ANALYSIS OF NONLINEAR DEFORMATION TASK OF LAYERED CYLINDRICAL SHELL BY LOCAL SURFACE FORCE AND TEMPERATURE FIELD

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Abstract

Based on one of the variants of improved theory, in the case of axisymmetric loading of layered cylindrical shell by local surface force and temperature field, for numerical solution of the nonlinear deformation task is obtained for this class the system of decision differential equations.

A particular example of deformation of cylindrical shell is considered. It is given an appropriate analysis based on the results obtained from numerical realization of the example.

Keywords and phrases: Layered shell, non-linear deformation, nonuniformity of in-plane shear deformation.

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The layered shells, as elements of the building structures that are subject to the wide range of surface forces and temperature field, have been widely applied in many fields of construction.

In this work it is considered the class of layered shells that are composed from the layers having substantially different mechanical properties. The study of mode of deformation of such class shells is desirable by application of one variant of the adjusted theory that is constructed on the base of the broken line hypothesis. The essence of the broken line hypothesis is as follows: the element of the shell arranged on the normal surface of the coordinate surface after deformation passes to the line, which gives the possibility of taking into account the heterogeneity of deformation of in-plane shear along the layered shell thickness.

Based on the proposed improved theory, the mode of deformation of orthotropic layered shell is considered in case of impact of acting on it surface forces and temperature filed. It is assumed that the deformation of the shells is elastic, i.e. connections between deformations and forces in each I layer are described by Hook's law, based on the hypothesis of the Duhamel-Neiman hypothesis [8], which has the following form in the α, β, γ curvilinear coordinate system:

$$\begin{aligned}
&\sigma_\alpha^i = B_{11}^i \varepsilon_{\alpha\alpha}^{(\gamma)} + B_{12}^i \varepsilon_{\beta\beta}^{(\gamma)} - \beta_1^i T,\\
&\sigma_\beta^i = B_{21}^i \varepsilon_{\alpha\alpha}^{(\gamma)} + B_{22}^i \varepsilon_{\beta\beta}^{(\gamma)} - \beta_2^i T,\\
&\tau_{\beta\gamma}^i = B_{44}^i \varepsilon_{\beta\gamma}^{(\gamma)}, \quad \tau_{\alpha\gamma}^i = B_{55}^i \varepsilon_{\alpha\gamma}^{(\gamma)}, \quad \tau_{\alpha\beta}^i = B_{66}^i \varepsilon_{\alpha\beta}^{(\gamma)},
\end{aligned} \tag{1}$$

where $T(\alpha, \beta, \gamma)$ is the temperature field.

In the case of improved theory, the basic equations and ratios of orthotropic layered shells are presented in the curvilinear coordinate system.

The tangential displacements in the layered shells layers tangent surface with taking into account the stresses and strains continuity conditions will be the following [4]:

$$\begin{aligned}
u_\alpha^{(i)} &= u + a_1^{(i)} \gamma_\alpha^{(0)} + \gamma(\psi_\alpha - a_2^{(i)} \gamma_\alpha^{(0)}),\\
u_\beta^{(i)} &= v + b_1^{(i)} \gamma_\beta^{(0)} + \gamma(\psi_\beta - b_2^{(i)} \gamma_\beta^{(0)}),
\end{aligned} \tag{2}$$

where u, v are the tangential displacement of coordinate surface, ψ_α, ψ_β are the angles of full rotation on normal of coordinate surface, $\gamma_\alpha^{(0)}$, $\gamma_\beta^{(0)}$ are represents the in-plane shear deformations, in that passes the coordinate surface. The design formulae for including in the expressions of tangential displacement (2) coefficients a_1^i, a_2^i, b_1^i, b_2^i are stated in the work [3].

In case of taking into account the tangential displacements (2) the components of deformation will be presented as:

$$\begin{aligned}
&\varepsilon_{\alpha\alpha}^{(\gamma)} = \varepsilon_{\alpha\alpha}^{(i)} + \gamma \varkappa_{\alpha\alpha}^{(i)}, \quad \varepsilon_{\alpha\beta}^{(\gamma)} = \varepsilon_{\alpha\beta}^{(i)} + 2\gamma \varkappa_{\alpha\beta}^{(i)},\\
&\varepsilon_{\beta\beta}^{(\gamma)} = \varepsilon_{\beta\beta}^{(i)} + \gamma \varkappa_{\beta\beta}^{(i)}, \quad \varepsilon_{\alpha\gamma}^{(\gamma)} = \gamma_\alpha^{(i)},\\
&\varepsilon_{\gamma\gamma}^{(\gamma)} = 0, \quad \varepsilon_{\beta\gamma}^{(\gamma)} = \gamma_\beta^{(i)}.
\end{aligned} \tag{3}$$

Expressions for $\varepsilon_{\alpha\alpha}^{(i)}$, $\varepsilon_{\beta\beta}^{(i)}$,..., $\varkappa_{\beta\beta}^{(i)}$ from (3) can be found in the works [4-6].

Taking into account (3) and Hooke's law (1), we have obtain the following expressions for elasticity relations:

$$\begin{aligned}
&N_\alpha = C_{11}\varepsilon_{\alpha\alpha} + C_{12}\varepsilon_{\beta\beta} + K_{11}\varkappa_\alpha + K_{12}\varkappa_\beta + A_{11}\frac{\partial \gamma_\alpha^{(0)}}{\partial \alpha}\\
&+A_{12}\gamma_\alpha^{(0)} + B_{11}\frac{\partial \gamma_\beta^{(0)}}{\partial \beta} + B_{12}\gamma_\beta^{(0)} - N_{\alpha T},\\
&N_\beta = C_{12}\varepsilon_{\alpha\alpha} + C_{22}\varepsilon_{\beta\beta} + K_{12}\varkappa_\alpha + K_{22}\varkappa_\beta + A_{21}\frac{\partial \gamma_\alpha^{(0)}}{\partial \alpha}\\
&+A_{22}\gamma_\alpha^{(0)} + B_{21}\frac{\partial \gamma_\beta^{(0)}}{\partial \beta} + B_{22}\gamma_\beta^{(0)} - N_{\beta T},
\end{aligned}$$

$$
\begin{aligned}
&N_{\alpha\beta} = C_{66}\varepsilon^*_{\alpha\beta} + 2K_{66}\varkappa_{\alpha\beta} + k_2(K_{66}\varkappa^*_{\alpha\alpha} + 2D_{66}\varkappa_{\alpha\beta})\\
&+(A_{16} + k_2E_{16})\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta} + (A_{26} + k_2E_{26})\gamma^{(0)}_\alpha + (B_{16} + k_2F_{16})\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha}\\
&+(B_{26} + k_2F_{26})\gamma^{(0)}_\beta,\\
&N_{\beta\alpha} = C_{66}\varepsilon^*_{\alpha\beta} + 2K_{66}\varkappa_{\alpha\beta} + k_1(K_{66}\varkappa^*_{\alpha\alpha} + 2D_{66}\varkappa_{\alpha\beta})\\
&+(A_{16} + k_1E_{16})\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta} + (A_{26} + k_1E_{26})\gamma^{(0)}_\alpha + (B_{16} + k_1F_{16})\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha}\\
&+(B_{26} + k_1F_{26})\gamma^{(0)}_\beta,\\
&M_\alpha = K_{11}\varepsilon_{\alpha\alpha} + K_{12}\varepsilon_{\beta\beta} + D_{11}\varkappa_\alpha + D_{12}\varkappa_\beta + E_{11}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha}\\
&+F_{11}\frac{\partial\gamma^{(0)}_\beta}{\partial\beta} + F_{12}\gamma^{(0)}_\beta - M_{\alpha T},\\
&M_\beta = K_{12}\varepsilon_{\alpha\alpha} + K_{22}\varepsilon_{\beta\beta} + D_{12}\varkappa_\alpha + D_{22}\varkappa_\beta + E_{21}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha}\\
&+F_{21}\frac{\partial\gamma^{(0)}_\beta}{\partial\beta} + F_{22}\gamma^{(0)}_\beta - M_{\beta T},\\
&M_{\alpha\beta} = M_{\beta\alpha} = K_{66}\varepsilon^*_{\alpha\beta} + 2D_{66}\varkappa_{\alpha\beta} + E_{16}\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta}\\
&+E_{26}\gamma^{(0)}_\alpha + F_{16}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha} + F_{26}\gamma^{(0)}_\beta,\\
&Q_\alpha = K_1\gamma^{(0)}_\alpha, \quad Q_\beta = K_2\gamma^{(0)}_\beta,
\end{aligned}
\tag{4}
$$

where N_α, N_β, $N_{\alpha\beta}$, $N_{\beta\alpha}$ are the tangential forces, Q_α, Q_β are the cutting forces, M_α, M_β are the torque moments, $N_{\alpha T}$, $N_{\beta T}$, $M_{\alpha T}$, $M_{\beta T}$ are the integral characteristics of temperature field; C_{ij}, K_{ij}, D_{ij}, K_1, K_2 are the characteristics of stiffness; A_{11}, A_{12}, ..., F_{16}, F_{26} are the values that are determined by geometrical and mechanical characteristics of layers of shells [4].

The including in the expressions (4) of elasticity dependencies variables $\varepsilon_{\alpha\alpha}$, $\varepsilon_{\beta\alpha}$, $\varepsilon^*_{\alpha\beta}$, $\gamma^{(0)}_\alpha$, $\gamma^{(0)}_\beta$ represents the characteristics of deformation of coordinate surface that will be as:

$$
\begin{aligned}
&\varepsilon_{\alpha\alpha} = \varepsilon_\alpha + \frac{1}{2}\theta^2_\alpha, \quad \varepsilon_{\beta\beta} = \varepsilon_\beta + \frac{1}{2}\theta^2_\beta, \quad \varepsilon^*_{\alpha\beta} = \varepsilon_{\alpha\beta} + \theta_\alpha\theta_\beta,\\
&\theta_\alpha = -\frac{1}{A}\frac{\partial\omega}{\partial\alpha} + k_1u, \quad \theta_\beta = -\frac{1}{B}\frac{\partial\omega}{\partial\beta} + k_2v,\\
&\gamma^{(0)}_\alpha = \psi_\alpha - \theta_\alpha, \quad \gamma^{(0)}_\beta = \psi_\beta - \theta_\beta,
\end{aligned}
\tag{5}
$$

there formulae for ε_α, ε_β, $\varepsilon_{\alpha\beta}$ from (5) can be found in the work [4].

The equilibrium equations of element of layered shell will be as:

$$
\begin{aligned}
&\frac{\partial BN_\alpha}{\partial\alpha}+\frac{\partial AN_{\beta\alpha}}{\partial\beta}+\frac{\partial A}{\partial\beta}N_{\alpha\beta}-\frac{\partial B}{\partial\alpha}N_\beta+ABk_1Q_\alpha^*+ABq_1=0,\\
&\frac{\partial AN_\beta}{\partial\beta}+\frac{\partial BN_{\alpha\beta}}{\partial\alpha}+\frac{\partial B}{\partial\alpha}N_{\beta\alpha}-\frac{\partial A}{\partial\beta}N_\alpha+ABk_2Q_\beta^*+ABq_2=0,\\
&\frac{\partial BQ_\alpha^*}{\partial\alpha}+\frac{\partial AQ_\beta^*}{\partial\beta}-ABk_1N_\alpha-ABk_2N_\beta+ABq_3=0,\\
&\frac{\partial BM_\alpha}{\partial\alpha}+\frac{\partial AM_{\beta\alpha}}{\partial\beta}+\frac{\partial A}{\partial\beta}M_{\alpha\beta}-\frac{\partial B}{\partial\alpha}M_\beta-ABQ_\alpha=0,\\
&\frac{\partial AM_\beta}{\partial\beta}+\frac{\partial AM_{\alpha\beta}}{\partial\alpha}+\frac{\partial B}{\partial\beta}M_{\beta\alpha}-\frac{\partial A}{\partial\beta}M_\alpha-ABQ_\beta=0,
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
Q_\alpha^*&=Q_\alpha-(N_\alpha+k_1M_\alpha)\theta_\alpha-(N_{\alpha\beta}+k_1M_{\alpha\beta})\theta_\beta,\\
Q_\beta^*&=Q_\beta-(N_{\beta\alpha}+k_2M_{\beta\alpha})\theta_\alpha-(N_\beta+k_2M_\beta)\theta_\beta.
\end{aligned}
\tag{7}
$$

Furthermore, the tasks of axisymmetric deformation of the rotation layered shells will be considered in case of impact of acting on it of surface forces and temperature field. In order to study the tasks of this class, let's introduce on the coordinate surface the system of curvilinear coordinates S, θ, where S is the arc length of the coordinate surface of the meridian from the parallel circle and θ represents the central angle in the parallel circle of the coordinate surface from the selected plane.

From basic equations and relations given in this work we obtain following system of nonlinear differential equations in the curvilinear coordinate system S, θ for the solution of above mentioned class of problems:

$$
\begin{aligned}
&\frac{dN_s}{ds}=a_{11}^*N_s+a_{12}^*Q_s^*+a_{13}^*M_s+a_{14}^*u+a_{15}^*w+a_{16}^*\psi_s\\
&+d_{11}(N_s+k_1M_s)+d_{12}\Phi+d_{11}^*N_{sT}+d_{13}^*N_{\theta T}+d_{14}^*\Phi_T+f_1,\\
&\frac{dQ_s^*}{ds}=a_{21}^*N_s+a_{22}^*Q_s^*+a_{23}^*M_s+a_{24}^*u+a_{25}^*w+a_{26}^*\psi_s\\
&+d_{21}(N_s+k_1M_s)\psi_s+d_{22}\Phi+d_{21}^*N_{sT}+d_{22}^*M_{sT}\\
&+d_{23}^*N_{\theta T}+d_{24}^*\Phi_T+f_2,\\
&\frac{dM_s}{ds}=a_{31}^*N_s+a_{32}^*Q_s^*+a_{33}^*M_s+a_{34}^*u+a_{35}^*w+a_{36}^*\psi_s\\
&+d_{31}(N_s+k_1M_s)\psi_s+d_{32}\Phi+d_{31}^*N_{sT}+d_{32}^*M_{sT}\\
&+d_{33}^*N_{\theta T}+d_{34}^*\Phi_T+d_{35}^*+f_3,\\
&\frac{du}{ds}=a_{11}^*N_s+a_{12}^*Q_s^*+a_{13}^*M_s+a_{14}^*u+a_{15}^*w+a_{16}^*\psi_s\\
&+d_{41}(N_s+k_1M_s)\psi_s+d_{42}\Phi_s^2+d_{43}^*Q_s^*\psi_s+d_{44}(N_s+k_1M_s)\psi_s\\
&+d_{45}Q_s^2+d_{46}(N_s+k_1M_s)Q_s^2\psi_s+d_{47}(N_s+k_1M_s)^2\psi_s^2+d_{48}\Phi\\
&+d_{41}^*N_{sT}+d_{42}^*N_{sT}+d_{43}^*N_{\theta T}+d_{44}^*\Phi_T+f_4,\\
&\frac{dw}{ds}=a_{52}^*Q_s^*+a_{54}^*u+a_{56}^*\psi_s+d_{51}(N_s+k_1M_s)\psi_s+f_5,
\end{aligned}
\tag{8}
$$

$$\begin{aligned}
\frac{d\psi_s}{ds} &= a^*_{61}N_s + a^*_{62}Q^*_s + a^*_{63}M_s + a^*_{64}u + a^*_{65}w + a^*_{66}\psi_s \\
&+d_{61}(N_s + k_1M_s)\psi_s + d^*_{62}\Phi^2_s + d^*_{63}Q^*_s\psi_s + d_{64}(N_s + k_1M_s)\psi^2_s \\
&+d^*_{65}Q^{*2}_s + d_{66}(N_s + k_1M_s)Q^*_s\psi_s + d_{67}(N_s + k_1M_s)^2\psi^2_s + d_{68}\Phi \\
&+d^*_{61}N_{sT} + d^*_{62}N_{sT} + d^*_{63}N_{\theta T} + d^*_{64}\Phi_T + f_6,
\end{aligned}$$

where

$$\begin{aligned}
\Phi &= \frac{1}{c_0 - c_1N_s - c_2M_s - c_3\psi_s}\left\{\frac{1}{c_0}\left(c_1N_s + c_2M_s + c_3\psi_s\right)\right. \\
&\times\left[a_1N_s + a_2Q^*_s + a_3M_s + a_4u + a_5w + (a_6 - q_4)\psi_s\right] \\
&+\frac{1}{c_0}\left(c_1N_s + c_2M_s + c_3\psi_S\right)^2 + \left(d_1N_s + d_2Q^*_s + d_3M_s + d_4u\right. \\
&\left.+d_5w + d_6\psi_s\right)\psi_s + \left(N_s + k_1M_s\right)\left(b_1N_s + b_2Q^*_s + b_3M_s + b_4u\right. \\
&\left.+b_5w + b_6\psi_s + b_7Q^{*2}_s + b_8Q^*_s\psi_s + b_9\psi^2_s\right) + \left(N_s + k_1M_s\right)^2 \\
&\left.\times\left(b_2\psi_s + 2b_7Q^*_s\psi_s + b_8\psi^2_s\right) + b_7\left(N_s + k_1M_s\right)^3 + M_s\psi_s\frac{dk_1}{ds}\right\}, \\
\Phi_T &= \frac{1}{c_0 - c_1N_s - c_2M_s - c_3\psi_s}\left\{\frac{k_2}{c_0}\left(c_1N_s + c_2M_s + c_3\psi_s\right)\right. \\
&\times\left(b_{31}N_{sT} + b_{32}M_{sT} - N_{\theta T}\right) + \left[(b_{31} + k_2b_{41})N_{sT}\right. \\
&\left.+(b_{32} + k_1b_{42})M_{sT} - N_{\theta T} - k_1M_{\theta T}\right]\psi_s \\
&\left.+(N_s + k_1M_s)(b_1N_{sT} + b_3M_{sT})\right\}.
\end{aligned} \tag{9}$$

Cowefficients from (8) and (9) are derived by means of geometric and mechanical characteristics of the shell [6, 7].

If we add to system of equations (8) the boundary conditions, we obtain the nonlinear boundary value problem.

As particular case is considered deformation tasks of the simply supported by edges orthotropic three-layered shell in the case of impact by acting on it contour compression force, local surface loadings and temperature field. The solution of task implies that shell is uniformly heated and coordinate surface passes in the middle of mid-surface.

Let's designate accordingly as h_1, h_2, h_3 the outer, middle and internal layers of shell; as E^i_1, E^i_2 are the elasticity modulus of i $(i = 1, 2, 3)$ layer of shell; v^i_{12}, v^i_{21} are the Poisson coefficients; G^i_{13} are the modulus of in-plane shear; α^i_{1T}, α^i_{2T} are the coefficient of thermal expansion; R is the radius of coordinate surface of cylindrical shell, and l is the length of cylinder. The task is solved for the following magnitudes of listed values: $R = 50$; $l = 60$; $h_1 = 0.3$; $h_2 = 2$; $h_3 = 0.3$; $E^1_1 = 1.5 \cdot 10^4$; $E^1_2 = 3 \cdot 10^4$; $E^2_1 = 2 \cdot 10^2$; $E^2_2 = 3 \cdot 10^2$; $E^3_1 = 1.5 \cdot 10^4$; $E^3_2 = 3 \cdot 10^4$; $\nu^1_{12} = 0.2$; $\nu^1_{21} = 0.34$; $\nu^2_{12} = 0.1$; $\nu^2_{21} = 0.15$; $\nu^3_{12} = 0.2$; $\nu^3_{21} = 0.34$; $G^1_{13} = 0.15 \cdot 10^4$; $G_13^2 = 0.15 \cdot 10^2$; $G_13^3 = 0.35 \cdot 10^4$; $\alpha^1_{1T} = 0.8 \cdot 10^{-4}$; $\alpha^1_{2T} = 1.2 \cdot 10^{-4}$; $\alpha^2_{1T} = 0.5 \cdot 10^{-2}$; $\alpha^2_{2T} = 1.2 \cdot 10^{-2}$; $\alpha^3_{1T} = 0.7 \cdot 10^{-4}$; $\alpha^3_{2T} = 0.16 \cdot 10^{-4}$.

Table

q	ω					
	T=0		T=50		T=100	
	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
	ε=10					
0	−0.17766	−0.17151	0.89130	0.84887	1.96031	1.86923
−10	−1.38211	−1.41122	−0.313162	−0.32811	0.75581	0.62361
−20	−2.58661	−2.65083	−1.51763	−1.63764	−0.44866	−0.61001
	ε=5					
0	−0.17766	−0.17151	0.89130	0.84887	1.96031	1.86923
−10	−1.15362	−1.34853	−0.08458	−0.328113	0.98446	0.69227
−20	−2.12941	−2.52553	−1.06052	−1.50514	0,00848	−0.48471
	ε=2.5					
0	−0.17766	−0.17151	0.89130	0.84887	1.96031	1.86923
−10	−0.84635	−1.18993	0.22261	−0. 16952	1.29165	0.85086
−20	−1.51150	−2.20836	−0.44608	−1.18722	0.62288	−0.16752

In the Table there are stated the numerical result of solution of proposed task in the $s = \frac{l}{2}$ point of ω function. The numerical data stated in the table presents constructed broken line hypotheses solutions obtained using linear as well as non-linear theories in the case of impact on cylinder of $N_s = -200$ compression contour force, acting on the cylindrical surface $s \in \left(\frac{l}{2} - \varepsilon, \frac{l}{2} + \varepsilon\right)$ as $q_3 = q \sin \pi \frac{s - \frac{l}{2} + \varepsilon}{2\varepsilon}$ the normal surface force and temperature filed T.

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