute of Mathematics, where, in 1953, he defended his candidate dissertation.
5. In 1956, he completed his doctoral dissertation "The Theory of Lebesgue Measure", which was successfully defended in 1959;
6. In 1961 he was awarded with degree of the Doctor of Sciences, in 1965 – with title of the Professor, in 1967 – with title of the Honored Scientist of Georgia.
7. In 1969, at the Vekua Institute of Applied Mathematics, he founded the Department of the Mathematical Logic and Theory of Algorithms with the aim of formation of the school of classical mathematical logic in Georgia.
8. In 1970, with the above already mentioned aims, at the Faculty of Mechanics and Mathematics of the Javakhishvili Tbilisi State University, he founded a faculty seminar and specialized courses in mathematical logic and set theory.
9. In 1977, he published a fundamental monograph "Some Problems of the Notation Theory", which laid the foundation for a completely new mathematical theory today is named as Shalva Pkhakadze's notation theory.
10. In 1978, he was awarded with Ivane Javakhishvili Order of Honor, in 1979 – with Gottlob Frege Order of Honor.
11. He died on August 8, 1994. He is buried in Saburtalo Pantheon of Public Figures;
12. On February 23, 1995, according to decision №23 of the local government board of Zestafoni district, №3 Secondary School of Zestafoni, where he studied in his childhood, in his honor was named after Shalva Pkhakadze, a honored scientist of Georgia.
13. His university textbook "Mathematical Logic – Foundations" in three parts was published after his death (in 1996 (part 1), in 1999 (part 2), in 2009 (part 3)).
14. In 2019, with financial and institutional support of Shota Rustaveli National Science Foundation of Georgia, Georgian Technical University and Georgian National Academy of Sciences, the First Tbilisi

International Summer School in "Logic, Language, Artificial Intelligence" was held, which was dedicated to the memory of the founder of the Georgian school of classical mathematical logic, honored scientist of Georgia, Shalva Pkhakadze.

1 Family

This paper, as well as this edition of the present journal is dedicated to the 100^{th} anniversary of the founder of the school of classical mathematical logic in Georgia, honored scientist of Georgia, Shalva Pkhakadze.

Shalva Pkhakadze was born in the village of Zeda Sakara of Zestafoni region in the family of Samson Pkhakadze and Nino Putkaradze. Besides him four more children grew up in the Pkhakadze family – his sister Tamara and his brothers – Mikheil, Vasil, and Petre.

On the photo from left to right are Shalva Pkhakadze's: sister Tamara Pkhakadze, mother Nino Putkaradze, Shalva Pkhakadze himself, wife Mary Tskhadadze, father Samson Pkhakadze.

All the five, due to their professional and social activities, became respected and highly rated public figures.

Tamar Pkhakadze (1914 - 1983) was an honored geologist of Georgia. She discovered and studied a medical spring in the Terjola region of Georgia, which in her honor was named as Tamara's spring.

Mikheil Pkhakadze (1917 - 2007), an honored physician of Georgia, a retired colonel, a medical officer, who participated in the second world war, was granted by several orders, had devoted all of his life to the care of public health.

Vasil Pkhakadze (1924 - 1994), an honored physician of Georgia and Abkhazia, author of a number of scientific publications, had been working for years in Abkhazia, where due to his particularly responsive nature and high professionalism he acquired many friends, who are respecting his memory even today.

Petre Pkhakadze (1928 - 1984), an honored meliorator of Georgia, was a scientific secretary of the melioration institute of the agricultural academy of Georgia. He was author of a number of scientific works. His doctoral thesis was dedicated to important problems of the drainage and utilization of Kolkheti lowland.

On the photo from left to right are brothers of Shalva Pkhakadze: Vasil, Petre, and Mikheil.

One could say much more about each of them since their merits are not limited to the above dry facts, but here the merits of their parents – Samson Pkhakadze (1888 - 1967) and Nino Putkaradze (1901 - 1987) must be stressed especially – in their family the cult of faith and honesty was a leading category – this was a main cause of that they raised their children with faithfulness, kindness and love to the humans and homeland.

2 Personal Life

Shalva Pkhakadze was born on April 7 – on the day of Annunciation – in 1919, in Zeda Sakara, a village of the Zestafoni region of Georgia. He graduated with honors from the primary school of Zeda Sakara in 1930, from the Zestafoni secondary school1 in 1933,[1] and from the Zestafoni high-

[1]In 1995, in honor of him, to this school was given his name

school in 1936.

In 1941 Shalva Pkhakadze graduated with honors from the department of physic and mathematics of the Tbilisi State University and later, in 1942-1952, he worked as a teacher of mathematics at various secondary schools of Zestafoni region. Simultaneously, in the same years, he was a Methodist of Zestafoni region department of education and the leader of mathematical section of schoolteachers of the region.

In 1949, under the guidance of Vladimer Chelidze, Shalva Pkhakadze began intensive investigations in the set theory and theory of integrals, and a year later he began his independent research in the measure theory. On January 1952 he delivered a report at the seminar of the department of function theory of Andria Razmadze Mathematical Institute of the Georgian Academy of Sciences. In the report he presented the obtained results, described the main directions of his researches and set up a series of interesting scientific problems in the theory of the Lebesgue measure. At the same session of the seminar he accepted the advice of Vladimer Chelidze to arrange as candidate dissertation a part of his results concerning to the validity of the Fubini theorem on the change of the integration order in iterated integrals of characteristic functions of plane sets.

As a result of above-mentioned report, the same year, he was invited to work at the Andria Razmadze mathematical institute as a Junior Researcher. Filling this position during a year he passed candidate exams and wrote final version of his candidate dissertation, which he defended on June 30, 1953. The same 1953 year, in October, Shalva Pkhakadze delivered several reports at Steklov Mathematical Institute at the seminar of Academician Pyotr Sergeyevich Novikov. In those reports he described the main direction of his research in detail, formulated main problems of this direction and presented obtained results. The positive opinion of the participants of the seminar and of the Academician Pyotr Novikov strengthened his belief in his forces. Thus, after this, he continued the fruitful investigations in the already chosen by him direction with increased motivation.

In December 1955 he filled the position of Senior Researcher and in 1957 he was given the official status of Senior Researcher in the speciality "Theory of function of a real variable". In 1956 Shalva Pkhakadze prepared the final version of his doctoral dissertation "Toward the Theory of Lebesgue Measure", which he has successfully defended in 1959. In January 1965 he was granted Academic Status of Professor.

Since at that stage of development of mathematics and automata theory the need of specialists in mathematical logic had strongly increased in general, Shalva Pkhakadze considered as his duty to devote all his forces to founding and developing mathematical logic in Georgia. Thus, in 1967 he began to study mathematical logic and from this year to the end of his life he delivered lectures and special courses in mathematical logic at the Tbilisi State University. Parallel to this, due to an active support of the management of the institute of Applied Mathematics of the Tbilisi State University, especially due to support of the director and founder of this institute Academician Ilia Vekua in 1969 Shalva Pkhakadze founded a department of Mathematical Logic and Theory of Algorithms, which was first research unit of such type in Georgia.[2] – It must be underlined here that Shalva Pkhakadze, as a founder of the Georgian School of the Classical Mathematical Logic, overcomes all difficulties which were connected with the change of the area of investigations and, even more, it was able for him to create a new direction in the Classical Mathematical Logic.

For years Professor Shalva Pkhakadze successfully combined intensive scientific research with pedagogical work at the University. He was a member of various scientific councils, among them of special councils for considering candidate and doctoral dissertations. Also, he was a member Georgian Scientific Council for Problems of Mechanics and Mathematics, a member of its Methodic Section and the Head of the Section of Mathematical Logic. At the same time, as the member of the board of the Tbilisi Physical-Mathematical school of talented pupils, he was systematically helping the teachers of this school to overcome difficulties connected with the new program in mathematics.

For long and fruitful scientific, pedagogical and social activities Shalva Pkhakadze was awarded with the title of Honored Scientist of Georgia in 1967. In 1978 he was awarded with Ivane Javakhishvili's Honorary Medal and, also, in 1979 – with Gottlob Frege's Honorary Medal.

He passed away on August 8, 1994. He is buried in the Saburtalo Pantheon of Public Figures. After his death, in 1996-1999, at the Ivane Javakhishvili Tbilisi State University, by his authorship and under the editorship of Konstantine Pkhakadze and Vakhtang Pkhakadze, the textbook for students "Mathematical Logic - Foundations" was published in three volumes. Also, On February 23, 1999, by decision №23 of the Local Government Council of the Zestafoni district, to the Zestafoni secondary school №3, where he studied in his childhood, was given the name of the Honored

[2]The first members of this newly established department were well known Georgian scientists Memed Rogava, Otar Chankvetadze, Otar Tskhadadze, Khimur Rukhaia, Rezo Tsakadze, Mikheil Tetruashvili, Nana Kalandarishvili, Alexsandre Kharazishvili, Roland Omanadze, Archil Kipiani, and Genadi Kobzev. At the same time, Most of them were members of the university seminar in mathematical logic led by Shalva Pkhakadze. Besides, among his followers – among his pupils and pupils of his pupils are also doctors of sciences Vakhtang Pkhakadze, Zurab Khasidashvili, Giorgi Pkhakadze, Temur Kutsia, Manana Pkhakadze, Lali Tibua, Besik Dundua, Mikheil Rukhaia, Konstantine Pkhakadze, Aleksandre Maskharashvili, Lasha Abzianidze, Merab Chikvinidze, Giorgi Chichua, and Shalva Malidze.

Scientist of Georgia Shalva Pkhakadze.

3 Scientific Life

3.1 Measure Theory

The first series of works at the first stage of Shalva Pkhakadze's scientific activities is dedicated to the theory of integrals [1, 2]. The results of these works are mainly included in the paper "On Iterate Integrals" [2], which is a short exposition of his candidate dissertation. Here the problem on the validity of the Fubini theorem on the change of the integration order is studied for iterated integrals of the characteristic functions of sets whose intersections with lines parallel to coordinate axes are Borel-measurable sets. Using a rather complicated arguments, the author obtains the following result: if the above-mentioned sets belong to the zero Borel class, then for the corresponding characteristic functions the Fubini theorem is valid. This result is final in some sense, namely, by means of an appropriate example it is shown that in the case where the intersection belongs to the first Borel class, the Fubini theorem, in general, is not valid.

Another series of the works of this stage is dedicated to the general set theory and measure theory [3–18]. A part of these works is included in his doctoral dissertation "The Theory of Lebesgue Measure" [9], which has become a classical monograph in the theory of invariant measure. It involves practically all results obtained earlier by the author in this direction. This monograph is a fundamental research in the theory of invariant measures. Therein a most productive notion – the notion of an absolutely null set is introduced. This notion is a basis of all the dissertation as well as of some research works carried on later by his followers.

It must be noted, that the introduction of this productive notion – the notion of an absolutely null set had required a very deep investigation. Really, the idea of introducing the notion of such sets for the Euclidean space R^n which may be neglected in the sense of measure, has occurred to several authors, for instance, there are a notion of a set of unconditional zero measure and others. However, they did not turn out to be productive and did not play any significant role in the development of the measure theory.

Unlike other authors, in defining absolutely null sets, Shalva Pkhakadze imposed restrictions not only on the set A under consideration, but also on any set which "naturally" has to be absolutely null along with A. Namely, he investigated four versions of the notion of absolutely null set and showed the unproductivity of the first three of them.

Thus, According to the first version, the restrictions "is of zero measure" and "is not of positive measure" are imposed only on the set $A \subset R$ under consideration, according to the second version – on any subset of A, according to the third, respectively, fourth versions, on any finite, respectively, countable configuration of A (A set A^* is said to be a finite, respectively countable, configuration of A, if it can be represented as a union of a finite, respectively, countable, number of sets congruent to a subset of A) and it is proved that in the case of the first three versions the union of two absolutely null sets can coincide with the whole space, when in the case of the fourth version (which Shalva Pkhakadze adopted as a final definition), absolutely null sets have a series of very important and productive properties.

In particular, the class of such null sets is closed with respect to the finite union and is not closed with respect to a countable union. Both facts play an important role while finding various extensions of Lebesgue type measures thus making it possible to solve Sierpinski's problems on extendibility of solvable classes and extendibility of Lebesgue type measures. The last problem is solved by Shalva Pkhakadze on the ground of a most probable hypothesis, namely on the ground of the hypothesis that there is no unattainable cardinal number less or equal to the continuum. Moreover, Shalva Pkhakadze discovered a series of important properties of his absolutely null sets formulated as a necessary and a sufficient condition and proved that each from these conditions can be taken as a basis for defining the notion of an absolutely null set. Two of those properties can be formulated as follows:

(1) For a set $A \subset R^n$ to be absolutely null, it is necessary and sufficient that any countable configuration of A be vanishing (a set $X \subset R$ is said to be vanishing if there is a set of zero Lebesgue measure which can be represented as $\Pi_{i=1}^{n} X_i$, where each X_i, is congruent to X);

(2) For a set $A \subset R^n$ to be absolutely null, it is necessary and sufficient that for any Lebesgue type measure μ there exists its extension μ_1, with $\mu_1(A) = 0$.

On the basis of the property (1), Shalva Pkhakadze constructed a non measurable absolutely null set in R^n for $n = 1, 2$. In this connection, he poses the problem of constructing a non measurable absolutely null set in R^n for $n = 3, 4, \ldots$. This problem was solved by A. Kharazishvili on the basis of the same property (1).

In the fundamental monograph "The Theory of Lebesgue Measure", Shalva Pkhakadze poses also other important problems. His followers were working successfully on these problems. For instance, M. Tetruashvili generalized Shalva Pkhakadze's results for topological groups, and

A. Kharazishvili along with the above-mentioned problem also solved three other ones (see the problems II, III, and IV in [9]).

The main item of the fundamental monograph under consideration is the elaboration of powerful methods for finding the Lebesgue measure extensions with various properties. However, the results in the general set theory obtained using the methods elaborated in this fundamental work are also of major interest. Namely, in [9] results of Sierpinski on the existence of a set of almost invariant zero measure and of a set of the first category are essentially generalized, some important examples are constructed and general covering theorems are proved. Besides this in [9], the notion of almost completely asymmetric set is introduced and a decomposition of the space as a union of completely asymmetric and almost Π_n invariant sets are found. On the basis of these facts, in [9], results of Paul Erdös on the number of solutions of some linear equations in R^n are essentially generalized and some other significant results are also obtained.

The second part of the second series of works of his first stage scientific activities consists of 9 papers [10–18]. Therein Shalva Pkhakadze continues research in the same direction and obtains deep results. One should especially note the paper "A general method of construction" [18], where a general method is elaborated for effective construction of such subgroups of the Abelian group of real numbers which have null measure and a continual set of symmetric residue classes. This method enables one to construct an infinite quantity of individual instances of such subgroups. It is proved that the cardinal number of the set of such subgroups is 2^n. Moreover, it is shown that each of such subgroups has a quite paradoxical property – the set obtained from it by two-times contraction can be decomposed into the continual quantity of sets congruent to it. There arises a natural question on the place of such sets among effective sets constructed by Lusin's school. Namely, it is very interesting to find out whether some of them lay out of the class of projective sets.

At the end of this part of the publication we are quoting an opinion of Academician Pyotr Novikov, which gives a better idea about the results of this series of works of Shalva Pkhakadze and which is mainly concentrated on his dissertation work, because he was one of the official opponents of this dissertation: *"In the theory of the Lebesgue measure there arose a series of problems among which the main one is the problem posed by Sierpinski on the existence for any extension of the Lebesgue measure of another extension which would be an extension of the first one. In the measure theory the problem of extension is of the fundamental ones and it naturally has been arising a great interest.*

However, there was only a little progress towards solution of the posed problems on the extension of the Lebesgue measure. As a matter of fact, *only separate examples of such extensions were obtained. The dissertation under consideration is a substantial forward step in the theory, and moreover, I would say that it shifted the theory away from the dead point. The numerous results of the author can be divided into the following groups:*

1. *Finding of general methods of extension of the Lebesgue measure and of extension of solvable classes of sets;*
2. *Generalizations of the measure in the sense that the usual requirements of invariance are replaced by the requirement of preservation of the measure under one-to-one transformations of the space forming a group.*
3. *Investigation of the structure of the measure by its decomposition into a sum of measures and establishing of a canonical decomposition.*
4. *Application of the methods developed by the author to problems lying outside of the measure theory.*

The first circle of the named results forms a rather general theory of extension of the Lebesgue measure. In these investigations the author introduces a series of important notions such as of absolutely null set, of a proper almost invariant set, etc. The germs of some of these notions can be found in works of previous authors. In the dissertation, however, they are developed at the full extent, they are systematically studied, and there is no doubt that in the future they will be repeatedly used. The theory of measure extension created in the dissertation would remain incomplete if therein there would be no progress in the direction of solving the above-mentioned problems. The central one is the problem of Sierpinski on existence of a nonextendable measure. In the dissertation the following answer is given to this problem: every Mn-measure is extendable if there is no unattainable cardinal number which would be less than continuum." Analyzing this answer, Pyotr Novikov further writes: *"I think that the problem of Sierpinski is solved fully enough. The author shows that if one adds to the axioms of the Mn-measure any of two quite natural axioms (A) and (B), then for such measures the problem of Sierpinski is solved completely without additional hypotheses. Likewise, without any additional hypothesis can be proved the extendability of any solvable class."* Finally, Pyotr Novikov concludes: *"The work under consideration is a substantial advance in one of the important areas of the set theory. Therein powerful methods are created which made possible to overcome serious difficulties."*

In addition to the above mentioned we underline that in 1960, in "Referential Journal of Mathematics" (N11456) famous Russian mathematician Vladimir Abramovich Rokhlin appreciates Shalva Pkhakadze's dissertation as follow: *"The work is a fundamental research dedicated to the investigation of invariant extensions of the Lebesgue measure. It is known that such continuations exist, but up to now this knowledge was very scarce and consisted mainly of separate examples. The author has managed to prove a series of structural and fundamental theorems on them."*

Also, it must be mentioned too, that Alexey Lyapunov, also very famous Russian mathematician, in his review about the dissertation has written: *"The work is a bright event in the set theory"*.

3.2 Mathematical Logic

Despite his important results in Measure Theory and not young age, in 1967-1969, Shalva Pkhakadze changes his scientific sphere and begins to study Mathematical Logic. Of course, it was not easy for him to change scientific sphere, however considering the necessity of founding and developing a very important field of mathematics – mathematical logic in Georgia, he decided to make this step. Thus, it can be said that the main result of Shalva Pkhakadze's non-standard career is the fact that together with other Georgian mathematical schools, the Georgian school of mathematical logic exists and develops till todays.

All these would not exist, if there were no significant results in his scientific activity at the second stage, stage of mathematical logic [19–26]. Namely, in his work "One Example of Intuitively Computable and Everywhere Defined Function and Church's Thesis" [24], published in 1984, where he described an intuitively computable everywhere defined function, which according to him is not recursive, have casted doubt on the Church's thesis, which was indubitable for that time.

In the mentioned work the author, using an original method called by himself the "complex diagonal method", constructs an intuitively computable everywhere defined function f and studies its properties. On the basis of these properties, the author, as a hypothesis, expresses his full belief that, contrary to Church's thesis, the function f is not recursive.

At that stage of development of mathematical logic and algorithm theory, when the work was published, a claim against the Church's thesis can be considered as a purely speculative act, because of the point, that almost every logician has considered as a doubtless fact impossibility to solve problem of Church's thesis. Therefore, there was no risk in proposing the hypothesis on invalidity of Church's thesis, but in the case of the author's hypothesis, the situation is completely different, because it was very audacious and risky to propose a hypothesis, that a concrete intuitively computable function f is not recursive.

The point is that one of the reasons in favor of the Church thesis reads as follows: for any intuitively computable function constructed up to now the proof of its recursiveness was easily found, and the belief that this will be so in the future is so strong that many authors don't consider it necessary to seek for the proof of recursiveness of a concrete intuitively computable function thinking that if needed this will be done without any difficulty. In this case it must be noted that there were attempts to prove the recursiveness of the above-mentioned function f and thereby the invalidity of Pkhakadze's hypothesis. Moreover, there were announced claiming the contrary, i.e. claiming that the function f, defined in Shalva Pkhakadze's above mentioned work, is recursive, though the proof of recursiveness of Pkhakadze's function is not published anywhere yet. Even more, nowadays, attitude of specialists towards Church's Thesis has radically changed and often there are published papers with disclaiming considerations about it, which makes correctness of the opposite thesis – Pkhakadze's thesis more trustworthy. However, this is not his main result in mathematical logic. Namely, refusing such an important thesis as Church's thesis, cannot be compared to what notation theory gave to mathematical logic [20]– [26]. Thus, with Shalva Pkhakadze's fundamental monograph "Some Problems of Notation Theory" [21] was founded the notation theory, which gives formal developing ability to the Frege's and Hilbert's classical formalism.[3] Obviously, taking into account the goals and the necessity of further development of formalistic approaches, this novelty is fundamentally significant.

Thus, as it was mentioned, the basic results of the second stage of scientific activities of Shalva Pkhakadze are mainly set forth in the monograph "Some Problems of the Notation Theory". Therein rules for definition of contracted symbols are introduced and studied. The monograph is in mathematical logic, but it is similar to the above considered one by the fact that from the very beginning a particular attention is paied to the development of the fundamental concepts. Thus, the conceptual aspect of the work is to be particularly stressed.

Thus, the development of the formal mathematics has led us to significant discoveries and results. They belong not only to the foundations of mathematics but also to its special fields. At the contemporary stage of the development of mathematics it is doubtful to obtain such results without using methods of formal mathematics. Therefore, for development of the whole mathematics it is most important to improve the methods of the formal mathematics. In spite of this, the majority of fields of mathematics is developing on the nonformal level. This is due to the fact that it is difficult to master in the methods of formal mathematics since valuable expositions of formal mathematical systems are not available. To change this existing

[3] We will consider this issue in more detail at the end of the article.

expositions, deep scientific research is needed.

One of the essential peculiarities of formal mathematical systems is the limitation of the alphabet of the theory and its reduction to minimum. This makes the class of the propositions of the theory available for the examination as a whole, the concept of the proof can be defined exactly, a series of mathematical problems can be formulated clearly and their solving is facilitated. Modern mathematical theories which are developing on informally level are influenced by methods of formal mathematical theories, they reduce their alphabet to minimum and associate formal mathematical concepts to the intuitive ones. For instance, the concept of algorithm and other concepts, related with it, are introduced in such a way.

Although the limitation of the alphabet of the theory is very advantageous theoretically, it causes substantial difficulties. Namely, one has to introduce extremely long forms (formulas and terms), and it is practically impossible to write them down and perceive their meaning. To get over these difficulties there are introduced contracted, in other words, abridging symbols.

In the case of a rather rich theory (as the set theory), the rules of definition of contracted symbols are just slightly restricted or are not restricted at all. So it is impossible to establish general properties of abridging symbols and abridged forms. Because of this the principle is adopted according to which an abridged form is considered as the form it denotes. But the complete realization of this principle is impossible due to the above-mentioned difficulties.

This implies the necessity of changing of the above principle and adopting of one which would make possible to draw conclusions about forms when we are operating directly on their abbreviations. This, in its turn, gives rise to the need for general rules of operations on abridged forms. To this aim, it is necessary to establish general properties of abridging symbols and abridged forms by restricting their intuitional concepts by exact mathematical notions, i.e. by defining the notion of the abridging symbol on the basis of restriction of the rules of their introduction. Moreover, for the aims of developing a formal mathematic to found a rational solution of the here posed problem is under the obligatory needing. In the considered work a rather rational system of definition rules is found for contracted (abridging) symbols in the cases of classical formal and unformal mathematical theories. By the rationality of a system of rules is meant the fact that, on the one hand, it is so general that with its aid almost all symbols used in classical formal theories can be derived, and on the other hand, the symbols introduced according to these rules possess properties rich enough to ensure a considerable liberty in operating on abridged forms. Namely, by means of contracted symbols mathematical judgments can be transferred into the enlarged theory using natural general laws.

The construction of a mathematical theory (formal or informal) with a limited alphabet and rational (in the above sense) rules of introducing of contracted symbols is of great theoretical and practical importance. It makes possible to study the main theory using effectively its various extensions obtained by adding contracted symbols; it reveals deep connections between various mathematical theories. It is practically necessary for the automation of mathematical research and the creation of special systems processing mathematical texts.

Moreover, in the work algorithmic processes of reconstruction of the form by its abbreviation are studied. In a rather naturally introduced class of algorithms, an algorithm is found for such a reconstruction with a minimal number of steps. Various numerical characteristics of contracted symbols and contracted forms are also studied and algorithms are found for their evaluation. It should be especially noted that a carefully thought out system of terms and concepts is used. Namely, there are introduced various types of abridging symbols and abridged forms, various reconstructing processes, various types of applications of definition of abridging symbols, various types of particular cases of abridging symbols, and so on.

It is clear from the above said that Shalva Pkhakadze's fundamental monograph "Some Problems of the Notation Theory" is a significant achievement in mathematical logic and a deep research in this field. It deals with the foundations of mathematics and has a great theoretical and practical value. It can be successfully used while writing monographs in those fields of formal and unformal mathematics which use a limited alphabet, namely, abridging (i.e. contracted) symbols of the types I-IV, II' and IV' are to be used. For them important properties are established facilitating to carry out mathematical reasoning and making possible to change intuitive judgment by mathematically exact ones.

The further study of the problems dealt within the monograph in the direction indicated by the author is of great importance. It should be also especially noted that the notation theory – a well formulated independent theory has been already created on the basis of the already obtained results.

Finally note that the results of the monograph arise interest not only of theoreticians, but also of specialists of applied mathematics working in the theory of automata. Presently an intensive work is being carried out aiming at the automation of mathematical research, the creation of automatic systems for processing mathematical texts. There exist rather convincing reasons according to which mathematical texts of only those mathematical theories can be processed with the help of automatic systems which use a restricted alphabet and are created on the basis of the well-developed notation theory. – To illustrate better the results of this work, below, we quoted some very famous Russian scientists.

Vladimir Mefodievich Matrosov: *"From our point of view, the representation of knowledge and the economy of "thinking" of a computer (in particular, a hierarchical organization of concepts and theories in the data structure of a computer and processing of mathematical texts) needs the application of results of the notation theory. In Shalva Pkhakadze's monograph "Some Problems of the Notation Theory" the first attempt is given of systematic development of the named theory. Here is considered theory in language of rather general type without concretizing axioms and are imposed restrictions on the rules of introducing of contracted symbols. This makes it possible to operate on contracted forms instead of forms they denote. General properties of contracted forms are studied and operations on them are defined. Also, here, the rules of introducing of contracted symbols are chosen so that rather good properties of contracted forms be obtained, which guaranteeing the homomorphism of the algebra of operations on contracted forms into the algebra of operations on forms of main theory. It is clear, that this is a very important result in the sence of developing and improving general formal approaches."*

Yuri Ivanovich Yanov: *"Although abridging notations, in other words, contracted notations have been constantly used in constructing mathematical theories, systematic study of problems arising in this connection was first made in the Shalva Pkhakadze's fundamental monograph "Some Problems of the Notation Theory". Therein a theoretical basis was laid down for introducing and using abridging notation in mathematical theories. The actuality of this problem is due to the fact that development and exposition of any, even rather simple, mathematical theory is practically impossible without introducing additional symbols, axioms and definitions playing a role of abridging notation. In this connection a series of important problems arise. Among them are conservativeness of the extension, i.e. whether the set of theorems which can be formulated in the language of the original theory remains unchanged, the problem of relation between properties of abridged and main forms and etc. The work of Shalva Pkhakadze, by means of a certain classification of abridging notation, makes it possible to obtain rather simply answers to mentioned problems. The basic ideas of the work were further developed in the works of the pupils of Sh. Pkhakadze. As a concrete application, the works in this direction may be used in problems of constructing an artificial intelligence system."*

Oleg Borisovich Lupanov: *"Professor Shalva Pkhakadze is a leading specialist in mathematical logic and set theory. During the last fifteen years, he has been developing an important direction in mathematical logic, connected with the notation theory. This direction formalizes the procedure of using abridging notation and is essential in constructing artificial mathematical* *languages, in problems of automated deriving of theorems, etc."*

Sergey Vsevolodovich Yablonsky: *"Shalva Pkhakadze is developing an original direction related to the theory of formal systems. In constructing mathematical theories, we usually start from a limited alphabet and therefore we have to use abridged i.e. contracted notation not contained within the framework of the initial theory. Shalva Pkhakadze gives a common approach to operations with contracted symbols and forms, that is of great importance for constructing mathematical theories according to general princeples of the mathematical logic and formal mathematics."*

Sergey Mikhailovich Nikolsky: *"The monograph not only opens an actual direction, but also contains large possibilities in the area of automated processing of mathematical texts."*

Thus, as it was already underline above, that the main contribution in mathematical logic, which was done by Shalva Pkhakadeze, is the theory of formal notation, shortly, notation theory founded by his fundamental monograph work "Some Problems of Notation Theory" and, also, due to the works of his pupils and followers. Here, it must be noted that, the scientific society is still in the process of comprehending the above-mentioned fundamental significance of Shalva Pkhakadze's notation theory. This might be caused by the fact that the results ahead of the time need more time to be understood.

However, it must be noted as well, that Shalva Pkhakadze's notation theory already significantly contributed in the development term rewriting systems, which, in turn, is a very important theory for the aims of formalization of mathematical and natural languages and theories. Namely, in the Femke Van Raamsdonk's doctoral thesis "Confluence and Normalization for Higher-Order Rewriting" Shalva Pkhakadze's notation theory is already appreciated as one of first sources of the higher-order rewriting systems. In particular, at the first stage of the development of the theory of the term rewriting systems, it was possible to compute only expressions without bound variables, but on the basis of Shalva Pkhakadze's notation theory and on the base of the different types of substitution operators, which are defined in this theory, there was created an expression reduction system, on the basis of which term rewriting systems were equipped with possibility compute expressions with bound variables too.[4] This fact, and it is clear, is a very clear confirmation of the high theoretical significance of

[4]Expression reduction systems as initial theory was created in Tbilisi by Zurab Khasidashvili, which defended his doctoral thesis in Tbilisi State University in 1991 by leadership of Shalva Pkhakadze (for more details see publications: 1. Femke Van Raamsdonk, Confluence and Normalization for Higher-Order Rewriting, Vrije University, Amsterdam, 1-212, 1996; 2. Z. Khazidashvili, Expresion reduction systems. Proceedings of VIAM TSU, Vol. 36, 200-220, 1992).

Shalva Pkhakadze's notation theory. However, below, we will try to present the deeper conceptual meanings of this theory, but beforehand we underline that below presented views are mainly based on the already sufficiently proven factual circumstance, that Georgian and Mathematical languages, in general, are languages of a one and same types. This very important factual circumstance is strengthened by the fact that almost any token of the Georgian language i.e. any word, any morpheme, and any punctuation symbol of the Georgian language are describable as a some kind symbol of the $\mathfrak{J}$ sufficiently general mathematical language, which was defined by Shalva Pkhakadze in his notation theory and which are very shortly overviewed below.[5]

Thus, one more time, we underline, that in general, notation theory is the system of the formal rules of the formal extensions of the formal theories and languages. At the same time, we call a formal language and theory without possibility to be formally extended, respectively, with possibility to be formally extended as a formally non developable, respectively, as a formally developable language and theory.

In notation theory, which is formed on the basis of Shalva Pkhakadze's $\mathfrak{J}$ sufficiently general mathematical language, are described different types of formal extension rules, which are called as contracted rules. With the help of these contracted rules one can extend any $\mathfrak{J}$ sufficiently general mathematical language in almost any case of extension needed. This means that notation theory gives us scientifically founded understanding of $\mathfrak{J}$ formally developable mathematical languages.

At the same time, based on the $\mathfrak{J}$ sufficiently general mathematical language there is defined $\mathfrak{J}$ sufficiently general mathematical theory and any $\mathfrak{J}$ sufficiently general mathematical theory together with the above mentioned contracted rules gives us scientifically founded understanding of $\mathfrak{J}$ formally developable mathematical theories.

In addition, if $\mathfrak{J}^*$ is any extension of any semantically completely under-

[5]The views, which will be presented below, are a result of the researches, which permanently goes on with leadership of Konstantine Pkhakadze. The researches have begun in 1999; since 2002 until 2010, researches were going in confine of the State Priority Program "Free and Complete Inclusion of a Computer in the Georgian Natural Language System" and, from 2010 till todays it goes on in confine of the long-term project "The Technological Alphabet of the Georgian language" of the center for the Georgian language technology of the Georgian technical university. As a result, more than 200 papers were published by K. Pkhakadze in diferent years together with diferent co-authors. They are: M. Ivanishvili, E. Soselia, L. Lekiashvili, G. Chankvetadze, L. Tibua, I. Beriashvili, R. Skhirtladze, K. Gabunia, N. Buadze, G. Kandelaki, V. Pkhakadze, T. Esitashvili, B. Tskhadadze, G. Chichua, A. Maskharashivili, L. Abzianidze, N. Pkhakadze, N. Vakhania, N. Labadze, B. Chikvinidze, L. Gurasashvili, M. Beriasvili, M. Chikvinidze, D. Kurtskhalia, S. Shinjikashvili, Sh. Malidze, C. Demurchev and N. Okroshiashvili.

stood, i.e. interpreted i.e. consistent $\mathfrak{J}$ formally developable mathematical theory and if this extension was made with the help of above overviewed contracted rules Pkhakadze, then $\mathfrak{J}^*$ is also a semantically completely understood, i.e. interpreted i.e. consistent theory.

Above, non-formally and very shortly we have described the general semantic program of foundation of mathematics, which is elaborated by Shalva Pkhakadze and which is the main result of his notation theory.

Thus, as a conclusion of above-mentioned, we underline the next: since Frege's mathematical language is Shalva Pkhakadze's $\mathfrak{J}$ sufficiently general mathematical language and Hilbert's mathematical theory is Pkhakadze's $\mathfrak{J}$ sufficiently general mathematical theory, we can conclude that Shalva Pkhakadze's notation theory gives formally developable abilities to the formally non developable Frege's mathematical language and Hilbret's mathematical theory.

It is clear, that above shortly described $\mathfrak{J}$ formally developable mathematical languages and $\mathfrak{J}$ formally developable mathematical theories of Shalva Pkhakadze give very fruitful new possibilities to construct non-simple intelligence systems. Also, they give us scientifically founded understanding on the human's lingual nature and lead us to formulate a particular and a general thesis of the Georgian Language:[6]

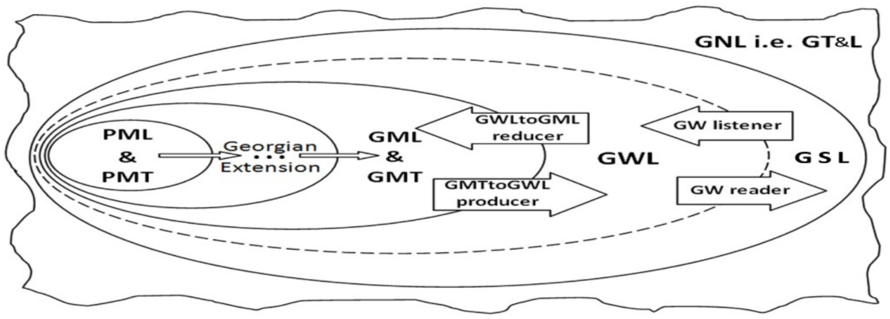

Figure 1: Gradual extension of PML to GT&L via GERs

A particular thesis of the Georgian language: Natural Georgian language is a result of that formal extension of the Primary Mathematical Language (PML), whose extension is made with the help of Contracted Rules (CRs) Shalva Pkhakadze's type, which we have called Georgian Extension Rules (GER).

Natural Georgian Thinking and Language System (GT&L), shortly

[6]The fact that we applied the Shalva Pkhakadze's notation theory to the Georgian language is based on the above underlined and already sufficiently confirmed fact, that Georgian language is a language of the mathematical type.

Georgian Natural Language System (GNL) is a result of that formal extension of the PMT, which is made with the help GERs (see, above, figure 1, where GML, respectively, GMT abbreviates the Georgian Mathematical Language, respectively, Georgian Mathematical Theory, which together with PML, respectively, PMT is a some $\mathfrak{J}$ formally developable mathematical language, respectively, $\mathfrak{J}$ formally developable mathematical theory of Shalva Pkhakadze's type, which exists in the Georgian-speaking peoples by nature in an innate way).

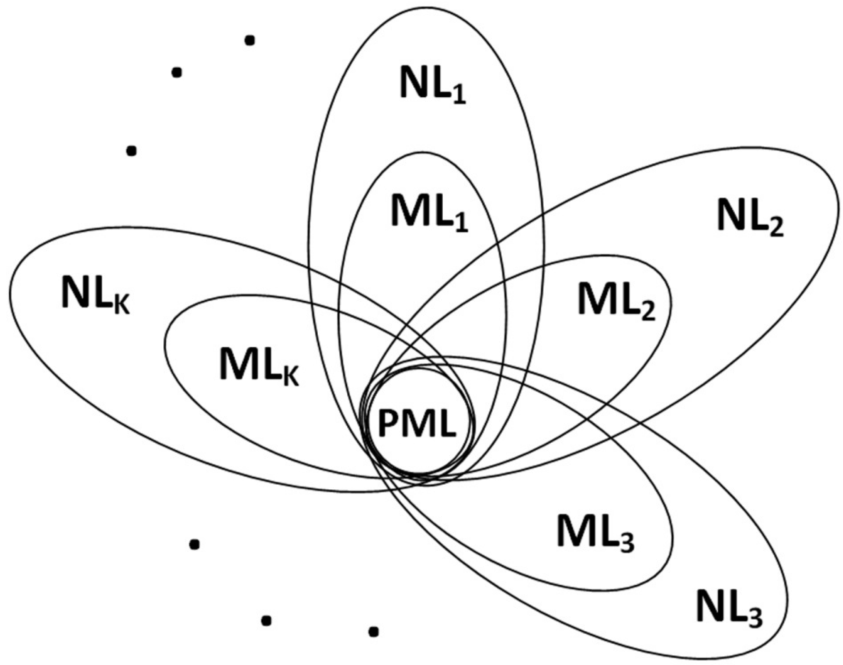

Figure 2: Graphical representation of the relation between PML and NLs

A general thesis of the Georgian language: According to us any Natural Language (NL) is a result of step by step extensions of the Primary Mathematical Theory (PMT), alphabet of which is called Primary Mathematical Language (PML) and axioms, inference and extension rules of which are called Primary Mathematical Concepts (PMC) (see, above, figure 2, where by $\mathbf{NL_K}$ is denoted **K**-th **NL** and by the $\mathbf{ML_K}$ the Mathematical Language, which stands between the PML and $\mathbf{NL_K}$). At the same time, we declare that the PML and PMCs are naturally and universally inborn in all humans and in confines of the PMT they act automatically, i.e. instinctually.[7]

[7]Thus, it can say, that we share Pascal's insight on existence of selfunderstandable

To summarize, it must be underlined, that the above presented new lingual views, which together with notation theory and general semantic program of Shalva Pkhakadze are mainly based on the very clear mathematical specifics of the Georgian language, differ with other today existing lingual views and gives us a new understanding of genesis of different natural languages. This new understanding is clearly shown on the above presented diagram (see figure 2), which we call "lingual flower with mathematical heart". Thus, on the basis of all above presented it becomes clear, that for us:

1. The primary mathematical language, which is naturally inborn in all humans, is the parent of all natural languages i.e. parent of all native languages, that means that this primary mathematical language is a native language for all humans. In other words, Shalva Pkhakadze's theory of formal notations dictates that mathematics together with linguistics is a part of natural science;

2. Shalva Pkhakadze's theory of formal notations changes a paradigm in formal mathematical and linguistic researches, because this theory dictates us, that for complete formal and technological foundation of the mathematics and natural languages, firstly, it is necessary to recover the deep subconscious part of natural languages, in other words, of conscious natural languages. In other words, Shalva Pkhakadze's theory of formal notations dictates that for construction of artificial intelligence system almost completely knowing a natural language and general mathematical skills it is necessary complete recovering of the primary mathematical language and primary mathematical concepts on the basis of which is constructed the primary mathematical theory, which, in turn, as primary and universally lingual knowledge is naturally inborn in all humans and gives birth to all different natural languages, in other words, to all different human languages.

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words, because of – according to his insight – in the case of nonexistence of such words, it would be impossible to understand as them, as well as all other words and phrases. Moreover we share completely also next old bible view that "in the beginning was the word" i.e. language and "all things were made by him" i.e. by language "and without him was not any thing made that was made", and, accordingly, we know, that "in him" i.e. "in language" is and "was life", because of for us life is a whole of divisible alike units (cells) that are related with each other informatically, in other words, lingually.

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