BOUNDARY-CONTACT PROBLEMS OF STATICS OF LINEAR THEORY ELASTICITY MIXTURES

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Abstract

The paper deals with boundary contact problems of the linear theory of mixture statics of two isotropic elastic materials. Research is carried out by the method of potential and singular integral equations.

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The paper deals with boundary contact problems of the linear theory of mixture statics of two isotropic elastic materials. Research is carried out by the method of potential and singular integral equations.[1], [2] Boundary value problems of statics and dynamics for a mixture are investigated with the similar method in [3].

Let $\mathbb{R}^3$ be a three-dimensional Euclidean space and let the inhomogeneous medium area a mixture of two isotropic elastic material occupies all of the space. Imagine that $\mathbb{R}^3$ consist of two homogenous parts of D_1 - final domain and $D_2 = \mathbb{R}^3 \backslash \overline{D}_1$ and D_1 and D_2 both are separated by the surface S, on which specific conditions are given.

Consider the so called main contact problem: Find in the areas D_1 and D_2 regular vectors $\overset{(1)}{U}$ and $\overset{(2)}{U}$ satisfying the equations

$$\overset{(l)}{A}(D_x)\overset{(l)}{U}(x) = 0, \quad x \in D_l, \quad l = 1, 2. \tag{1}$$

and the contact conditions

$$\begin{cases} [\overset{(1)}{U}(z)]^+ - [\overset{(2)}{U}(z)]^- = f(z), \\ z \in S \\ [\overset{(1)}{\mathcal{P}}(D_z, n)\overset{(1)}{U}(z)]^+ - [\overset{(2)}{\mathcal{P}}(D_z, n)\overset{(2)}{U}(z)]^- = F(z), \end{cases} \tag{2}$$

where

$$\overset{(l)}{U} = (\overset{(l)}{u'},\ \overset{(l)}{u''}), \qquad l = 1, 2,$$
$$\overset{(2)}{U}_j(x) = O(|x|^{-1}), \qquad \frac{\partial \overset{(2)}{U}_j(x)}{\partial x_i} = o(|x|^{-1}), \tag{3}$$

f and F vectors set on S,

$$A(D_x) = \begin{Vmatrix} A^{(1)}(D_x) & A^{(2)}(D_x) \\ A^{(3)}(D_x) & A^{(4)}(D_x) \end{Vmatrix}_{6\times 6}, A^{(i)}(D_x) = \left\| A^{(i)}_{kp}(D_x) \right\|_{3\times 3},$$
$$A^{(1)}_{kp}(\xi) = a_1|\xi|^2\delta_{kp} + b_1\xi_k\xi_p, \quad A^{(2)}_{kp}(\xi) = A^{(3)}_{kp}(\xi) = c|\xi|^2\delta_{kp} + d\xi_k\xi_p, \tag{4}$$
$$A^{(4)}_{kp}(\xi) = a_2|\xi|^2\delta_{kp} + b_2\xi_k\xi_p, \quad \xi = (\xi_1,\ \xi_2,\ \xi_3), \quad i = 1, 2, 3.$$

The operator $\mathcal{P}$ is called a generalized operator of stress and has the form [3]

$$\mathcal{P} = \begin{Vmatrix} \overset{(1)}{\mathcal{P}} & \overset{(2)}{\mathcal{P}} \\ \overset{(3)}{\mathcal{P}} & \overset{(4)}{\mathcal{P}} \end{Vmatrix}_{6\times 6}, \qquad \overset{(k)}{\mathcal{P}} = \left\| \overset{(k)}{\mathcal{P}}_{ij} \right\|, \quad k = \overline{1, 4},$$
$$\overset{(1)}{\mathcal{P}}_{ij}(D, n) = (\mu_1 - \lambda_5)\delta_{ij}\frac{\partial}{\partial n} + (\mu_1 + \lambda_5)n_j D_i + \left(\lambda_1 - \frac{\alpha_2\rho_2}{\rho}\right) n_i D_j,$$
$$\overset{(2)}{\mathcal{P}}_{ij}(D, n) = (\mu_3 + \lambda_5)\delta_{ij}\frac{\partial}{\partial n} + (\mu_3 - \lambda_5)n_j D_i + \left(\lambda_3 - \frac{\alpha_2\rho_1}{\rho_1}\right) n_i D_j, \tag{5}$$
$$\overset{(3)}{\mathcal{P}}_{ij}(D, n) = (\mu_3 + \lambda_5)\delta_{ij}\frac{\partial}{\partial n} + (\mu_3 - \lambda_5)n_j D_i + \left(\lambda_4 + \frac{\alpha_2\rho_2}{\rho}\right) n_i D_j,$$
$$\overset{(4)}{\mathcal{P}}_{ij}(D, n) = (\mu_2 - \lambda_5)\delta_{ij}\frac{\partial}{\partial n} + (\mu_2 + \lambda_5)n_j D_i + \left(\lambda_2 + \frac{\alpha_2\rho_1}{\rho}\right) n_i D_j,$$

Note that a $\overset{(2)}{\mathcal{P}}_{ij} = \overset{(3)}{\mathcal{P}}_{ij}$, $\mathcal{P} \neq \mathcal{P}'$, $\mathcal{P}(\xi, \xi) = A(\xi)$.

Theorem 1. *The homogeneous problem corresponding to Problem (1), (2) has only the trivial solution.*

Green formulas for D_1 and D_2 have the form

$$\int\limits_{D_1} \overset{(1)}{W}(\overset{(1)}{U}, \overset{(1)}{U})dx = \int\limits_{S} [\overset{(1)}{U}]^+ [\overset{(1)}{\mathcal{P}}\overset{(1)}{U}]^+ \, ds, \tag{6}$$

$$\int_{D_2} \overset{(2)}{W}(\overset{(2)}{U}, \overset{(2)}{U})dx = -\int_S [\overset{(2)}{U}]^- [\overset{(2)}{\mathcal{P}}\overset{(2)}{U}]^- \, ds, \tag{7}$$

where [3]

$$\begin{aligned} W(U,U) &= \varepsilon'_{rr}\varepsilon'_{ii}\left(\lambda_1 - \frac{\alpha_2\rho_2}{\rho}\right) + \varepsilon''_{rr}\varepsilon''_{ii}\left(\lambda_2 + \frac{\alpha_2\rho_1}{\rho}\right) \\ &+ 2\mu_1\varepsilon'_{ij}\varepsilon'_{ij} + 2\mu_2\varepsilon''_{ij}\varepsilon''_{ij} + 4\mu_3\varepsilon'_{ij}\varepsilon''_{ij} + 2\left(\lambda_3 - \frac{\rho_1\alpha_2}{\rho}\right)\varepsilon'_{ii}\varepsilon''_{rr} \\ &- 2\lambda_5 h_{ij}h_{ij}. \end{aligned} \tag{8}$$

Here ε'_{ij} and ε''_{ij} are two strain tensors, h_{ij} are rotating components. From (6) and(7) we have

$$\begin{aligned} &\int_{D_1} \overset{(1)}{W}(\overset{(1)}{U}, \overset{(1)}{U})dx + \int_{D_2} \overset{(2)}{W}(\overset{(2)}{U}, \overset{(2)}{U})dx \\ &= \int_D \left\{ [\overset{(1)}{U}]^+ [\overset{(1)}{\mathcal{P}}\overset{(1)}{U}]^+ - [\overset{(2)}{U}]^- [\overset{(2)}{\mathcal{P}}\overset{(2)}{U}]^- \right\} ds \\ = \int_S &\left\{ \left[[\overset{(1)}{U}]^+ - [\overset{(2)}{U}]^- \right] [\overset{(1)}{\mathcal{P}}\overset{(1)}{U}]^+ + \left[[\overset{(1)}{\mathcal{P}}][\overset{(1)}{U}]^+ - [\overset{(2)}{\mathcal{P}}\overset{(2)}{U}]^- \right] [\overset{(2)}{U}]^- \right\} ds. \end{aligned}$$

By virtue of the homogeneity of the contact conditions of the last equation we get

$$\overset{(1)}{W}(\overset{(1)}{U}, \overset{(1)}{U}) = 0, \quad x \in D_1, \quad \overset{(2)}{W}(\overset{(2)}{U}, \overset{(2)}{U}) = 0, \quad x \in D_2.$$

The general solution of these equations has the form

$$\overset{(1)}{U} = (\overset{(l)}{u'}, \overset{(l)}{u''}), \quad \overset{(l)}{u'} = [a'_l \cdot x] + b'_l, \quad \overset{(l)}{u''} = [a'_l \cdot x] + b''_l,$$

where a'_1, a'_2, b'_l, b''_l are arbitrary three component vectors (arbitrary unrestricted). Considering the conditions (3) we obtain

$$\overset{(1)}{U} = 0, \quad x \in D_l \quad (l = 1, 2),$$

what was required to prove.

The solution to the problem (1), (2) will be sought in the form of a simple layer potential:

$$\overset{(1)}{U}(x) = \int\limits_S \overset{(1)}{\Psi}(x-y)\overset{(1)}{\mathrm{g}}(y)d_yS, \quad x \in D_1, \tag{9}$$

$$\overset{(2)}{U}(x) = \int\limits_S \overset{(2)}{\Psi}(x-y)\overset{(2)}{\mathrm{g}}(y)d_yS, \quad x \in D_2, \tag{10}$$

where $\overset{(1)}{\mathrm{g}}$, $\overset{(2)}{\mathrm{g}}$ - are the unknowns of six-component vectors class $C^{o,\gamma'}(S), \gamma' > 0$.

From the condition (2) we obtain

$$\int\limits_S \overset{(1)}{\Psi}(z-y)\overset{(1)}{\mathrm{g}}(y)d_yS - \int\limits_S \overset{(2)}{\Psi}(z-y)\overset{(2)}{\mathrm{g}}(y)d_yS = f(z), \tag{11}$$

$$\begin{aligned} &\overset{(1)}{\mathrm{g}}(z) + \int\limits_S \left[\overset{(1)}{\mathcal{P}}(D_z,n)\overset{(1)}{\Psi}(z-y)\right]\overset{(1)}{\mathrm{g}}(y)d_yS + \overset{(2)}{\mathrm{g}}(z) \\ &- \int\limits_S \left[\overset{(2)}{\mathcal{P}}(D_z,n)\overset{(2)}{\Psi}(z-y)\right]\overset{(2)}{\mathrm{g}}(y)d_yS = F(z), \end{aligned} \tag{12}$$

Let's rewrite (11), (12) in the form

$$\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}} - \overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}} = f, \tag{13}$$

$$(\mathcal{J} + \overset{(1)}{\mathcal{K}})\overset{(1)}{\mathrm{g}} - (-\mathcal{J} + \overset{(2)}{\mathcal{K}})\overset{(2)}{\mathrm{g}} = F, \tag{14}$$

where

$$\mathcal{H}\mathrm{g}(z) = \int\limits_S \Psi(z-y)\mathrm{g}(y)d_yS, \quad z \in S, \tag{15}$$

$$\mathcal{K}\mathrm{g}(z) = \int\limits_S \left[\mathcal{P}(D_z,n)\Psi(z-y)\right]\mathrm{g}(y)d_yS, \quad z \in S, \tag{16}$$

Let's set notation [3]

$$\mathcal{K}^* \mathrm{g}(z) = \int\limits_S [\mathcal{P}(D_z, n)\Psi(y-z)]^* \, \mathrm{g}(y) d_y S, \quad z \in S, \tag{17}$$

$$\mathcal{L}^{\pm}\mathrm{g}(z) = \lim_{D^{\pm} \ni x \to z \in S} [\mathcal{P}(D_x, n)W(x; \mathrm{g})] = [\mathcal{P}(D_z, n)W(z; \mathrm{g})]^{\pm}, \tag{18}$$

where $\mathcal{H}$ is the self-conjugate completely continuous operator, and $\mathcal{K}$ and $\mathcal{K}^*$ are mutually conjugate singular integral operators, a $\mathcal{J} \pm \mathcal{K}$ and $\mathcal{J} \pm \mathcal{K}^*$ are normal type operators with an index of zero.

The equations $(-\mathcal{J} + \mathcal{K})\varphi = 0$ and $(-\mathcal{J} + \mathcal{K}^*)\Psi = 0$ have only trivial solutions, the equation $(\mathcal{J} + \mathcal{K})\varphi = 0$ and $(\mathcal{J} + \mathcal{K}^*)\Psi = 0$ have six linearly independent solutions each. The complete system solution of equation $(\mathcal{J} + \mathcal{K}^*)\Psi = 0$ is written clearly

$$\begin{aligned}
\overset{(1)}{\Psi}(z) &= (1,0,0,0,0,0), & \overset{(4)}{\Psi}(z) &= (0,-z_1,z_2,1,0,0),\\
\overset{(2)}{\Psi}(z) &= (0,1,0,0,0,0), & \overset{(5)}{\Psi}(z) &= (z_3,0,-z_1,0,1,0),\\
\overset{(3)}{\Psi}(z) &= (0,0,1,0,0,0), & \overset{(6)}{\Psi}(z) &= (-z_2,z_1,0,0,0,1).
\end{aligned} \tag{19}$$

Let us apply the operator $\overset{(1)}{\mathcal{L}}$ to equation (13), then the system (13), (14) can be rewritten as

$$\overset{(1)}{\mathcal{L}}\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}} - \overset{(1)}{\mathcal{L}}\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}} = \overset{(1)}{\mathcal{L}} f \tag{20}$$

$$(\mathcal{J} + \overset{(1)}{\mathcal{K}})\overset{(1)}{\mathrm{g}} - (-\mathcal{J} + \overset{(2)}{\mathcal{K}})\overset{(2)}{\mathrm{g}} = F, \tag{21}$$

If we consider that [3] $\mathcal{L}\mathcal{H}\varphi = -\varphi + \mathcal{K}^2\varphi$, we will get:

$$(-\mathcal{J} + \overset{(1)}{\mathcal{K}^2})\overset{(1)}{\mathrm{g}} - \overset{(1)}{\mathcal{L}}\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}} = \overset{(1)}{\mathcal{L}} f, \tag{22}$$

$$(\mathcal{J} + \overset{(1)}{\mathcal{K}})\overset{(1)}{\mathrm{g}} - (-\mathcal{J} + \overset{(2)}{\mathcal{K}})\overset{(2)}{\mathrm{g}} = F, \tag{23}$$

System (22), (23) is a system of singular integral equations and it is obvious that if the system (13), (14) is solvable in the class of $C^{0,\gamma'}(S)$ then in the same class is solvable system (22), (23).

Lemma 1. *If system(13),(14) is solvable in the class $C^{0,\gamma'}(S)$ then it is uniquely solvable.*

Indeed, suppose that the homogeneous system corresponding to the system (13), (14) has a nontrivial solution $g_0 = (\overset{(1)}{g}_0, \overset{(2)}{g}_0)$, then by the uniqueness of the solution of the problem, we find that $\overset{(1)}{U}_0(x)$ and $\overset{(2)}{U}_0(x)$ certain to (9) and (10) and are identically equal to zero in D_1 and D_2, but then $\overset{(1)}{U}_0(x)$ and $\overset{(2)}{U}_0(x)$ will be equal identically in $\mathcal{R}^3$, which implies that $\overset{(1)}{g}_0 = \overset{(2)}{g}_0 = 0$, which proves the Lemma.

Theorem 2. *If system (20), (21) is solvable, then $\overset{(2)}{g}$ is uniquely determined, and $\overset{(1)}{g}$ - with precisely composed form $\sum\limits_{l=1}^{6} a_l \overset{(l)}{\varphi}$, $\{\overset{(l)}{\varphi}\}_{l=1}^{6}$ is a complete system of linearly independent solutions of $(\mathcal{J} + \overset{(1)}{\mathcal{K}})\varphi = 0$.*

Let's show that the general solution of the homogeneous system corresponding to system (20),(21), takes the form

$$\overset{(1)}{g} = \sum_{l=1}^{6} a_l \overset{(l)}{\varphi}, \quad \overset{(2)}{g} = 0.$$

Indeed from relations

$$\overset{(1)}{\mathcal{L}}[\overset{(1)}{\mathcal{H}}\overset{(1)}{g} - \overset{(2)}{\mathcal{H}}\overset{(2)}{g}] = 0, \quad (\mathcal{J} + \overset{(1)}{\mathcal{K}})\overset{(1)}{g} - (-\mathcal{J} + \overset{(2)}{\mathcal{K}})\overset{(2)}{g} = 0, \tag{24}$$

By Lemma 1, we should get

$$\begin{cases} \overset{(1)}{\mathcal{H}}\overset{(1)}{g} - \overset{(2)}{\mathcal{H}}\overset{(2)}{g} = \sum\limits_{j=1}^{6} b_l \overset{(j)}{\varphi}, \\ (\mathcal{J} + \overset{(1)}{\mathcal{K}})\overset{(1)}{g} - \left(-\mathcal{J} + \overset{(2)}{\mathcal{K}}\right)\overset{(2)}{g} = 0, \end{cases} \tag{25}$$

Here b_j are arbitrary constants, $\overset{(j)}{\Psi}$ $(j = \overline{1,6})$ are vectors a determined by formulas (19), they constitute a complete system of linearly independent solutions of $(\mathcal{J} + \overset{(1)}{\mathcal{K}^*})\Psi = 0$.

We have $(\mathcal{J} + \overset{(1)}{\mathcal{K}^*})\overset{(1)}{\mathcal{H}}\overset{(j)}{\varphi}_1 = 0$, $\quad j = \overline{1,6}$, $\{\overset{(j)}{\varphi}\}_{j=1}^{6}$ a complete system of linearly independent solutions of the equation $(\mathcal{J} + \overset{(1)}{\mathcal{K}})\varphi = 0$, And system $\{\overset{(j)}{\psi}\}_{j=1}^{6}$, and system $\{\overset{(1)}{\mathcal{H}}\overset{(j)}{\varphi}\}_{j=1}^{6}$ - are linearly independent, so each of these vector from this systems is expressed by a linear combination of the vectors of the second system, i.e.

$$\overset{(j)}{\psi}(z) = \sum_{j=1}^{6} c_{jl} \overset{(1)}{\mathcal{H}} \overset{(l)}{\varphi}_1(z).$$

Then from (25) we have

$$\begin{cases} \overset{(1)}{\mathcal{H}} \left[\overset{(1)}{\mathrm{g}} - \sum\limits_{j,l=1}^{6} c_{jl} b_j \overset{(l)}{\varphi}_1 \right] - \overset{(2)}{\mathcal{H}} \overset{(2)}{\mathrm{g}} = 0, \\ (\mathcal{J} + \overset{(1)}{\mathcal{K}}) \left[\overset{(1)}{\mathrm{g}} - \sum\limits_{j,l=1}^{6} c_{jl} b_j \overset{(l)}{\varphi}_1 \right] - (-\mathcal{J} + \overset{(2)}{\mathcal{K}}) \overset{(2)}{\mathrm{g}} = 0. \end{cases} \tag{26}$$

Lemma 1 implies that (26) has only the trivial solution, i.e. $\overset{(1)}{\mathrm{g}} = \sum\limits_{l=1}^{6} a_l \overset{(l)}{\varphi}_1$, $\overset{(2)}{\mathrm{g}} = 0$, where $a_l = \sum\limits_{l=1}^{6} c_{jl} b_j$ are arbitrary constants, i.e. we have found that if $\mathrm{g}_0 = (\overset{(1)}{\mathrm{g}}_0, \overset{(2)}{\mathrm{g}}_0)$ is a solution of system (25), then for arbitrary a_1 the solution will be a vector

$$\mathrm{g} = \left(\overset{(1)}{\mathrm{g}}_0 + \sum_{j=1}^{6} a_l \overset{(1)}{\varphi}_1, \ \overset{(2)}{\mathrm{g}}_0 \right). \tag{27}$$

Q.E.D.

Theorem 3. *If (20), (21) be solvable, then its solution when choosing a_1 will satisfy the system (13), (14).*

Indeed, let (20), (21) is solvable and its solution is $\mathrm{g}_0 = (\overset{(1)}{\mathrm{g}}_0, \overset{(2)}{\mathrm{g}}_0)$, then

$$\left(-\mathcal{J} + \overset{(1)}{\mathcal{K}^2} \right) \overset{(1)}{\mathrm{g}}_0 - \overset{(1)}{\mathcal{L}} \overset{(2)}{\mathcal{H}} \overset{(2)}{\mathrm{g}}_0 \equiv \overset{(1)}{\mathcal{L}} \overset{(1)}{\mathcal{H}} \overset{(1)}{\mathrm{g}}_0 - \overset{(1)}{\mathcal{L}} \overset{(2)}{\mathcal{H}} \overset{(2)}{\mathrm{g}}_0 = \overset{(1)}{\mathcal{L}} f, \tag{28}$$

from

$$\overset{(1)}{\mathcal{L}} \left[\overset{(1)}{\mathcal{H}} \overset{(1)}{\mathrm{g}}_0 - \overset{(2)}{\mathcal{H}} \overset{(2)}{\mathrm{g}}_0 - f \right] = 0. \tag{29}$$

From here, considering that [3]$-\psi(z) + (\mathcal{K}^*)^2 \psi(z) = \mathcal{H}\mathcal{L}\psi(z)$, from (29) we obtain

$$\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathcal{L}}\left[\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f\right]$$
$$=-\left(\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f\right)+(\mathcal{K}^*)^2\left(\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f\right)=0.$$

by the virtue of the operators $\mathcal{H}$ and $\mathcal{L}^{\pm}$, is equivalent to equation (29), which means that $\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f$ is a solution of equation $(-\mathcal{J}+(\mathcal{K}^*)^2)h=0$ and therefore it is expressed by a linear combination of vectors

$$\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f=\sum_{j=1}^{6}b_j\overset{(j)}{\psi}, \tag{30}$$

where b_j are permanents, uniquely determined by $\mathrm{g}_0=(\overset{(1)}{\mathrm{g}}_0,\overset{(2)}{\mathrm{g}}_0)$.

As $(\mathcal{J}+\overset{(1)}{\mathcal{K}})\overset{(l)}{\varphi}_1=0$, $\;l=\overline{1,6}$, $\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1+\overset{(l)}{\mathcal{H}}\overset{(1)}{\mathcal{K}}\overset{(l)}{\varphi}_1=0$, where considering that $\mathcal{H}\mathcal{K}\varphi(z)=\mathcal{K}^*\mathcal{H}\varphi(z)$, we obtain $\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1+\overset{(1)}{\mathcal{K}^*}\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1=0$; Therefore, $(\mathcal{J}+\overset{(1)}{\mathcal{K}^*})\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1=0$ and $\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1$ $(l=\overline{1,6})$, is a solution of equation $(\mathcal{J}+\overset{(1)}{\mathcal{K}^*})h=0$, however $\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1$ can be expressed as follows

$$\overset{(1)}{\mathcal{H}}\overset{(l)}{\varphi}_1=\sum_{j=1}^{6}a_{jl}\overset{(l)}{\psi},\quad l=\overline{1,6},\;\; a_{jl}=const \tag{31}$$

where $\det\parallel a_{lj}\parallel\neq 0$.

Let us assume that $\mathrm{g}=(\overset{(1)}{\mathrm{g}},\overset{(2)}{\mathrm{g}})$ is the solution of system (22), (23), defined by the formula (27), where a_1 are arbitrary constants. Taking into account (30), (31), let's consider the expression

$$\begin{aligned}\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}-f&=\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0+\sum_{l=1}^{6}a_l\overset{(1)}{\mathcal{H}}\overset{(1)}{\varphi}_1-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f\\&=\overset{(1)}{\mathcal{H}}\overset{(1)}{\mathrm{g}}_0-\overset{(2)}{\mathcal{H}}\overset{(2)}{\mathrm{g}}_0-f+\sum_{l=1}^{6}\sum_{j=1}^{6}a_{jl}a_l\psi^{(j)}\\&=\sum_{l=1}^{6}b_j\overset{(j)}{\psi}+\sum_{j=1}^{6}\left(\sum_{l=1}^{6}a_{jl}a_l\right)\overset{(j)}{\psi}=\sum_{j=1}^{6}\left(\sum_{l=1}^{6}a_{jl}a_l+b_j\right)\overset{(j)}{\psi},\end{aligned}$$

Since det $\| a_{jl} \| \neq 0$.

then a_l can be chosen so, that

$$\sum_{l=1}^{6} a_{jl}a_l + b_j = 0, \quad j = \overline{1,6}. \tag{32}$$

The solution of this system is denoted by $\overset{(0)}{a}_l$, then

$$\overset{(l)}{\mathrm{g}}(z) = \overset{(l)}{\mathrm{g}}_0(z) + \sum_{l=1}^{6} \overset{(0)}{a}_l \overset{(l)}{\varphi}_1(z), \quad \overset{(2)}{\mathrm{g}}(z) = \overset{(2)}{\mathrm{g}}_0(z) \tag{33}$$

will be a solution of system (13), (14). Theorem 3 is proved.

We prove that the determinant of the symbolic matrix of the system (20) (21) ((22) (23)) is different from zero, and the system is normally a solvable system of singular integral equations and for it Noether's theorem is valid.

Finally, we have the following. If $s \in \mathcal{L}_2(\gamma)$, $f \in C^{1,\gamma'}(s)$, $F \in C^{0,\gamma'}(s)$, $0 < \gamma' < \gamma < 1$, problem (1), (2) is uniquely solvable in the class of regular vectors and the solution is represented in the form of (9), (10), where the vector $\mathrm{g} = (\overset{(1)}{\mathrm{g}}, \overset{(2)}{\mathrm{g}}) \in C^{0,\gamma'}(s)$ is the solution of uniquely solvable system (11), (12).

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