THE FLOW OF WEAKLYELECTROCONDUCTIVE LIQUID BETWEEN POROUS WALLS WITH HEAT TRANSFER

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Abstract

Is studied the pulsating flow of viscous incompressible liquid between porous walls with head transfer when in perpendicular of walls is applied external uniform magnetic field. The flow of liquid is caused due the pulsation drop of pressure and the pulsation movement of the porous walls. The physical characteristics of liquid flow are found.

Keywords and phrases: Weakly conducting; liquid; pulsation; fluid; heat transfer.

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1 Introduction

Is studied the pulsating flow of viscous incompressible liquid between porous walls with head transfer when in perpendicular of walls is applied external uniform magnetic field. The flow of liquid is caused due the pulsation movement of the porous walls and pulsation drop of pressure that is given by the formula: $-\frac{1}{\rho}\frac{\partial P}{\partial z} = Ae^{-i\omega t}$. The temperature change on porous walls of tube and in the tube is carried out by pulsating. In the heat transfer equation is taken not account the dissipation of caused due friction energy $\eta\left(\frac{\partial V}{\partial x}\right)$, as well as Joules heat σV^2.

Are obtained exact solutions of Navier-Stokes and heat transfer equations in the case of non-stationary motion of weak electroconductive viscous incompressible liquid fluid is. The physical characteristics of motion and heat transfer are studied by taking into account the impact of changes of Hartman, Prandtl, Reynolds numbers and pulsating flow to the criteria of similarity.

Accordingly to the assessed problem are studied in the [3, 5, 7, 8] articles and in the [4, 6, 10] works is considered laminar flow of fluid in pipe without a heat transfer when on the walls carried out the intensive inflow or leakage.

2 Basic Part

Let's consider the weak electroconductive viscous incompressible liquid flow in planar porous pipe with taking into account the heat transfer when in perpendicular of motion is applied the external homogeneous (H_0) magnetic field. The internal induction in comparison to external magnetic field is comparatively small and due it is neglected. Is implied that the liquid velocity has components $\vec{V}(u_0^*, 0, v_z(x,t))$ along the oz and ox axles, and the temperature $T(x,t)$ represents a function of x and t. $u_0^* = const$ is the leakage rate.

The motion and heat transfer equations in the non-inductive approximation generally has the following form [1, 2, 9]:

$$\begin{gathered}
\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\Delta\right)\vec{V} = -\frac{1}{\rho}\mathrm{grad}p + \nu\Delta\vec{V} - \frac{\sigma}{\rho}\left[H\left[\vec{V}\cdot\vec{H}\right]\right], \\
\rho C_\nu\left(\frac{\partial T}{\partial t} + (V\nabla)T\right) = k\Delta T + \Phi + \sigma\left[\vec{V}\cdot\vec{H}\right]^2, \\
\mathrm{div}\vec{V} = 0, \quad \mathrm{div}\vec{H} = 0,
\end{gathered} \tag{1}$$

where $\left[\vec{V}\cdot\vec{H}\right]^2$ is the Joules heat and Φ is the energy dissipation due the friction that is equal to

$$\begin{aligned}
\Phi = 2\eta\Bigg\{&\frac{1}{2}\left[\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)^2 + \left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial y}\right)^2\right] \\
&+ \left(\frac{\partial V_x}{\partial x}\right)^2 + \left(\frac{\partial V_y}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial z}\right)^2\Bigg\}.
\end{aligned}$$

If we take into account the above mentioned from the (1) system would receive in the dimensionless quantities

$$\begin{gathered}
\frac{\partial U}{\partial \tau} - \frac{\partial^2 U}{\partial \xi^2} - R\frac{\partial U}{\partial \xi} + M^2U = f(\tau), \\
p_r\frac{\partial \theta}{\partial \tau} - \frac{\partial^2 \theta}{\partial \xi^2} - p_rR\frac{\partial \theta}{\partial \xi} = \left(\frac{\partial U}{\partial \xi}\right)^2 + M^2U^2,
\end{gathered} \tag{2}$$

where $\xi = \frac{x}{L}$, $\tau = \frac{\nu}{L^2}t$, $U = \frac{V}{V_0^*}$, $\theta = \frac{k}{V_0^{*2}}T$, are the dimensionless quantities, and V_0^* and L in are accordingly characteristic velocity and characteristic length. $M = H_0L\sqrt{\frac{\sigma}{\eta}}$ is the Hartman's number, $\alpha = \frac{\omega L^2}{\nu}$ is the similarity criteria of steady pulsation motion, $P_r = \dfrac{\eta C_\nu}{k}$ is the Prandtl number, $R = \dfrac{u_0^* L}{\nu}$ is the liquid leakage characteristic Reynolds number, σ is the conductivity ratio, ν is the kinematic coefficient of viscosity, η is the dynamic coefficient of viscosity, ω is the frequency, C_ν is the specific heat capacity, k is the coefficient of conductivity.

the (2) system the initial and boundary conditions generally would be given in the following form:

$$\begin{gathered} U(\xi,0)=0, \quad U(1,\tau)=\varphi_1(\tau), \quad U(-1,\tau)=\varphi_2(\tau), \\ \theta(\xi,0)=\theta_1(\xi,0)+\theta_2(\xi,0)=0, \end{gathered} \tag{3}$$

$$\begin{gathered} \theta(1,\tau)=\theta_1(1,\tau)+\theta_2(1,\tau)=q_1^{(1)}(\tau)+q_1^{(2)}(\tau)=q_1(\tau), \\ \theta(-1,\tau)=\theta_1(-1,\tau)+\theta_2(-1,\tau)=q_2^{(1)}(\tau)+q_2^{(2)}(\tau)=q_2(\tau), \end{gathered} \tag{4}$$

where $\theta_1(\xi,\tau)$ is the temperature, when in the heat conductivity equation is considered only the friction heat, and $\theta_2(\xi,\tau)$ is the temperature, when in the heat conductivity equation is considered only the Joule's heat.

We imply that the liquid immediately begins to move (i.e. $U(\xi,0)=0$) and the temperature change of the planar tube walls in the initial moment is equal to zero.

If for the solution of (2)-(3) boundary problem we apply Laplase integral transformation we will obtain

$$\overline{U}'' + R\overline{U}' - (M^2+s)\overline{U} = -\overline{f}(s) \tag{5}$$

$$\overline{U}(1,s)=\overline{\varphi}_1(s), \quad \overline{U}(-1,s)=\overline{\varphi}_2(s), \tag{6}$$

where

$$\overline{U}(\xi,s)=\int_0^\infty U(\xi,\tau)e^{-s\tau}d\tau, \quad \overline{f}(s)=\int_0^\infty f(\tau)e^{-s\tau}d\tau,$$

$$\overline{\varphi}_1(s)=\int_0^\infty \varphi_1(\tau)e^{-s\tau}d\tau, \quad \overline{\varphi}_2(s)=\int_0^\infty \varphi_2(\tau)e^{-s\tau}d\tau.$$

The solution of (5)-(6) boundary conditions for velocity transformation receives con

$$\begin{gathered} \overline{U}(\xi,s)=\left(\overline{\varphi}_1(s)-\frac{\overline{f}(s)}{M^2+s}\right)e^{\frac{R(1-\xi)}{2}}\frac{\mathrm{sh}\overline{\beta}(1+\xi)/2}{\mathrm{sh}\overline{\beta}} \\ +\left(\overline{\varphi}_2(s)-\frac{\overline{f}(s)}{M^2+s}\right)e^{\frac{R(1+\xi)}{2}}\frac{\mathrm{sh}\overline{\beta}(1-\xi)/2}{\mathrm{sh}\overline{\beta}}+\frac{\overline{f}(s)}{M^2+s}, \end{gathered} \tag{7}$$

where

$$\overline{\beta}=\sqrt{R^2+4(M^2+s)}.$$

Let's study the liquid flow that is caused due pulsating motion of porous walls: $U(\pm 1,\tau)=\varphi_{1,2}(\tau)=Ae^{-i\alpha\tau}$, $\overline{\varphi}_{1,2}(s)=\dfrac{A_{1,2}}{s+i\alpha}$ and pulsating drop of pressure: $-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}=f_1(t)=A^{-i\omega t}$, $\overline{f}_1(s)=\dfrac{A}{s+i\alpha}$.

If we consider the above mentioned in the (7) formula it would take the following form:

$$\begin{gathered}\overline{U}(\xi,s)=\frac{A_1e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\overline{\beta}(1+\xi)/2}{(s+i\alpha)\mathrm{sh}\overline{\beta}}+\frac{A_2e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\overline{\beta}(1-\xi)/2}{(s+i\alpha)\mathrm{sh}\overline{\beta}}\\ +\frac{D}{(M^2+s)(s+i\alpha)\mathrm{sh}\overline{\beta}}\\ \times\left(\mathrm{sh}\overline{\beta}-e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\overline{\beta}(1+\xi)/2-e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\overline{\beta}(1-\xi)/2\right),\end{gathered}\tag{8}$$

where $D=\dfrac{AL_2}{\nu V_0^*}$ is the amplitude of pulsating drop of pressure and A_1 and A_2 - are the amplitudes of walls motion.

If the (8) formula would be written down in originals then for the calculation of velocity we receive the following formula:

$$\begin{gathered}U(\xi,\tau)=\left\{\left(A_1-\frac{D}{M^2-i\alpha}\right)e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\overline{\beta}(1+\xi)/2\right.\\ \left.+\left(A_2-\frac{D}{M^2-i\alpha}\right)e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\overline{\beta}(1-\xi)/2+\frac{D\mathrm{sh}\beta}{M^2-i\alpha}\right\}\frac{e^{-i\alpha\tau}}{\overline{sh\beta}}\\ +\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_ne^{-s_n\tau}}{i\alpha-s_n}\left[\left(A_1+\frac{4D}{R^2+\mu_n^2}\right)e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}\right.\\ \left.+\left(A_2+\frac{4D}{R^2+\mu_n^2}\right)e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right]=U_1(\xi,\tau)+U_2(\xi,\tau),\end{gathered}\tag{9}$$

where $s_n=-\dfrac{\mu_n^2+R^2+4M^2}{4}$, $\mu_n=\pi n$, $U_1(\xi,\tau)$ describes the steady pulsating flows between the porous walls and $U_2(\xi,\tau)$ describes the oscillation caused due porous walls pulsating motion and pulsating drop of pressure in the liquid.

After the rather large interval in the liquid oscillations would be damped $(U(\xi,\tau)\to\infty)$, thus for the velocity the formula (9) takes the following form:

$$\begin{gathered}U(\xi,\tau)=U_1(\xi,\tau)=\left\{\left(A_1-\frac{D}{M^2-i\alpha}\right)e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\beta(1+\xi)/2\right.\\ \left.+\left(A_2-\frac{D}{M^2-i\alpha}\right)e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\beta(1-\xi)/2+\frac{D\mathrm{sh}\beta}{M^2-i\alpha}\right\}\frac{e^{-i\alpha\tau}}{\mathrm{sh}\beta}.\end{gathered}\tag{10}$$

If in the equation of heat conductivity (2) neglected firstly the Joule's heat and then the friction heat we accordingly will obtain the following equations:

$$P_r \frac{\partial \theta_1}{\partial \tau} - \frac{\partial^2 \theta_1}{\partial \xi^2} - P_r R \frac{\partial \theta_1}{\partial \xi} = \left(\frac{\partial U}{\partial \xi} \right)^2, \tag{11}$$

$$P_r \frac{\partial \theta_2}{\partial \tau} - \frac{\partial^2 \theta_2}{\partial \xi^2} - P_r R \frac{\partial \theta_2}{\partial \xi} = M^2 U^2. \tag{12}$$

If we in the equations (11)-(12) will consider the formula (10) of velocity and apply the Laplace integral transformation formula then by taking into account the (4) boundary conditions the temperature in the transformations would be expressed as:

$$\begin{gathered} \overline{\theta}_m(\xi,\tau) = \left(\bar{q}_1^{(m)}(s) - \bar{q}_m(1)\right) e^{\frac{P_r R(1-\xi)}{2}} \frac{\mathrm{sh}\overline{\gamma}(1+\xi)/2}{\mathrm{sh}\overline{\gamma}} \\ + \left(\bar{q}_2^{(m)}(s) - \bar{q}_m(1)\right) e^{-\frac{P_r R(1+\xi)}{2}} \frac{\mathrm{sh}\overline{\gamma}(1-\xi)/2}{\mathrm{sh}\overline{\gamma}} + \bar{q}_m(\xi), \end{gathered} \tag{13}$$

where $m = 1, 2$; $\overline{\gamma} = \sqrt{P_r^2 R^2 + 4P_r s}$,

$$\begin{gathered} \beta_{1,2} = -\frac{R}{2} \pm \frac{1}{2}\sqrt{R^2 + 4(M^2 - i\alpha)}, \\ 2\beta_3 = \beta_1 + \beta_2, \quad 2\beta_4 = \beta_1, \quad 2\beta_5 = \beta_2, \end{gathered} \tag{14}$$

$$\bar{q}_1(\xi) = -\frac{1}{s + 2i\alpha} \left\{ \sum_{k=1}^{2} \frac{\beta_k^2 a_k e^{2\beta_k \xi}}{4\beta_k^2 + 2P_r R\beta_k - sP_r} + \frac{(M^2 - i\alpha) a_3 e^{-R\xi}}{R^2(1 - P_r) - sP_r} \right\},$$

$$\bar{q}_2(\xi) = -\frac{M^2}{s + 2i\alpha} \left\{ \sum_{k=1}^{5} \frac{a_k e^{2\beta_k \xi}}{4\beta_k^2 + 2P_r R\beta_k - sP_r} + \frac{a_6}{sP_r} \right\}, \tag{15}$$

$$a_{1,2} = \left[\frac{\left(A_1 - \frac{D}{M^2 - i\alpha}\right) e^{-\beta_{2,1}} - \left(A_2 - \frac{D}{M^2 - i\alpha}\right) e^{\beta_{2,1}}}{2\mathrm{sh}(\beta_1 - \beta_2)} \right]^2, \tag{16}$$

$$\begin{gathered} a_3 = \frac{1}{2\mathrm{sh}^2(\beta_1 - \beta_2)} \left[\left(A_2 - \frac{D}{M^2 - i\alpha}\right) e^{2\beta_3} + \left(A_1 - \frac{D}{M^2 - i\alpha}\right)^2, \right. \\ \left. -2\left(A_2 - \frac{D}{M^2 - i\alpha}\right)\left(A_1 - \frac{D}{M^2 - i\alpha}\right) \mathrm{ch}(\beta_1 - \beta_2) \right], \end{gathered} \tag{17}$$

$$\begin{gathered} a_{4,5} = \frac{D}{(M^2 - i\alpha)\mathrm{sh}(\beta_1 - \beta_2)} \left[\left(A_2 - \frac{D}{M^2 - i\alpha}\right) e^{\beta_{2,1}} \right. \\ \left. - \left(A_1 - \frac{D}{M^2 - i\alpha}\right) e^{-\beta_{2,1}} \right], \\ a_6 = \left(\frac{D}{M^2 - i\alpha} \right)^2. \end{gathered} \tag{18}$$

Let's study the temperature changes at steady pulsating motion of liquid, when the temperature change in the initial moment is equal to zero, and on the planar walls of pipe changes by pulsating law:

$$q_{1,2}^{(1)}(\tau) = B_{1,2}^{(1)} e^{-2i\alpha\tau}, \quad q_{1,2}^{(2)}(\tau) = B_{1,2}^{(2)} e^{-2i\alpha\tau}.$$

If we considered the above mentioned in the (13) formula, then for the temperature in originals accordingly would be obtained the following expressions:

$$\begin{aligned}
&\theta_m(\xi,\tau) = \Big[\Big(B_1^{(m)} - q_m(-1)\Big) e^{\frac{P_r R(1-\xi)}{2}} \mathrm{sh}\gamma(1+\xi)/2 \\
&+ \Big(B_2^{(m)} - q_m(-1)\Big) e^{-\frac{P_r R(1+\xi)}{2}} \mathrm{sh}\gamma(1-\xi)/2 + q_m(\xi)\mathrm{sh}\gamma\Big] \frac{e^{-2i\alpha\tau}}{\mathrm{sh}\gamma} \\
&+\frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)} \Big[\Big(B_1^{(m)} - q_m^*(1)\Big) e^{\frac{P_r R(1-\xi)}{2}} \sin\mu_n(1+\xi)/2 \\
&+ \Big(B_2^{(m)} - q_m^*(-1)\Big) e^{-\frac{P_r R(1+\xi)}{2}} \sin\mu_n(1-\xi)/2\Big].
\end{aligned} \tag{19}$$

where $m = 1, 2$; $\mu_n = \pi n$, $s_n = -\dfrac{\mu_n^2 + P_r^2 R^2}{4P_r}$, $\gamma = \sqrt{P_r^2 R^2 - 8i\alpha P_r}$,

$$q_1(\xi) = -\sum_{k=1}^{2} \frac{\beta_k^2 a_k e^{2\beta_k\xi}}{4\beta_k^2 + 2P_r R\beta_k + 2i\alpha P_r} - \frac{(M^2 - i\alpha)a_3 e^{-R\xi}}{R^2(1-P_r) + 2i\alpha P_r},$$

$$q_2(\xi) = -M^2 \sum_{k=1}^{5} \frac{a_k e^{2\beta_k\xi}}{4\beta_k^2 + 2P_r R\beta_k + 2i\alpha P_r} + \frac{M^2 a_6}{2i\alpha P_r}, \tag{20}$$

$$q_1^*(\xi) = -\sum_{k=1}^{2} \frac{\beta_k^2 a_k e^{2\beta_k\xi}}{4\beta_k^2 + 2P_r R\beta_k - s_n P_r} - \frac{(M^2 - i\alpha)a_3 e^{-R\xi}}{R^2(1-P_r) - s_n P_r},$$

$$q_2^*(\xi) = -M^2 \sum_{k=1}^{5} \frac{a_k e^{2\beta_k\xi}}{4\beta_k^2 + 2P_r R\beta_k - s_n P_r} + \frac{M^2 a_6}{2s_n P_r},$$

Let's mention that at the $q_2(\xi)$ and $q_2^*(\xi)$ computation before the a_3 and a_4 coefficients would be implied the sign "-".

I. let's consider the pulsating flow of liquid that is caused by the pulsating motion of walls. Let's say the pulsating motion of walls occurs in the same phase, by the same amplitude ($A_1 = A_2 = U_0$), the temperature change on the walls of tube carried out pulsating in the same phase, by same amplitude, ($B_{1,2}^{(1)} = \theta_1^{(1)} = const$, $B_{1,2}^{(2)} = \theta_1^{(2)} = const$), and the drop of pressure is equal to zero ($D = 0$).

Due the consideration of above mentioned, for the velocity and temperature by the (9) and (19) formulae we will obtain that

$$\begin{aligned}
&\frac{U^I(\xi,\tau)}{U_0} = \frac{e^{-i\alpha\tau}}{\mathrm{sh}\beta}\left(e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\beta(1+\xi)/2 + e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\beta(1-\xi)/2\right)\\
&+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{-(M^2+\frac{R^2+\mu_n^2}{4})\tau}}{i\alpha - s_n}\left(e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}\right.\\
&\left.+e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right) = U_1^I(\xi,\tau) + U_2^I(\xi,\tau),
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\theta_m^I(\xi,\tau) = \left[\left(\theta_1^{(m)} - q_m(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\mathrm{sh}\gamma(1+\xi)/2\right.\\
&\left.\left(\theta_1^{(m)} - q_m(-1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\mathrm{sh}\gamma(1-\xi)/2 + q_m(\xi)\mathrm{sh}\gamma\right]\frac{e^{-2i\alpha\tau}}{\mathrm{sh}\gamma}\\
&+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)}\left[\left(\theta_1^{(m)} - q_m^*(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\sin\mu_n(1+\xi)/2\right.\\
&\left.+\left(\theta_1^{(m)} - q_m^*(-1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\sin\mu_n(1-\xi)/2\right]\\
&= \theta_m^*(\xi,\tau) + \theta_m^{**}(\xi,\tau),
\end{aligned} \tag{22}$$

where $m = 1, 2$, and $q_{1,2}(\xi)$ and $q_{1,2}^*(\xi)$ would be calculated due the (20) formula.

If we calculate the friction force in liquid and on planar pipe's walls, we accordingly will obtain the following formulae:

$$\begin{aligned}
F^I = \frac{U_0 e^{-i\alpha\tau}}{2\mathrm{sh}\beta}&\left[\beta\left(e^{\frac{R(1-\xi)}{2}}\mathrm{ch}\frac{\beta(1+\xi)}{2} - e^{\frac{R(1+\xi)}{2}}\mathrm{ch}\frac{\beta(1-\xi)}{2}\right)\right.\\
&\left.-R\left(e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\frac{\beta(1+\xi)}{2} + e^{\frac{R(1+\xi)}{2}}\mathrm{sh}\frac{\beta(1-\xi)}{2}\right)\right]
\end{aligned}$$

$$F_{1,2}^I = \frac{U_0 e^{-i\alpha\tau}}{2\mathrm{sh}\beta}\left[\beta\left(\pm\mathrm{ch}\beta\mu e^{\mu R}\right) - R\mathrm{sh}\beta\right],$$

and for the flow rate and average velocity we will have:

$$\theta^I = \frac{U_0\beta(\mathrm{ch}\beta/2 - \mathrm{ch}R)e^{-i\alpha\tau}}{(M^2 - i\alpha)\mathrm{sh}(\beta_1 - \beta_2)},$$

$$U^I = \frac{1}{2}\frac{U_0\beta(\mathrm{ch}\beta/2 - \mathrm{ch}R)e^{-i\alpha\tau}}{(M^2 - i\alpha)\mathrm{sh}\beta}.$$

When the pulsating fluid flow is caused by the porous walls pulsating motion (the walls are moved in the same phase with the same amplitude), the friction force on the pile walls takes the different, and the maximum value of velocity makes at the pipe's axis.

II. Let's say that walls pulsating motion and temperature changes on the pipe walls is made by different signs of amplitude ($A_1 = V_0$, $A_2 = -V_0$, $B_1^{(1)} = B_2^{(1)} = \theta_2^{(1)} = const$, $B_1^{(2)} = B_2^{(2)} = \theta_2^{(2)} = const$). The drop of pressure in the pipe is still equal to zero ($D = 0$).

If we consider the above mentioned for the velocity and temperature we will obtain

$$\begin{aligned}\frac{U''(\xi,\tau)}{V_0} &= \frac{e^{-i\alpha\tau}}{\mathrm{sh}\beta}\left(e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\beta(1+\xi)/2 - e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\beta(1-\xi)/2\right)\\ &+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{-\left(M^2+\frac{R^2+\mu_n^2}{4}\right)\tau}}{i\alpha - M^2 - \frac{R^2+\mu_n^2}{4}}\left(e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}\right.\\ &\left.-e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right) = U_1^{II}(\xi,\tau) + U_2^{II}(\xi,\tau),\end{aligned} \tag{23}$$

$$\begin{aligned}\theta''_m(\xi,\tau) &= \left[\left(\theta_2^{(m)} - q_m(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\mathrm{sh}\gamma(1+\xi)/2\right.\\ &\left.\left(\theta_2^{(m)} - q_m(-1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\mathrm{sh}\gamma(1-\xi)/2 + q_m(\xi)\mathrm{sh}\gamma\right]\frac{e^{-2i\alpha\tau}}{\mathrm{sh}\gamma}\\ &+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)}\left[\left(\theta_2^{(m)} - q_m^*(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\sin\mu_n(1+\xi)/2\right.\\ &\left.-\left(\theta_2^{(m)} - q_m^*(-1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\sin\mu_n(1-\xi)/2\right]\\ &= \theta_{m+2}^*(\xi,\tau) + \theta_{m+2}^{**}(\xi,\tau),\end{aligned} \tag{24}$$

where $q_{1,2}(\xi)$ and $q_{1,2}^*(\xi)$ would be calculated due the formula (20).

If we calculate the friction force in fluid and on the planar pipe walls accordingly we will obtain the following formulae:

$$\begin{aligned}F^{II} &= \frac{V_0e^{-i\alpha\tau}}{2\mathrm{sh}\beta}\left[\beta\left(e^{\frac{R(1-\xi)}{2}}\mathrm{ch}\frac{\beta(1+\xi)}{2} + e^{-\frac{R(1+\xi)}{2}}\mathrm{ch}\frac{\beta(1-\xi)}{2}\right)\right.\\ &\left.-R\left(e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\frac{\beta(1+\xi)}{2} - e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\frac{\beta(1-\xi)}{2}\right)\right],\\ F_{1,2}^{II} &= \frac{V_0e^{-i\alpha\tau}}{2\mathrm{sh}\beta}\left[\beta(\pm\mathrm{ch}\beta + e^{\mu R})\mu R\mathrm{sh}\beta\right],\end{aligned}$$

and for the flow rate and average velocity we will have:

$$\theta^{II} = \frac{V_0e^{-i\alpha\tau}}{(M^2 - i\alpha)\mathrm{sh}}\left[\beta(\mathrm{ch}\beta + \mathrm{ch}R) + R\mathrm{sh}R\right],$$

$$U^{II} = \frac{1}{2}\frac{V_0e^{-i\alpha\tau}}{(M^2 - i\alpha)\mathrm{sh}}\left[\beta(\mathrm{ch}\beta + \mathrm{ch}R) + R\mathrm{sh}R\right].$$

When the fluid pulsating flow is caused by the walls pulsating motion (the walls pulsating motion is carried out by reverse direction of sign), then on the planar pipes axis the friction force didn't reach the maximal value, and on the pipe walls don't make the same values as in the case of non-porous walls.

At pulsating motion of porous walls (I-II case) the transfer of pulsation occurs throughout the whole liquid. As calculations show, the localization of pulsating fluid flow becomes in adjacent of the walls. Thus the stabilization of pulsating flow of liquid occurs very quickly. The fluid's rate that describes the process of pulsating stabilization, would be defined by the (21) and (23) formulae; the smaller is the distance between the porous wall, the sooner will be stabilized the pulsating motion of fluid and, conversely, the greater is the distance between the walls, the more time is required for stabilization of fluid pulsating motion.

The stabilization of fluid pulsating flow occurs after the long time from fluid motion start i.e. when $\tau \to \infty$, then the values

$$U_2^I = \frac{U_0}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{-\left(M^2+\frac{R^2+\mu_n^2}{4}\right)\tau}}{i\alpha - M^2 - \frac{R^2+\mu_n^2}{4}} \times\left(e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2} + e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right),$$

$$U_2^{II} = \frac{V_0}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{-\left(M^2+\frac{R^2+\mu_n^2}{4}\right)\tau}}{i\alpha - M^2 - \frac{R^2+\mu_n^2}{4}} \times\left(e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2} - e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right),$$

tends to zero, and fluid steady pulsating flow would be calculated by following formula:

$$\frac{U^I(\xi,\tau)}{U_0} = \frac{e^{-i\alpha\tau}}{\mathrm{sh}\beta}\left(e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\beta(1+\xi)/2 + e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\beta(1-\xi)/2\right),$$

$$\frac{U^{II}(\xi,\tau)}{V_0} = \frac{e^{-i\alpha\tau}}{\mathrm{sh}\beta}\left(e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\beta(1+\xi)/2 - e^{-\frac{R(1+\xi)}{2}}\mathrm{sh}\beta(1-\xi)/2\right).$$

At the steady pulsating fluid flow (I-II case) after some time the temperature change in fluid occurs in pulsating mode, i.e. when $\tau \to \infty$, then the values

$$\theta_m^{**}(\xi,\tau) = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)}\left[\left(\theta_2^{(m)} - q_m^*(1)\right)e^{\frac{P_rR(1-\xi)}{2}} \times\sin\mu_n(1+\xi)/2 + \left(\theta_2^{(m)} - q_m^*(1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\sin\mu_n(1-\xi)/2\right],$$

$$\theta_{m+2}^{**}(\xi,\tau)=\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)}\left[\left(\theta_2^{(m)}-q_m^*(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\right.$$
$$\left.\times\sin\mu_n(1+\xi)/2-\left(\theta_2^{(m)}-q_m^*(1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\sin\mu_n(1-\xi)/2\right],$$

tends to zero, and at steady pulsating motion of fluid (that is caused by the pulsating motion of porous walls) the fluid temperature changes law will had the pulsating character that would be calculated by the following formulae:

$$\theta_m^{**}(\xi,\tau)=\left[\left(\theta_1^{(m)}-q_m(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\mathrm{sh}\gamma(1+\xi)/2\right.$$
$$\left.+\left(\theta_1^{(m)}-q_m(-1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\mathrm{sh}\gamma(1-\xi)/2+q_m(\xi)\mathrm{sh}\gamma\right]\frac{e^{-2i\alpha\tau}}{\mathrm{sh}\gamma},$$

$$\theta_{m+2}^{**}(\xi,\tau)=\left[\left(\theta_2^{(m)}-q_m(1)\right)e^{\frac{P_rR(1-\xi)}{2}}\mathrm{sh}\gamma(1+\xi)/2\right.$$
$$\left.-\left(\theta_2^{(m)}-q_m(-1)\right)e^{-\frac{P_rR(1+\xi)}{2}}\mathrm{sh}\gamma(1-\xi)/2+q_m(\xi)\mathrm{sh}\gamma\right]\frac{e^{-2i\alpha\tau}}{\mathrm{sh}\gamma}.$$

When the fluid motion is caused by the pulsating motion of porous walls, then the friction heat impact on the weak conductive fluid will be more important than at pulsating motion of non-porous walls, and the action of Joule heat action in both cases is almost similar.

III. Let's consider the pulsating flow of fluid that is caused only by pulsating drop of pressure ($D\neq 0$). The pipe walls are motionless ($A_1 = A_2 = 0$).

Let's imply that temperature change on the porous pipe walls isn't occurring by pulsating law ($B_1^{(1)}=B_2^{(1)}=B_1^{(2)}=B_2^{(2)}=0$).

If we consider the above mentioned, then for velocity and temperature from (9) and (19) formulae we will obtain.

$$\frac{U^{III}(\xi,\tau)}{D/(M^2-i\alpha)}=\frac{e^{-i\alpha\tau}}{\mathrm{sh}\beta}\left[\mathrm{sh}\beta-e^{\frac{R(1-\xi)}{2}}\mathrm{sh}\beta(1+\xi)/2-e^{\frac{R(1+\xi)}{2}}\right.$$
$$\left.\times\mathrm{sh}\beta(1-\xi)/2\right]+2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(M^2-i\alpha)\mu_n e^{-\left(M^2+\frac{R^2+\mu_n^2}{4}\right)\tau}}{\left(i\alpha-M^2-\frac{R^2+\mu_n^2}{4}\right)(R^2+\mu_n^2)} \tag{25}$$
$$\times\left[e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right]$$
$$=U_1^{III}(\xi,\tau)+U_2^{III}(\xi,\tau),$$

$$\theta_m^{III}(\xi,\tau) = \frac{e^{-2i\alpha\tau}}{\text{sh}\beta}\Big[q_m(\xi,\tau)\text{sh}\gamma - q_m(1)e^{\frac{P_rR(1-\xi)}{2}}\text{sh}\gamma(1+\xi)/2$$
$$-q_m(-1)e^{-\frac{P_rR(1+\xi)}{2}}\text{sh}\gamma(1-\xi)/2\Big] - \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)} \tag{26}$$
$$\times\Big[q_m^*(1)e^{\frac{P_rR(1-\xi)}{2}}\sin\mu_n(1+\xi)/2 + q_m^*(-1)e^{-\frac{P_rR(1+\xi)}{2}}$$
$$\times\sin\mu_n(1-\xi)/2] = \theta^*_{m+4}(\xi,\tau) + \theta^* *_{m+4}(\xi,\tau),$$

where the $q_m(\xi)$ and $q_m^*(\xi)$ would be calculated from the (20) formulae.

Pulsating drop of pressure in fluid generates the pulsating flow and oscillating motion that will be expressed by the (25) formula.

The stabilization of pulsating flow in liquid (that is caused by a pulsating drop of pressure) occurs after the rather long time from fluid oscillating motion start, i.e. when $\tau \to \infty$.

Then

$$\frac{U_2^{III}(\xi,\tau)}{D/(M^2-i\alpha)} = 2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(M^2-i\alpha)\mu_n e^{-\left(M^2+\frac{R^2+\mu_n^2}{4}\right)\tau}}{\left(i\alpha - M^2 - \frac{R^2+\mu_n^2}{4}\right)(R^2+\mu_n^2)}$$
$$\left[e^{\frac{R(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2} + e^{-\frac{R(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right]$$

the sum trends to zero and steady pulsating flow will be calculated by following formula:

$$\frac{U_2^{III}(\xi,\tau)}{D/(M^2-i\alpha)} = \frac{e^{-i\alpha\tau}}{\text{sh}\beta}\left[\text{sh}\beta - e^{\frac{R(1-\xi)}{2}}\text{sh}\beta(1+\xi) - e^{-\frac{R(1+\xi)}{2}}\text{sh}\beta(1-\xi)\right].$$

When in the porous pipe will be stabilized pulsating flow of liquid, then the temperature change still occurs by pulsating law, as well as by oscillating law, after a rather long time, i.e. when $\tau \to \infty$. The values

$$\theta^{**}_{m+4} = -\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{s_n\tau}}{P_r(s_n+2i\alpha)}\Big[q_m^*(1)e^{\frac{P_rR(1-\xi)}{2}}\sin\mu_n(1+\xi)/2$$
$$+q_m^*(-1)e^{-\frac{P_rR(1+\xi)}{2}}\sin\mu_n(1-\xi)/2\Big]$$

trends to zero, and the temperature change occurs by the pulsating law and will be calculated by (26) formula.

If we calculate the friction force in fluid, on the pipe porous walls accordingly obtain the following formula:

$$F^{III} = \frac{De^{-i\alpha\tau}}{2(M^2-i\beta)\text{sh}\beta}\left[\left(R\text{sh}\frac{\beta(1+\xi)}{2} - R\text{ch}\frac{\beta(1+\xi)}{2}\right)e^{\frac{R(1-\xi)}{2}}\right.$$
$$\left.+\left(R\text{sh}\frac{\beta(1-\xi)}{2} + R\text{ch}\frac{\beta(1-\xi)}{2}\right)e^{\frac{R(1+\xi)}{2}}\right],$$
$$F_{1,2}^{III} = \frac{De^{-i\alpha\tau}}{2(M^2-i\beta)\text{sh}\beta}[\beta(\pm e^{\mp R} \mp \text{ch}) + R\text{sh}\beta],$$

and for flow rate and average velocity we will have:

$$\theta^{III} = \frac{2De^{-i\alpha\tau}}{M^2 - i\alpha}\left[1 - \frac{\beta(\mathrm{ch}\beta - \mathrm{ch}R)}{2(M^2 - i\alpha)\mathrm{sh}\beta}\right],$$

$$U^{III} = \frac{De^{-i\alpha\tau}}{M^2 - i\alpha}\left[1 - \frac{\beta(\mathrm{ch}\beta - \mathrm{ch}R)}{2(M^2 - i\alpha)\mathrm{sh}\beta}\right].$$

When the fluid pulsating flow is caused by pulsating drop of pressure, then the on porous pipe axis ($\xi = 0$) the velocity and temperature can not reach the maximum value, and the friction force is not equal to zero on the axis, as is the case, as it takes place in the case of non-porous pipe.

3 Conclusion

Conducted by the above mentioned formulae calculations show that the action of external homogeneous magnetic field slows down fluid pulsating flow. At increasing of magnetic field the fluid flow rate on the planar pipe axis decreases, but at the walls is increasing, while the value of average velocity in the planar pipe cross section is not changed.

The velocity, temperature, friction and flow rate related to the time has periodic properties.

Each period starts with a strong fluid flow in front direction after that occurs reverse flow, and then we have motionless state and repeatedly weak counter flow.

1. When the fluid flow is caused by pulsating drop of pressure or walls pulsating motion, then the increasing of external homogeneous magnetic field and increasing of leakage Reynolds $\left(R = \dfrac{U_0^* L}{\nu}\right)$ number causes the increasing of friction and reduction of flow rate.

2. When the fluid pulsating flow is caused by the walls pulsating motion and pulsating drop of pressure, then the increasing of leakage Reynolds number causes deceleration of fluid pulsating flow stabilization, and the reduction of the leakage Reynolds numbers causes the acceleration of fluid pulsating flow stabilization, and the temperature change in both cases is slightly different from each other.

3. When the fluid flow is caused by the pulsating drop of pressure, then the increasing of Hartman number causes reducing of temperature in porous pipes. This result corresponds to obtained in the previous case results according of that the increasing of Hartman causes the deceleration of fluid pulsating flow.

In general, we can make such conclusion:

a) The stabilization of fluid pulsating flow and temperature change by pulsating law in non-porous pipes occurs faster than in porous pipes.

b) The impact of external magnetic field on fluid pulsating flow generally causes increasing of temperature in planar pipe.

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