IRREVERSIBLE DEFORMATION UNDER THERMOMECHANICAL LOADING OF SOLIDS

O.G. Kikvidze

Mechanical Engineering Department, Akaki Tsereteli State University
59 Tamar Mepe Str., Kutaisi 4600, Georgia
omar.kikvidze@atsu.edu.ge

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Abstract

The article considers irreversible deformation of solid under thermomechanical loading, using the phenomenological approach. It is assumed that the strains are small. On the basis of the dilatometric curves and the stress-strain curves, the condition was formulated for the stability of material, and the major inequality and constitutive equations for the irreversible strains under thermo-mechanical loading were obtained. These equations describe the pattern of inelastic deformation of a wide class of metallic materials in the temperature ranges of the phase transformations.

Keywords and phrases: Irreversible strain, major inequality, condition for stable deformation, constitutive equation.

AMS subject classification (2010): 74C05, 74D10.

1 Introduction

Thermomechanical loading of a number of metallic materials under certain conditions causes inelastic deformation, characterized by irreversible volume change. Under torsion of tubular specimens from shape memory materials with the change in temperature within the temperature ranges of direct martensitic transformations, there are accumulated inelastic shear strains.

Irreversible volume change is observed during welding and heat treatment of metallic materials accompanied by phase changes. During the martensitic transformation in iron-based alloys, the volume changes are significant (about 4%), which causes plastic deformation in the surrounding parent phase. Multiple thermal cycling of uranium specimens causes irreversible form change [1]. In addition to uranium and its alloys, the form change was also reliably recorded on metals such as α-iron, titanium, copper, aluminum, nickel, and zinc. In specimens, whose material has a texture, irreversible thermal changes were recorded even in the absence of stress.

The constitutive equations for plastic deformations were obtained on the basis of the stability condition proposed by Drucker [2, 3]. This idea was further extended by Koiter [4], Naghdi [5], who proposed constitutive equations based on Drucker's inequalities.

2 A Condition for the Stability of Material and Major Inequality

The phenomenological theory of the deformation of materials, as is well-known, is built on the basis of the experimental data of testing of specimens, regardless of whether the constitutive equations are derived from the general postulates or through the generalization of the simplest relationships. Let us build a theory of the irreversible deformation of materials under thermomechanical loading based on two series of the experimental data: tension of specimens at a constant temperature and fluctuating stress, and the deformation of specimens at a constant stress and variable temperature.

At each step of stress and temperature, the specimen is under steady state, and therefore, the deformation process is also steady. Considering the steady irreversible process of deformation, it is sufficient to indicate the behavior of material during loading and unloading. It is obvious that during thermomechanical loading, this mode applies both to mechanical and thermal factors. Deformations are considered small.

On the basis of these experimental data, using the well-known method of combining two inequalities of different dimensions used in thermodynamics of irreversible processes [7], the paper [8] formulates the following condition for a closed path of thermomechanical loading - "loading-unloading-heating cooling":

$$\oint \left(\sigma_{ij} - \sigma_{ij}^{0}\right) d\varepsilon_{ij} + a_{ij} \left(\varepsilon_{ij} - \varepsilon_{ij}^{0}\right) dT \geq 0, \tag{1}$$

where σ_{ij} is a true stress state, ε_{ij} are their corresponding strains, σ_{ij}^{0} is a some permissible stress state; ε_{ij}^{0} are their corresponding strains, T is a temperature, a_{ij} is a tensor, the components of which measured in Pa$/^{0}$C, inserted into (1) to align the dimensions of the additive components.

True stress state satisfies the condition $f(\sigma_{ij}, T, Q) = 0$, while permissible stress state satisfies inequality $f(\sigma_{ij}, T, Q) < 0$ (Q - some structural parameter, f - loading surface). Integration is carried out in a space "temperature-stress", along a path that comes out of the point and returns back to the same point. The integral value in (1) for the reversible processes on a closed path of thermomechanical loading equals to zero.

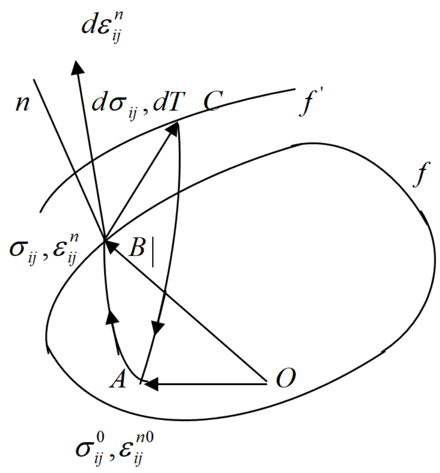

Figure 1: On formulating the condition for the stability of deformation and major inequality

For the components of full strain, we shall consider fair the additivity concept:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{T} + \varepsilon_{ij}^{n}, \quad d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{T} + d\varepsilon_{ij}^{n}, \tag{2}$$

where ε_{ij}^{e}, ε_{ij}^{T}, ε_{ij}^{n} are the components of elastic, temperature and irreversible deformations, respectively.

In order to interpret restrictions imposed on material by the condition (1), let us examine the element of work-hardening solids (Fig. 1). Suppose is a current position of a loading surface, while f' - is a new, infinitely near position of a loading surface.

Consider some loading path $A \to B \to C$. A starting point A represents the initial stress-strain state σ_{ij}^{0}, ε_{ij}^{0}. From a point B, infinitesimal thermo-mechanical additional loading is carried out causing theincremental elastic, temperature and irreversible strains. Let us go back to the point A following some path CA. Over the cycle $ABCA$, the condition of the stability of material (1) is fulfilled.

The elastic and temperature strains are reversible. Therefore, the closed path $ABCA$, considering the additivity conditions (2) in the integral of (1), leaves only the irreversible strains, and therefore, it can be written as [8, 9]:

$$\oint \left(\sigma_{ij} - \sigma_{ij}^{0}\right) d\varepsilon_{ij}^{n} + a_{ij}\left(\varepsilon_{ij}^{n} - \varepsilon_{ij}^{0n}\right) dT > 0. \tag{3}$$

Since the irreversible strain occurs only on the infinitesimally small

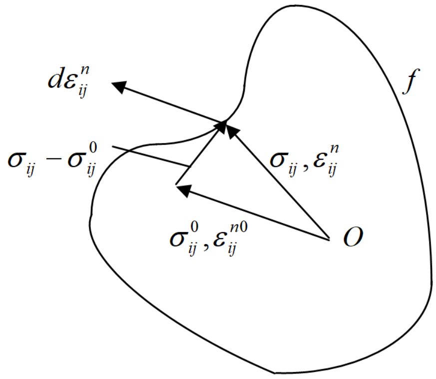

Figure 2: The possible shape of the loading surface

section BC (Fig.1), the last inequality becomes:

$$\left(\sigma_{ij}-\sigma^0_{ij}\right)d\varepsilon^n_{ij}+a_{ij}\left(\varepsilon^n_{ij}-\varepsilon^{0n}_{ij}\right)dT>0, \tag{4}$$

from which the major inequality follows:

$$\sigma_{ij}\varepsilon^n_{ij}+a_{ij}\varepsilon^n_{ij}dT>\sigma^0_{ij}d\varepsilon^n_{ij}+a_{ij}\varepsilon^{0n}_{ij}dT. \tag{5}$$

Inequality (5) is necessary for establishing constitutive equations. When the initial stress-strain state is a state σ_{ij}, ε^n_{ij}, matching the point B on a loading surface f, then for the cycle BCB, the following inequality can be written:

$$\sigma_{ij}\varepsilon^n_{ij}+a_{ij}\varepsilon^n_{ij}dT>0. \tag{6}$$

Inequality (6) is a condition for stable deformation of material.

Drucker's postulate is built on the basis of the stress-strain curve, from which the major inequality of plasticity and the condition of the stability of deformation follow [2]. It follows that the surface of plasticity is convex [2, 11]. The relationship (4) allows for the case when the additive components satisfy the following inequalities: $(\sigma_{ij}-\sigma^0_{ij})d\varepsilon^n_{ij}<0$, $a_{ij}(\varepsilon^n_{ij}-\varepsilon^{0n}_{ij})dT>0$, the first of which geometrically means that the loading surface can be concave, and the angle between the vectors $\overline{\sigma}_{ij}-\overline{\sigma}^0_{ij}$ and $\bar{d}\varepsilon^n_{ij}$ can be obtuse (Fig. 2). In principle, such a state for metallic materials takes place in the temperature range of phase transformations, when the temperature factor prevails in the development of the irreversible strains.

3 Constitutive Equations

In H. Ziegler's work [10], the extreme principles are widely used to obtain constitutive equations. According to major inequality (4), the increment function $d\Omega = \sigma_{ij} d\varepsilon_{ij}^n + a_{ij}\varepsilon_{ij}^n dT$ at prescribed increments of the irreversible strains and temperatures has a maximum on the real stresses and irreversible strains. Irreversible strains are a function of stress and temperature, therefore, the function $d\Omega$, is ultimately a function of the components of stress and temperature, which, during the irreversible deformation satisfy the condition:

$$f(\sigma_{ij}, T, Q) = 0. \tag{7}$$

As a structural parameter Q, during the deformation of shape memory alloys in the temperature range of the direct martensitic transformations, we should take the fraction of the martensitic phase [17], while during the deformation of structural steel beyond the limits of elasticity, the measure of hardening is taken as a structural parameter (the Odqvist parameter, or the work on the plastic strains).

We shall use the Lagrange method to find the conditional extremum of a function of several variables, which are connected by an additional condition (7). Let us generate an auxiliary function $dL = d\Omega - d\lambda \cdot f$, and from the extremum condition, we shall obtain an expression for the increments of the irreversible strains [11]:

$$d\varepsilon_{ij}^n = -\frac{\partial(a_{kl}\varepsilon_{kl}^n)}{\partial\sigma_{ij}} dT + d\lambda \frac{\partial f}{\partial \sigma_{ij}}, \tag{8}$$

where $d\lambda$ is Lagrange's indefinite scalar multiplier.

Let us introduce the following notation:

$$de_{ij}^n = d\varepsilon_{ij}^n + \frac{\partial(a_{kl}\varepsilon_{kl}^n)}{\partial\sigma_{ij}} dT. \tag{9}$$

Applying (9) to the expression for the equivalent quantity of the incremental strains $d\bar{\varepsilon}_l^n = (2de_{ij}^n de_{ij}^n/3)^{1/2}$ [11], we shall determine the Lagrange multiplier value:

$$d\lambda = \sqrt{\frac{3}{2}} d\bar{\varepsilon}_l^n \Big/ \sqrt{\frac{\partial f}{\partial\sigma_{ij}}\frac{\partial f}{\partial\sigma_{ij}}}. \tag{10}$$

With the appropriate choice of function f, the equation (8), taking into account the relationships (10), connects the increments of the components of the irreversible strain, the temperature increment and stress components.

If we apply the formula (8) to the second expression (2), then a thermal coefficient of linear expansion can be represented as:

$$\alpha_{ij} = \alpha\delta_{ij} - \partial(a_{kl}\varepsilon_{kl}^n)/\partial\sigma_{ij},$$

where α is mean value of a thermal coefficient of linear expansion, which is used typically in calculating the temperature stresses, δ_{ij} is Kronecker delta.

Equation (8) geometrically means that the vector de^n with the components de^n_{ij}, and the gradient vector of function f, with the components $\partial f/\partial\sigma_{ij}$ are collinear, while the full vector increment of the irreversible strain is not directed perpendicularly to the loading surface. In general, determination of the direction of the vector increment of the irreversible strain is difficult enough. Similar difficulties arise during the visco-plastic flow of material, when the vector increment of the plastic strains directed at an angle to the normal of the surface of visco-plasticity. However, according to the assumption of Naghdi and Murch [6], it is believed that the vector increment of the plastic strains directed along the normal to the surface of visco-plasticity. Similar allegation is also used in the simplest theories of creep. The adoption of a particular loading law also makes a change in the development of the irreversible strains. The issue of the direction of the vector of irreversible flow has not always found its solution. This is partly due to the fact that, constitutive equations, which most fully describe the properties of material, remain critical for the practical calculations.

The formulas (8) and (10) are used in [16] to calculate the increments of the inelastic strains associated with the phase transformations, when calculating a shape memory cylinder in the temperature range of the direct martensitic transformations. During plastic flow of isotropic material, the most acceptable and simplest is a law of hardening (loading) used in the flow theory [11]: $f = 3S_{ij}S_{ij}/2 - 2[\phi(\chi,T)]^2 = 0$, where S_{ij} - are the components of the stress deviator, χ - a hardening measure. Then, from the formula (10), we have $d\lambda = d\bar{\varepsilon}^p_l/(2\sigma_l)$ (σ_l - equivalent stress) and the equation (8) becomes:

$$d\varepsilon^p_{ij} = -\frac{\partial(a_{kl}\varepsilon^p_{kl})}{\partial\sigma_{ij}}dT + \frac{3}{2}\frac{d\bar{\varepsilon}^p_l}{\sigma_l}S_{ij}, \tag{11}$$

where $d\bar{\varepsilon}^p_l$ - equivalent quantity of the plastic strain increments.

It is believed that the components of the tensor a_{ij} do not depend on the stress and are the parameters of material. For isotropic structural metallic materials, the tensor a_{ij} is a spherical tensor, whose components we denote by a_0. Then, from the equations (3.5) we have:

$$d\varepsilon^p_{ij} = -3\frac{\partial(a_0\varepsilon^p_0)}{\partial\sigma_{ij}}dT + \frac{3}{2}\frac{d\bar{\varepsilon}^p_l}{\sigma_l}S_{ij}, \tag{12}$$

where ε^p_0 is a mean plastic strain $\varepsilon^p_0 = \varepsilon^p_{ii}/3$, the value a_0 is determined through processing the dilatometric curves in the temperature range of the phase transformations and the stress-strain curves beyond the range of elasticity.

Using the above formulas (2), (12) and the relationships of thermoelasticity, the increment of the relative volume change is determined as:

$$d\varepsilon_0 = \frac{d\sigma_0}{3K} + \alpha dT - \left[\frac{\partial(a_0\varepsilon_0^p)}{\partial\sigma_{11}} + \frac{\partial(a_0\varepsilon_0^p)}{\partial\sigma_{22}} + \frac{\partial(a_0\varepsilon_0^p)}{\partial\sigma_{33}}\right] dT, \qquad (13)$$

where ε_0 - a mean strain, σ_0 - a mean normal stress, K - a volumetric modulus of elasticity.

Let us represent the condition (7) as follows:

$$f(\sigma_{ij}, T, Q) = \psi(\sigma_{ij}, T) - H(Q) = 0.$$

With the developing irreversible strain, the loading and unloading conditions can be written in the form of a non-isothermal theory of plasticity [12]:

$f = 0,\ \ \partial\psi/\partial\sigma_{ij} d\sigma_{ij} + \partial\psi/\partial T dT < 0,\ \ dH = 0$, unloading
$f = 0,\ \ \partial\psi/\partial\sigma_{ij} d\sigma_{ij} + \partial\psi/\partial T dT = 0,\ \ dH = 0$, neutral process
$f = 0,\ \ \partial\psi/\partial\sigma_{ij} d\sigma_{ij} + \partial\psi/\partial T dT > 0,\ \ dH \neq 0$, loading

With neglect of the plastic volume deformation, and if we suppose that the tensor components $a_{ij} = 0,\ i \neq j$, then from the relationships (11), we obtain the constitutive equations of thermoplasticity, given in paper [13].

4 Determination of the Components of Tensor a_{ij}

For material, the components of tensor a_{ij} are determined on the basis of the experimental data of testing specimens with the certain paths of thermo-mechanical loading, which are shown in Figure 3 for pure shear and uniaxial tension.

For shape memory alloys, the accumulation of inelastic strains is associated with thermoelastic martensitic transformations, in which the change in volume is very small. It has been established experimentally [14] that the inelastic phase shear strains are accumulated with temperature change in the temperature range of the direct martensitic transformations, even with a complete stress relief.

Therefore, the tensor components $a_{ij},\ i = j$ can be considered as zero, while in order to determine the components $a_{ij},\ i \neq j$, it is necessary to use the experimental data on torsion test of tubular specimens [14]. With an arbitrarily small constant value of shearing stress, with the change in temperature, the inelastic phase shear strains are accumulated in specimen. In this case, from the relationships (11), we obtain the following equation (the plane coincides with the pivoting plane of the cross section):

$$d\gamma_{xy}^n = a_{xy} \frac{\partial\gamma_{xy}^n}{\partial\tau} dT. \qquad (14)$$

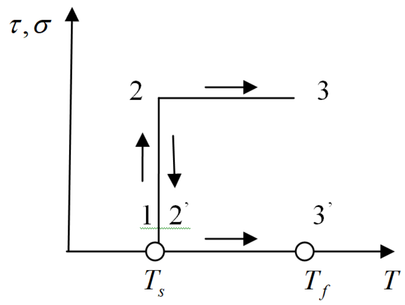

Figure 3: The types of thermo-mechanical loading in specimens testing

The value of a derivative on the right side of (14) is calculated as follows:

$$\partial\gamma_{xy}^n/\partial\tau \approx (\gamma_2^n - \gamma_1^n)/(\tau_2 - \tau_1), \tag{15}$$

where γ_1^n, γ_2^n are the irreversible shear strains for a given temperature at shearing stresses τ_1 and τ_2.

It is easy to see that $\partial\gamma_{xy}^n/\partial\tau \approx (\gamma_2 - \gamma_1)/(\tau_2 - \tau_1) - 1/G$, where γ_1, γ_2 are total shear strains at stresses τ_1 and τ_2, G is shearing modulus of elasticity.

The value $d\gamma_{xy}^n/dT$ is calculated upon the formula:

$$d\gamma_{xy}^n/\partial\tau \approx (\gamma_{i+1}^n - \gamma_{i-1}^n)/(2\Delta T), \tag{16}$$

where ΔT is temperature increment, γ_i^n is the value of inelastic shearing strain at the i - th point of curve.

It is obvious that $\gamma_{i+1}^n - \gamma_{i-1}^n = \gamma_{i+1} - \gamma_{i-1}$ (γ_i is total shear strain at the i - th point of curve). Then, $d\gamma_{xy}^n/\partial\tau \approx (\gamma_{i+1}^n - \gamma_{i-1}^n)/(2\Delta T)$. Using the relationships (15) and (16), from the formula (14) we determine the value a_{xy}.

For isotropic structural steel, tensor a_{ij} is a spherical tensor, the components of which (a_0) characterize a volume change of an inelastic nature. An inelastic volume change is associated with the structural transformations with the formation of a bainitic, martensitic or bainitic-martensitic structure, accompanied by the maximum strains $\varepsilon_0^n \approx 0,5???$ [15].

To determine the value a_0, the dilatometric curves and the stress-strain curves are used at a constant temperature in the temperature range of the phase transformations. When processing the dilatometric curves, it is necessary to take into account that the inelastic strain associated with stress does not arise $d\lambda\partial f/\partial\sigma_{ij} = 0$. Then, it follows from the equation (8) for uniaxial stress:

$$d\varepsilon^n = -a_0 dT \partial\varepsilon^n/\partial\sigma. \tag{17}$$

Irreversible strain is equivalent to the difference between the total and temperature strains. The derivative $d\varepsilon^n/dT \approx (\varepsilon^n_{i+1} - \varepsilon^n_{i-1})/(2\Delta T)$, where ε^n_i is the irreversible strain at the i - th point of curve. Processing of dilatograms in the temperature ranges of the phase transformations for technological processes of welding is given in the paper [15]. The determination of the value $\partial\varepsilon^n/\partial\sigma$ at a constant temperature is well-known in the theory of plasticity. Upon calculating the values $d\varepsilon^n/dT$ and $\partial\varepsilon^n/\partial\sigma$, from the formula (17) we find a_0.

Thus, the constitutive equations obtained above allow taking into in to account in the calculations an irreversible volume change associated with a temperature change in the ranges of phase transformation and the inelastic shearing strains that occur the cooling of shape memory alloys in the temperature range of the direct martensitic transformations.

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