THE BOUNDARY VALUE PROBLEM OF PLATES WITH DOUBLE POROSITY BY THE VEKUA METHOD FOR APPROXIMATIONS N=1

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Abstract

The purpose of this paper is to consider the linear theory of elasticity for solids with double porosity. From this system of the equations, using a method of a reduction of I. Vekua, we receive the equilibrium equations. Using the analytic functions of a complex variable and solutions of the Helmholtz equation. The Dirichlet boundary value problem are solved explicitly for approximation $N = 1$.

Key words and phrases: Double porosity, the Dirichlet boundary value problem.

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1 Introduction

In the last decade there has been interest in investigation of problems of the theories of elasticity and thermoelasticity for solids with double porosity based on the Darcys law. The first theory of consolidation for elastic materials with double porosity was presented in [1-3]. The Aifantis theory unifies the earlier proposed models of Barenblatt et al. [4] for porous media with double porosity and Biot [5] for porous media with single porosity. The fundamental solutions in the theories of elasticity and thermoelasticity for materials with double porosity are constructed by Scarpetta et al. [6], Svanadze [7], Svanadze and De Cicco [8].

2 Basic Equations

Let an elastic body with double porosity occupy the domain $\overline{\Omega} \subset R^3$. Denote by (x^1, x^2, x^3) a point of the domain $\overline{\Omega}$ in the arbitrary curvilinear system of coordinates. Let the domain $\overline{\Omega}$ be filled with an elastic isotropic homogenous medium having double porosity. The considered solid body is characterized by the displacement vector $\mathbf{u} = (u^1, u^2, u^3)$, and also by the fluid pressures $p_1(x^1, x^2, x^3)$ and $p_2(x^1, x^2, x^3)$ occurring respectively in the pores and fissures of the porous medium.

Then a homogeneous system of static equilibrium equations is written in the form [7]

$$\partial_i \sigma_{ij} = 0, \tag{1}$$

where $\partial_i \equiv \frac{\partial}{\partial x_i}$, σ_{ij} are components of stress tensor, the summation over the recurring index i is assumed to be made from 1 to 3.

Formulas that interrelate the stress components, the displacement vector components and the pressures p_1, p_2 have the form [7]

$$\sigma_{ij} = (\lambda \partial_k u_k - \beta_1 p_1 - \beta_2 p_2)\delta_{ij} + \mu(\partial_i u_j + \partial_j u_i). \tag{2}$$

where λ and μ are the Lamé parameters; β_1 and β_2 are the effective stress parameters; δ_{ij} is the Kronecker delta.

In the stationary case, the values p_1 and p_2 satisfy the following system of equations

$$\begin{cases} (k_1\Delta_3 - \gamma)p_1 + (k_{12}\Delta_3 + \gamma)p_2 = 0, \\ (k_{21}\Delta_3 + \gamma)p_1 + (k_2\Delta_3 - \gamma)p_2 = 0 \end{cases} \quad \text{in} \quad \Omega, \tag{3}$$

where $k_1 = \frac{\kappa_1}{\mu'}, k_2 = \frac{\kappa_2}{\mu'}, k_{12} = \frac{\kappa_{12}}{\mu'}, k_{21} = \frac{\kappa_{21}}{\mu'}$; μ' is fluid viscosity; κ_1 and κ_2 are the macroscopic intrinsic permeabilities associated with matrix and fissure porosity; k_{12} and k_{21} are the cross-coupling permeabilities for fluid flow at the interface between the matrix and fissure phases; $\gamma > 0$ is the internal transport coefficient and corresponds to fluid transfer rate with respect to the intensity of flow between the pore and fissures; Δ_3 is the three-dimensional Laplace operator.

It is easy to show that if $\gamma > 0$, $k_1k_2 - k_{12}k_{21} > 0$, then the system of equations (3) is equivalent to two independent equations: to the Laplace equation

$$\Delta\widetilde{p}_1 = 0 \quad in \quad \Omega \tag{4}$$

and to the Helmholtz equation

$$\Delta\widetilde{p}_2 - \zeta^2\widetilde{p}_2 = 0 \quad in \quad \Omega, \tag{5}$$

where

$$\widetilde{p}_1 := (k_1 + k_{21})p_1 + (k_2 + k_{12})p_2, \quad \widetilde{p}_2 := p_1 - p_2,$$

$$\zeta^2 := \frac{\gamma(k_1+k_2+k_{12}+k_{21})}{k_1k_2-k_{12}k_{21}} > 0.$$

3 Approximation $N=1$

In [9] we apply I. Vekua's method for a reduction of the equations (1-5) [10].

Consider approximation of the order $N=1$.

We introduce the complex variable $z = x_1 + ix_2$ ($i^2=-1$) and the operators $\partial_z = 0.5(\partial_1 - i\partial_2), \partial_{\bar z} = 0.5(\partial_1 + i\partial_2)$ ($\bar z = x_1 - ix_2$). The two-dimensional Laplace operator is expressed as $\Delta = 4\partial_z\partial_{\bar z}$.

The homogenous system of equation of the elastic plate may be written in the following complex form [10]:

$$\begin{cases} \mu\Delta \overset{(0)}{u}_+ + 2(\lambda+\mu)\partial_{\bar z}\overset{(0)}{\vartheta} - \dfrac{2\lambda}{h}\partial_{\bar z}\overset{(1)}{u}_3 - 2\partial_{\bar z}\left(\beta_1^*\overset{(0)}{\widetilde{p}}_1 + \beta_2^*\overset{(0)}{\widetilde{p}}_2\right) = 0, \\ \mu\Delta\overset{(1)}{u}_3 - \dfrac{3\lambda}{h}\overset{(0)}{\vartheta} - \dfrac{3(\lambda+2\mu)}{h^2}\overset{(1)}{u}_3 + \dfrac{3}{h}\left(\beta_1^*\overset{(0)}{\widetilde{p}}_1 + \beta_2^*\overset{(0)}{\widetilde{p}}_2\right) = 0, \end{cases} \tag{6}$$

$$\begin{cases} \mu\Delta\overset{(1)}{u}_+ + 2(\lambda+\mu)\partial_{\bar z}\overset{(1)}{\vartheta} - \dfrac{6\mu}{h}\partial_{\bar z}\overset{(0)}{u}_3 - \dfrac{3\mu}{h^2}\overset{(1)}{u}_+ \\ -2\partial\bar z\left(\beta_1^*\overset{(1)}{\widetilde{p}}_1 + \beta_2^*\overset{(1)}{\widetilde{p}}_2\right) = 0, \\ \mu\Delta\overset{(0)}{u}_3 + \dfrac{\mu}{h}\overset{(1)}{\vartheta} = 0. \end{cases} \tag{7}$$

$$\Delta\overset{(0)}{\widetilde{p}}_1 - \frac{3}{h^2}\overset{(0)}{\widetilde{p}}_1 = 0, \quad \Delta\overset{(0)}{\widetilde{p}}_2 - \left(\frac{3}{h^2}+\zeta^2\right)\overset{(0)}{\widetilde{p}}_2 = 0, \tag{8}$$

$$\Delta\overset{(1)}{\widetilde{p}}_1 - \frac{15}{h^2}\overset{(1)}{\widetilde{p}}_1 = 0, \quad \Delta\overset{(1)}{\widetilde{p}}_2 - \left(\frac{15}{h^2}+\zeta^2\right)\overset{(1)}{\widetilde{p}}_2 = 0, \tag{9}$$

where

$$\overset{(k)}{\widetilde{p}}_\alpha(x^1,x^2) = \left(k+\frac{1}{2}\right)\frac{1}{h}\int_{-h}^{h}\widetilde{p}_\alpha(x^1,x^2,x^3)P_k\left(\frac{x^3}{h}\right)dx^3,$$

$$\overset{(k)}{u}_i = \left(k+\frac{1}{2}\right)\frac{1}{h}\int_{-h}^{h}u_i(x^1,x^2,x^3)P_k\left(\frac{x^3}{h}\right)dx^3,$$

$$\overset{(k)}{u}_+ = \overset{(k)}{u}_1 + i\overset{(k)}{u}_2, \quad \overset{(k)}{\vartheta} = \partial_z\overset{(k)}{u}_+ + \partial_{\bar z}\overline{\overset{(k)}{u}}_+, \;\; k=0,1,$$

$$\beta_1^* = \frac{\beta_1+\beta_2}{k_0},\ \ \beta_2^* = \frac{\beta_1(k_2+k_{12})-\beta_2(k_1+k_{21})}{k_0},$$
$$k_0 = k_1+k_2+k_{12}+k_{21},$$

$P_k\left(\frac{x^3}{h}\right)$ is the Legendre polynomials of order k.

The general solution of the equations (6-9) have the following form

$$\overset{(0)}{\widetilde{p}_1} = \overset{(0)}{\chi_1}(z,\bar z),\ \ \overset{(0)}{\widetilde{p}_2} = \overset{(0)}{\chi_2}(z,\bar z),\ \ \overset{(1)}{\widetilde{p}_1} = \overset{(1)}{\chi_1}(z,\bar z),\ \ \overset{(1)}{\widetilde{p}_1} = \overset{(1)}{\chi_1}(z,\bar z), \tag{10}$$

$$\begin{aligned} 2\mu\overset{(0)}{u_+} &= \varkappa^*\varphi(z) - z\overline{\varphi' z} - \overline{\psi(z)} - \frac{\lambda h}{6(\lambda+\mu)}\partial_{\bar z}\chi(z,\bar z) \\ &+\frac{4h^2}{3}a_0\partial_z\overset{(0)}{\chi_1}(z,\bar z) + \frac{4h^2}{3+\zeta^2h^2}b_0\partial_z\overset{(0)}{\chi_2}(z,\bar z), \end{aligned} \tag{11}$$

$$2\mu\overset{(1)}{u_3} = \chi(z,\bar z) - \frac{2\lambda h}{3\lambda+2\mu}(\varphi'(z)+\overline{\varphi'(z)}) + a_1\overset{(0)}{\chi_1}(z,\bar z) + a_2\overset{(0)}{\chi_2}(z,\bar z), \tag{12}$$

$$\begin{aligned} \overset{(1)}{u_+} &= i\partial_{\bar z}\tau + \frac{4h^2(\lambda+2\mu)}{3\mu}\overline{f''(z)} + z\overline{f'(z)} + f(z) - 2h\overline{\mathrm{g}'(z)} \\ &+\frac{1}{2(\lambda+2\mu)h}\left(\frac{4h^2}{15}\beta_1^*\partial_{\bar z}\overset{(1)}{\chi_1}(z,\bar z) + \frac{4h^2}{15+\zeta^2h^2}\beta_2^*\partial_{\bar z}\overset{(1)}{\chi_2}(z,\bar z)\right), \end{aligned} \tag{13}$$

$$\begin{aligned} \overset{(0)}{u_3} &= -\frac{1}{2h}(\bar z f(z) + z\overline{f(z)}) + \mathrm{g}(z) + \overline{\mathrm{g}(z)} \\ &-\frac{1}{4(\lambda+2\mu)}\left(\frac{4h^2}{15}\beta_1^*\overset{(1)}{\chi_1}(z,\bar z) + \frac{4h^2}{15+\zeta^2h^2}\beta_2^*\overset{(1)}{\chi_2}(z,\bar z)\right) \end{aligned} \tag{14}$$

where $\varkappa^* = \frac{5\lambda+6\mu}{3\lambda+2\mu}.$, $\varphi(z)$, $\psi(z)$, $f(z)$, $g(z)$ are an arbitrary analytic function of z, $a_\alpha = \frac{12\mu h}{9\lambda+6-\delta_{\alpha 2}\zeta^2(\lambda+2\mu)h^2}\beta_\alpha^*$, $(\alpha=1,2)$, $a_0 = \frac{4\mu h^2}{3(3\lambda+2\mu)}\beta_1^*$, $b_0 = \frac{4\mu h^2}{3+h^2\zeta^2}\frac{3-h^2\zeta^2}{9\lambda+6\mu-\zeta^2(\lambda+\mu)h^2}\beta_2^*$, $\chi(z,\bar z)$, $\tau(z,\bar z)$, $\overset{(0)}{\chi_1}(z,\bar z)$, $\overset{(0)}{\chi_2}(z,\bar z)$, $\overset{(1)}{\chi_1}(z,\bar z)$, $\overset{(1)}{\chi_2}(z,\bar z)$ are the general solutions of the following Helmholtz equations

$$\Delta\chi - \eta^2\chi = 0,\ \ \Delta\tau - \gamma^2\tau = 0,\ \ \Delta\overset{(0)}{\chi_1} - \eta_1^2\overset{(0)}{\chi_1} = 0,\ \ \Delta\overset{(1)}{\chi_1} - \eta_2^2\overset{(1)}{\chi_1} = 0,$$

$$\Delta\overset{(1)}{\chi_1} - \eta_3^2\overset{(1)}{\chi_1} = 0,\ \ \Delta\overset{(1)}{\chi_2} - \eta_4^2\overset{(1)}{\chi_2} = 0,\ \ \eta^2 = \frac{12(\lambda+\mu)}{(\lambda+2\mu)h^2},$$

$$\gamma^2 = \frac{3}{h^2},\ \ \eta_1^2 = \frac{3}{h^2},\ \ \eta_2^2 = \left(\frac{3}{h^2}+\zeta^2\right),\ \ \eta_3^2 = \frac{15}{h^2},\ \ \eta_4^2 = \left(\frac{15}{h^2}+\zeta^2\right).$$

The constructed general solution enables one to solve analytically a sufficiently wide class of boundary value problems of the elastic equilibrium of porous plates with double porosity.

Let's consider the following boundary value problems for system (6)-(9).

Find the solutions of the homogeneous system of equations (6)-(9) compitible with the kinematic boundary conditions:

$$\begin{array}{ll} \overset{(0)}{u}_+ |_{|z|=R} = F_+, & \overset{(1)}{u}_3 |_{|z|=R} = F_3 \\ \overset{(0)}{\widetilde{p}}_1 |_{|z|=R} = H_1, & \overset{(0)}{\widetilde{p}}_2 |_{|z|=R} = H_2, \end{array} \tag{15}$$

$$\begin{array}{ll} \overset{(1)}{u}_+ |_{|z|=R} = G_+, & \overset{(0)}{u}_3 |_{|z|=R} = G_3 \\ \overset{(1)}{\widetilde{p}}_1 |_{|z|=R} = H_3, & \overset{(1)}{\widetilde{p}}_2 |_{|z|=R} = H_4, \end{array} \tag{16}$$

where $F_+,\ F_3,\ G_+,\ G_3,\ H_i$ are the known functions.

Let us introduce the functions $\varphi(z),\ \psi(z),\ \chi(z,\bar{z}),\ F_+$ and F_3 by the series:

$$\varphi(z) = \sum_{n=1}^{\infty} a_n z^n,\ \ \psi(z) = \sum_{n=0}^{\infty} b_n z^n,\ \ F_+ = \sum_{-\infty}^{\infty} A'_n e^{in\theta},\ \ F_3 = \sum_{-\infty}^{\infty} B'_n e^{in\theta},$$
$$\chi = \sum_{-\infty}^{\infty} \alpha_n I_n(\eta r) e^{in\theta},\ \ \overset{(0)}{\chi}_j = \sum_{-\infty}^{\infty} \alpha_{jn} I_n(\eta_1 r) e^{in\theta},\ \ H_j = \sum_{-\infty}^{\infty} A_{jn} e^{in\theta},$$

where $I_n(\cdot r)$ are Bessel's modificed functions, $j = 1, 2$.

By substituting (10-12) into (15) we obtain the system of algebraic equations:

$$\begin{aligned}
&\frac{5\lambda + 6\mu}{3\lambda + 2\mu} R a_1 - R\bar{a}_1 - \frac{\lambda \eta h}{12(\lambda + \mu)} I_1(\eta R)\alpha_0 = A_1,\\
&I_0(\eta R)\alpha_0 - \frac{2\lambda h}{3\lambda + 2\mu}(a_1 + \bar{a}_1) = B_0,\\
&\frac{5\lambda + 6\mu}{3\lambda + 2\mu} R^n a_n - \frac{\lambda \eta h}{12(\lambda + \mu)} I_n(\eta R)\alpha_{n-1} = A_n \quad (n \geq 2),\\
&-(n+2)R^{n+2}\bar{a}_{n+2} - R^n \bar{b}_n - \frac{\lambda \eta h}{12(\lambda + \mu)} I_n(\eta R)\alpha_{-n-1} = A_{-n} \quad (n \geq 0),\\
&I_n(\eta R)\alpha_n - \frac{2\lambda h}{3\lambda + 2\mu}(n+1)R^n a_{n+1} = B_n \quad (n \geq 1).
\end{aligned}$$

where

$$A_n = A'_n - \frac{2h^2 \eta_1 a_2}{3} \alpha_{1n} I_n(\eta_1 R) - \frac{2h^2 \eta_2 b_0}{3 + \zeta^2 h^2} \alpha_{2n} I_n(\eta_2 R),$$

$$B_n = B'_n - a_1 \alpha_{1n} I_n(\eta_1 R) - a_2 \alpha_{2n} I_n(\eta_2 R).$$

For coefficients $a_n,\ b_n$ and α_n we have:

$$\alpha_{1n} = \frac{A_{1n}}{I_n(\eta_1 R)},\quad \alpha_{2n} = \frac{A_{2n}}{I_n(\eta_2 R)},\quad (n = 0, \pm 1, \pm 2, ...),$$

$$a_n = \frac{(3\lambda+2\mu)\left(A_n + \dfrac{\lambda\eta h I_n(\eta R)}{12(\lambda+\mu)I_{n-1}(\eta R)}B_{n-1}\right)}{R^{n-1}\left((5\lambda+6\mu)R - \dfrac{\lambda^2\eta h^2(n+1)I_n(\eta R)}{6(\lambda+\mu)I_{n-1}(\eta R)}\right)} \quad (n\geq 2),$$

$$\begin{aligned}
\alpha_n &= \frac{1}{I_n(\eta R)}\left(B_n + \frac{2\lambda\left(A_{n+1} + \dfrac{\lambda\eta h I_{n+1}(\eta R)}{12(\lambda+\mu)I_n(\eta R)}B_{n+1}\right)(n+1)}{(5\lambda+6\mu)R - \dfrac{\lambda^2 h^2\eta I_{n+1}(\eta R)}{6(\lambda+\mu)I_n(\eta R)}(n+1)}\right)\\
b_n &= -(n+2)\frac{(3\lambda+2\mu)\left(A_{n+2} + \dfrac{\lambda\eta h I_{n+2}(\eta R)}{12(\lambda+\mu)I_{n+1}(\eta R)}B_{n+1}\right)}{R^{n-1}\left((5\lambda+6\mu)R - \dfrac{\lambda^2 h^2\eta I_{n+2}(\eta R)}{6(\lambda+\mu)I_{n+1}(\eta R)}(n+2)\right)}\\
&- \frac{\lambda\eta h I_n(\eta R)}{12(\lambda+\mu)R^n I_{n+1}(\eta R)}\Bigg(B_{n+1}\\
&+ \frac{2\lambda\left(A_{n+2} + \dfrac{\lambda\eta h I_{n+2}(\eta R)}{12(\lambda+\mu)I_{n+1}(\eta R)}B_{n+1}\right)(n+2)}{(5\lambda+6\mu)R - \dfrac{\lambda^2 h^2\eta I_{n+2}(\eta R)}{6(\lambda+\mu)I_{n+1}(\eta R)}(n+2)}\Bigg) - \frac{\overline{A}_{-n}}{R^n} \quad (n\geq 1),\\
a_1 &= \frac{ReA_1 + \dfrac{\lambda\eta h I_1(\eta R)}{12(\lambda+\mu)I_0(\eta R)}B_0}{\dfrac{2(\lambda+2\mu)}{3\lambda+2\mu}R - \dfrac{\lambda^2 h^2\eta I_1(\eta R)}{3(\lambda+\mu)(3\lambda+2\mu)I_0(\eta R)}} + \frac{3\lambda+2\mu}{8(\lambda+\mu)}i\mathrm{Im}A_1,\\
\alpha_0 &= \frac{B_0}{I_0(\eta R)} + \frac{4\lambda\left(ReA_1 + \dfrac{\lambda\eta h I_1(\eta R)}{12(\lambda+\mu)I_0(\eta R)}B_0\right)}{2(\lambda+2\mu)I_0(\eta R)R - \dfrac{\lambda^2 h^2\eta I_1(\eta R)}{3(\lambda+\mu)}}.
\end{aligned}$$

Let

$$f(z) = \sum_{n=1}^{\infty} c_n z^n,\ \ g(z) = \sum_{n=0}^{\infty} d_n z^n,\ \ G_+ = \sum_{-\infty}^{\infty} M'_n e^{in\theta},\ \ G_3 = \sum_{-\infty}^{\infty} N'_n e^{in\theta},$$

$$\tau = \sum_{-\infty}^{\infty} \beta_n I_n(\gamma r)e^{in\theta},\ \ \overset{(1)}{\chi_j} = \sum_{-\infty}^{\infty} \beta_{jn} I_n(\eta_3 r)e^{in\theta},\ \ H_{j+2} = \sum_{-\infty}^{\infty} A_{j+2\,n}e^{in\theta}.$$

We now find the coefficients c_n, d_n and β_n from following system of algebraic equations:

$$\frac{i\gamma}{2}I_1(\gamma R)\beta_0 + R(c_1+\bar{c}_1) = M_1,\quad d_0 + \bar{d}_0 - \frac{R^2}{2h}(c_1+\bar{c}_1) = N_0,$$

$$\begin{aligned}
&\frac{i\gamma}{2}I_n(\gamma R)\beta_{n-1}+R^n c_n=M_n \quad (n\geq 2),\\
&\frac{i\gamma}{2}I_n(\gamma R)\beta_{-n-1}-2(n+1)R^{n+1}\bar{d}_{n+1}+(n+2)\bar{c}_{n+2}+\\
&+\frac{4h^2(\lambda+2\mu)}{3\mu}(n+1)(n+2)R^n\bar{c}_{n+2}=M_{-n} \quad (n\geq 0),\\
&R^n d_n-\frac{R^{n+2}}{2h}c_{n+1}=N_n \quad (n\geq 1),
\end{aligned} \tag{17}$$

where

$$M_n=M_n'-\frac{h}{\lambda+2\mu}\left(\frac{\eta_3\beta_1^*\beta_{1n}}{15}I_n(\eta_3 R)+\frac{\eta_4\beta_2^*\beta_{2n}}{15+\zeta^2h^2}I_n(\eta_4 R)\right),$$

$$N_n=N_n'+\frac{h^2}{\lambda+2\mu}\left(\frac{\beta_1^*\beta_{1n}}{15}I_n(\eta_3 R)+\frac{\beta_2^*\beta_{2n}}{15+\zeta^2h^2}I_n(\eta_4 R)\right).$$

The solutions of the system (17) have the following forms:

$$\beta_{1n}=\frac{A_{3n}}{I_n(\eta_3 R)},\quad \beta_{2n}=\frac{A_{4n}}{I_n(\eta_4 R)},\quad (n=0,\pm 1,\pm 2,...),$$

$$c_n=\frac{\bar{M}_{-n+2}+\dfrac{I_{n-2}(\gamma R)}{I_n(\gamma R)}M_n+\dfrac{2(n-1)}{R}N_{n-1}}{\left(\dfrac{I_{n-2}(\gamma R)}{I_n(\gamma R)}+1\right)R^n+\dfrac{4h^2(\lambda+2\mu)}{3\mu}(n-1)nR^{n-2}} \quad (n\geq 2),$$

$$d_n=\frac{N_n}{R^n}+\frac{R^2\left(\bar{M}_{-n+1}+\dfrac{I_{n-1}(\gamma R)}{I_{n+1}(\gamma R)}M_{n+1}+\dfrac{2n}{R}N_n\right)}{2\left(\dfrac{I_{n-1}(\gamma R)}{I_{n+1}(\gamma R)}+1\right)R^{n+1}+\dfrac{8h^2(\lambda+2\mu)}{3\mu}n(n+1)R^{n-1}} \quad (n\geq 1),$$

$$\beta_n=\frac{2}{i\gamma I_{n+1}(\gamma R)}\left(M_{n+1}-\frac{\left(\bar{M}_{-n+1}+\dfrac{I_{n-1}(\gamma R)}{I_{n+1}(\gamma R)}M_{n+1}+\dfrac{2n}{R}N_n\right)R^2}{\left(\dfrac{I_{n-1}(\gamma R)}{I_{n+1}(\gamma R)}-1\right)R^2+\dfrac{4h^2(\lambda+2\mu)}{3\mu}n(n+1)}\right),$$
$$(n\geq 1),$$

$$c_1+\bar{c}_1=\frac{\mathrm{Re}M_1}{R},\quad d_0+\bar{d}_0=\frac{\mathrm{Re}M_1}{2h}R+N_0,\quad \beta_0=\frac{2\mathrm{Im}M_1}{\gamma I_1(\gamma R)}.$$

It is easy to prove the absolute and uniform convergence of the series obtained in the circular ring (including the contours) when the functions set on the boundaries have sufficient smoothness.

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