THE STATIONARY FLOW OF LAMINAR LIQUID IN AN CIRCULAR PIPE OF INFINITE LENGTH

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(Received: 07.01.2018; accepted: 22.05.2018)

Abstract

In the article is considered stationary flow of viscous incompressible electrically conducting liquid in infinite length pipe at existence of transversal magnet field. The motion is originated due applied in the initial moment of time constant longitudinal drop of pressure. The exact solution of problem in the general form is obtained.

Key words and phrases: Stationary flow, conducting, circular pipe, liquid.
AMS subject classification: 85A30, 76W05.

1 Introduction

Subject of presented work is presented by research of stationary motion of electrically conducting viscous incompressible liquid in infinite length pipe, arranged in external uniform magnet field in perpendicular to axis of pipe. Is accepted that motion is originated by applied in the initial moment of time constant transverse drop of pressure, although is not difficult to generalize problem on case of existence of initial distribution of velocity as well as on the case of moving walls.

2 Basic Part

Hartmann flows currently are rather detailed studied [1-4, 9,12], and possibilities of obtaining new analytical exact solutions appears as quite limited. Nevertheless, these possibilities at the same time are existing and are possible to found, as it was expressed below, to found simple new solution that simultaneously have the rather interesting quality singularities.

It is know [1-8] that if viscous incompressible liquid is moving in the direction, perpendicular to uniform field (H_0), then the equations of magnetohydrodynamics are reduced to the following system

$$\Delta h + \frac{\partial u}{\partial \xi} = 0, \quad \Delta u + M^2 \frac{\partial h}{\partial \xi} = -Q. \tag{1}$$

There is introduced the following dimensionless values

$$u = \frac{v}{v_0}, \quad h = \frac{H}{H_0 R_m}, \quad Q = \frac{Pa^2}{v \cdot v_0}, \quad \xi = \frac{x}{a}, \tag{2}$$

where v_0 is the certain characteristic velocity, a is the characteristic dimension, R_m is the magnetic Reynolds number, M is the Hartmann number, v is the coefficient of viscosity, P is the drop of pressure applied in the direction of motion.

Is assumed that applied magnetic field is directed along the axis Ox, and velocity and induced magnetic field have the components $v = v_z(x, y)$ and $H = H_z(x, y)$ only along the axis z.

Due the introductions of

$$F = e^{\frac{M}{2}\xi} \left(u + Mh + \frac{Q\xi}{M} \right), \quad \Phi = e^{-\frac{M}{2}\xi} \left(u - Mh - \frac{Q\xi}{M} \right) \tag{3}$$

the main system (1) would be reduced to two separate following equations

$$\Delta F - \mu^2 F = 0, \quad \Delta \Phi - \mu^2 \Phi = 0, \quad \mu = \frac{M}{2}. \tag{4}$$

If the flow is carried out in the given profile pipe (Γ), walls of that would be assumed as non- conducting, then the boundary condition of such problem will be as

$$F|_{(\Gamma)} = \left[e^{\mu\xi} \left(\frac{\bar{v}}{v_0} + \frac{Q\xi}{2\mu} \right) \right]_{(\Gamma)}, \quad \Phi|_{(\Gamma)} = \left[e^{-\mu\xi} \left(\frac{\bar{v}}{v_0} - \frac{Q\xi}{2\mu} \right) \right]_{(\Gamma)} \tag{5}$$

(is accepted that walls of pipe are moving with constant velocity $\bar{v}$).

For the case of rectangular cross-section the exact solution of problem (at $\bar{v} = 0$) is obtained by Shercliff. In the presented paper is considered the circular cross-section of radius a (such problem is naturally to consider as generalized of known one-dimensional Hartmann problem).

The solution of equation (4) on axis of pipe is represented by trigonometric series

$$\begin{aligned} F &= \frac{A_0}{2} I_0(\mu\rho) + \sum_{n=0}^{\infty} A_n I_n(\mu\rho) \cos(n\theta), \\ \Phi &= \frac{B_0}{2} I_0(\mu\rho) + \sum_{n=0}^{\infty} B_n I_n(\mu\rho) \cos(n\theta), \end{aligned} \tag{6}$$

where $\rho = \frac{r}{a}$; r, θ are the polar coordinates ($\xi = \rho\cos\theta$); $I_n(x)$ are the modified cylindrical functions [11].

The boundary conditions

$$F|_{\rho=1} = \frac{Q}{2\mu} e^{\mu\cos\theta}\cos\theta, \quad \Phi|_{\rho=1} = -\frac{Q}{2\mu} e^{-\mu\cos\theta}\cos\theta \tag{7}$$

gives the possibility to found the coefficients A_n and B_n:

$$A_n = (-1)^n B_n = \frac{Q}{\pi\mu I_n(\mu)} \int_0^{\pi} e^{\mu\cos\theta}\cos n\theta\cos\theta d\theta = \frac{Q}{2\pi\mu}(C_{n+1} + C_{n-1}),$$

where

$$C_m = \int_0^{\pi} e^{\mu\cos\theta}\cos m\theta d\theta = \pi I_m(\mu).$$

The final solution of this problem is given by following formulae

$$\begin{aligned} \frac{\mu}{Q}u &= ch(\mu\rho\cos\theta)\left[\frac{I_0'(\mu)}{2I_0(\mu)} I_0(\mu\rho) + \sum_{k=0}^{\infty} \frac{I_{2k}'(\mu)}{I_{2k}(\mu)} I_{2k}\cos 2k\theta\right] \\ &- sh(\mu\rho\cos\theta)\sum_{k=0}^{\infty}\frac{I_{2k+1}'(\mu)}{I_{2k+1}(\mu)} I_{2k+1}\cos(2k+1)\theta, \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{2\mu^2}{Q}h &= -\frac{\rho}{2}\cos\theta + ch(\mu\rho\cos\theta)\sum_{k=0}^{\infty}\frac{I_{2k+1}'(\mu)}{I_{2k+1}(\mu)} I_{2k+1}\cos(2k+1)\theta \\ &- sh(\mu\rho\cos\theta)\left[\frac{I_0'(\mu)}{2I_0(\mu)} I_0(\mu\rho) + \sum_{k=0}^{\infty} \frac{I_{2k}'(\mu)}{I_{2k}(\mu)} I_{2k}\cos 2k\theta\right]. \end{aligned} \tag{9}$$

Similarly would be stated the exact solution of according problem for circular cross-section pipe for case of external flow past of cylinder and so on.

Lets state, in particular, formula for distribution of velocities in the flow, surrounding non- conducting cylinder, moving with constant velocity v_0.

$$\begin{aligned} u &= ch(\mu\rho\cos\theta)\left[\frac{K_0(\mu\rho)}{2K_0(\mu)} I_0(\mu) + 2\sum_{k=0}^{\infty} \frac{K_{2k}(\mu\rho)}{K_{2k}(\mu)} K_{2k}\cos 2k\theta\right] \\ &- 2sh(\mu\rho\cos\theta)\sum_{k=0}^{\infty}\frac{K_{2k+1}(\mu\rho)}{K_{2k+1}(\mu)} I_{2k+1}\cos(2k+1)\theta, \end{aligned} \tag{10}$$

where $K_n(x)$ is the function of MacDonalds.

3 Conclusion

Thus, the obtained solution (8, 9, 10) in contrary of results in the works [1-3, 9, 11, 12], is convenience for calculations at large values of parameter $\mu = \frac{M}{2}$. It should be noticed that in routine hydrodynamics such problem has only trivial solution $v \equiv v_0$, while in the magnetic hydrodynamics the velocity vanish at $r \to \infty$.

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