ABOUT ONE BOUNDARY VALUE PROBLEM FOR THE NON-SHALLOW SPHERICAL SHELLS

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Abstract

In the present paper we consider the geometrically nonlinear and non-shallow spherical shells, when components of the deformation tensor have nonlinear terms. Using complex variable functions and the method of the small parameter approximate solutions are constructed for $N = 2$ in the hierarchy by I. Vekua. Concrete problem has been solved.

Key words and phrases: Non-shallow shells, geometrically nonlinear theory, small parameter, spherical shells.

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1 Introduction

I. Vekua has constructed the refined theory of shallow shells [1],[2]. This method for non-shallow shells in case of the geometrical and physical non-linear theory was generalized by T.Meunargia [3],[4].

In the present paper we consider the system of equilibrium equations of the two dimensional geometrically nonlinear non-shallow spherical shells which are obtained from the three-dimensional problems of the theory of elasticity for isotropic and homogeneous shell by the method of I. Vekua.

2 Approximation of Order $N = 2$

The displacement vector $\boldsymbol{U}(x^1, x^2, x^3)$ are expressed by the following formula [1, 2] (approximation $N = 2$)

$$\boldsymbol{U}(x^1, x^2, x^3) = \mathbf{u}(x^1, x^2) + \frac{x^3}{h}\mathbf{v}(x^1, x^2) - \frac{1}{2}\left(\frac{3(x^3)^2}{h^2} - 1\right)\mathbf{w}(x^1, x^2).$$

Here $\mathbf{u}(x^1,x^2)$, $\mathbf{v}(x^1,x^2)$ and $\mathbf{w}(x^1,x^2)$ are the vector fields on the middle surface $x^3=0$, $2h$ is the thickness of the shell, x^3 is a thickness coordinate $(-h\le x^3\le h)$, x^1 and x^2 are isometric coordinates on the spherical surface

$$x^1=\tan\frac{\theta}{2}\cos\varphi,\quad x^2=\tan\frac{\theta}{2}\sin\varphi,$$

where θ and φ are the geographical coordinates.

Let us construct the solutions of the form [2, 5]

$$u_i=\sum_{k=1}^{\infty}\overset{k}{u}_i\varepsilon^k,\qquad v_i=\sum_{k=1}^{\infty}\overset{k}{v}_i\varepsilon^k,\qquad w_i=\sum_{k=1}^{\infty}\overset{k}{w}_i\varepsilon^k,\quad (i=1,2,3),$$

where u_i, v_i and w_i are the components of the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ respectively, $\varepsilon=\frac{h}{R_0}$ is a small parameter, R_0 is the radius of the midsurface of the sphere.

Using I. Vekua's method and complex variable functions the system of equilibrium equations can be represented in the form

$$\begin{aligned}
&4\mu\frac{\partial}{\partial\bar z}\left(\frac{1}{\Lambda}\frac{\partial\overset{k}{u}_+}{\partial\bar z}\right)+2(\lambda+\mu)\frac{\partial\overset{k}{\theta}}{\partial\bar z}+2\lambda\frac{\partial\overset{k}{v}_3}{\partial\bar z}=\overset{k}{X}_+,\\
&4\mu\frac{\partial}{\partial\bar z}\left(\frac{1}{\Lambda}\frac{\partial\overset{k}{w}_+}{\partial\bar z}\right)+2(\lambda+\mu)\frac{\partial\overset{k}{\Theta}}{\partial\bar z}-5\mu\left[2\frac{\partial\overset{k}{v}_3}{\partial\bar z}+\frac{3}{h}\overset{k}{w}_+\right]=\overset{k}{Z}_+,\\
&\mu\left(\nabla^2\overset{k}{v}_3+3\overset{k}{\Theta}\right)-3\left[\lambda\overset{k}{\theta}+(\lambda+2\mu)\overset{k}{v}_3\right]=\overset{k}{Y}_3,
\end{aligned}\tag{1}$$

$$\begin{aligned}
&4\mu\frac{\partial}{\partial\bar z}\left(\frac{1}{\Lambda}\frac{\partial\overset{k}{v}_+}{\partial\bar z}\right)+2(\lambda+\mu)\frac{\partial\overset{k}{\vartheta}}{\partial\bar z}+6\lambda\frac{\partial\overset{k}{w}_3}{\partial\bar z}-3\mu\left(2\frac{\partial\overset{k}{u}_3}{\partial\bar z}+\overset{k}{v}_+\right)=\overset{k}{Y}_+,\\
&\mu\left(\nabla^2\overset{k}{u}_3+\overset{k}{\vartheta}\right)=\overset{k}{Y}_3,\\
&\mu\nabla^2\overset{k}{w}_3-5\left[\lambda\overset{k}{\vartheta}+3(\lambda+2\mu)\overset{k}{w}_3\right]=\overset{k}{Z}_3,\quad (k=1,2,...),
\end{aligned}\tag{2}$$

where $z=x^1+ix^2,\Lambda=\dfrac{4R_0^2}{(1+z\bar z)^2},\nabla^2=\dfrac{4}{\Lambda}\dfrac{\partial^2}{\partial z\partial\bar z}$ and

$$\overset{k}{u}_+=\overset{k}{u}_1+i\overset{k}{u}_2,\quad \overset{k}{v}_+=\overset{k}{v}_1+i\overset{k}{v}_2,\quad \overset{k}{w}_+=\overset{k}{w}_1+i\overset{k}{w}_2,$$

$$\overset{k}{\theta}=\frac{1}{\Lambda}\left(\frac{\partial\overset{k}{u}_+}{\partial z}+\frac{\partial\overset{k}{\overline{u}}_+}{\partial\bar z}\right),\quad \overset{k}{\vartheta}=\frac{1}{\Lambda}\left(\frac{\partial\overset{k}{v}_+}{\partial z}+\frac{\partial\overset{k}{\overline{v}}_+}{\partial\bar z}\right),$$

$$\overset{k}{\Theta}=\frac{1}{\Lambda}\left(\frac{\partial\overset{k}{w}_{+}}{\partial z}+\frac{\partial\overline{\overset{k}{w}}_{+}}{\partial\bar z}\right).$$

Introducing the well-known differential operators

$$\frac{\partial}{\partial z}=\frac{1}{2}\left(\frac{\partial}{\partial x^1}-i\frac{\partial}{\partial x^2}\right),\quad \frac{\partial}{\partial\bar z}=\frac{1}{2}\left(\frac{\partial}{\partial x^1}+i\frac{\partial}{\partial x^2}\right).$$

$\overset{k}{X}_{+}$, $\overset{k}{Y}_{+}$, $\overset{k}{Z}_{+}$, $\overset{k}{X}_{3}$, $\overset{k}{Y}_{3}$, $\overset{k}{Z}_{3}$ are the components of external force and well-known quantity, defined by functions $\overset{0}{u}_{i},...,\overset{k-1}{u}_{i},\overset{0}{v}_{j},...,\overset{k-1}{v}_{j}$, $\overset{0}{w}_{i},...,\overset{k-1}{w}_{i}$ $(i,j=1,2,3)$.

The complex representation of a general solutions of systems (1) end (2) are written in the following form

$$\begin{aligned}
\overset{k}{u}_{+}=&-\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{1}{\pi}\iint_D\frac{\Lambda(\zeta,\bar\zeta)\varphi'(\zeta)d\xi d\eta}{\bar\zeta-\bar z}+\left(\frac{1}{\pi}\iint_D\frac{\Lambda(\zeta,\bar\zeta)d\xi d\eta}{\bar\zeta-\bar z}\right)\overline{\varphi'(z)}\\
&-\overline{\psi(z)}-\frac{2\lambda}{\lambda+2\mu}\left(\frac{1}{\gamma_1}\frac{\partial\chi_1(z,\bar z)}{\partial\bar z}+\frac{1}{\gamma_2}\frac{\partial\chi_2(z,\bar z)}{\partial\bar z}\right),\\
\overset{k}{w}_{+}=&\frac{2}{3}\left(\frac{\gamma_2}{\gamma_1}\frac{\partial\chi_1(z,\bar z)}{\partial\bar z}+\frac{\gamma_1}{\gamma_2}\frac{\partial\chi_2(z,\bar z)}{\partial\bar z}+i\frac{\partial\chi_3(z,\bar z)}{\partial\bar z}+\frac{2\lambda}{3\lambda+2\mu}\overline{\varphi''(z)}\right),\\
\overset{k}{v}_{3}=&\chi_1(z,\bar z)+\chi_2(z,\bar z)-\frac{2\lambda}{3\lambda+2\mu}\left(\varphi'(z)+\overline{\varphi'(z)}\right),
\end{aligned}$$

$$\begin{aligned}
\overset{k}{v}_{+}=&i\frac{\partial\chi_4(z,\bar z)}{\partial\bar z}-\frac{\lambda}{10(\lambda+\mu)}\frac{\partial\chi_5(z,\bar z)}{\partial\bar z}+\frac{16(\lambda+\mu)}{3(\lambda+2\mu)}\overline{f''(z)}\\
&-\frac{1}{\pi}\iint_D\frac{\Lambda(\zeta,\bar\zeta)f'(\zeta)d\xi d\eta}{\bar\zeta-\bar z}-\left(\frac{1}{\pi}\iint_D\frac{\Lambda(\zeta,\bar\zeta)d\xi d\eta}{\bar\zeta-\bar z}\right)\overline{f'(z)}-2\overline{g'(z)},\\
\overset{k}{u}_{3}=&\frac{\lambda}{20(\lambda+\mu)}\chi_5(z,\bar z)+g(z)+\overline{g(z)}\\
&-\frac{1}{\pi}\iint_D\Lambda(\zeta,\bar\zeta)\left[f'(z)+\overline{f'(z)}\right]\ln|\zeta-z|d\xi d\eta,\\
\overset{k}{w}_{3}=&\chi_5(z,\bar z)-\frac{2\lambda}{3(\lambda+2\mu)}\left(f'(z)+\overline{f'(z)}\right),
\end{aligned}$$

where $\zeta=\xi+i\eta$, $\varphi(z)$,$\psi(z)$,$f(z)$ and $g(z)$ are any analytic functions of z, $\chi_1(z,\bar z)$, $\chi_2(z,\bar z)$, $\chi_3(z,\bar z)$, $\chi_4(z,\bar z)$ and $\chi_5(z,\bar z)$, are the general solutions of the following Helmholtz's equations, respectively:

$$\Delta\chi_\alpha(z,\bar z)-\gamma_\alpha^2\chi_\alpha(z,\bar z)=0,\quad \alpha=1,2,$$

$$\gamma_\alpha = \frac{6(\lambda+\mu)}{\lambda+2\mu}\left[1 \pm \sqrt{\frac{\lambda-4\mu}{\lambda+\mu}}\right],$$

$$\Delta\chi_3(z,\bar z) - 15\chi_3(z,\bar z) = 0, \quad \Delta\chi_4(z,\bar z) - 3\chi_4(z,\bar z) = 0,$$
$$\Delta\chi_5(z,\bar z) - \gamma^2\chi_5(z,\bar z) = 0, \quad \gamma^2 = \frac{60(\lambda+\mu)}{\lambda+2\mu}.$$

D is the domain of the plane Ox^1x^2 onto which the midsurface S of the shell is mapped topologically.

Here we present a general scheme of solution of boundary problems when the domain D is the circular ring with radius R_1 and R_2 [6–11].

The second boundary problem (in displacements) for any k takes the form

$$\overset{k}{u}_+ = -\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\bar\zeta)\varphi'(\zeta)d\xi d\eta}{\bar\zeta-\bar z} + \frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\bar\zeta)d\xi d\eta}{\bar\zeta-\bar z}$$
$$\times\overline{\varphi'(z)} - \overline{\psi(z)} - \frac{2\lambda}{\lambda+2\mu}\left(\frac{1}{\gamma_1}\frac{\partial\chi_1(z,\bar z)}{\partial\bar z} + \frac{1}{\gamma_2}\frac{\partial\chi_2(z,\bar z)}{\partial\bar z}\right) \tag{3}$$
$$= \begin{cases} \overset{(k)}{G}{}'_1, & |z| = R_1, \\ \overset{(k)}{G}{}''_1, & |z| = R_2, \end{cases}$$

$$\overset{k}{w}_+ = \frac{2}{3}\left(\frac{\gamma_2}{\gamma_1}\frac{\partial\chi_1(z,\bar z)}{\partial\bar z} + \frac{\gamma_1}{\gamma_2}\frac{\partial\chi_2(z,\bar z)}{\partial\bar z} + i\frac{\partial\chi_3(z,\bar z)}{\partial\bar z}\right)$$
$$+\frac{4\lambda}{3(3\lambda+2\mu)}\overline{\varphi''(z)} = \begin{cases} \overset{(k)}{G}{}'_2, & |z| = R_1, \\ \overset{(k)}{G}{}''_2, & |z| = R_2, \end{cases} \tag{4}$$

$$\overset{k}{v}_3 = \chi_1(z,\bar z) + \chi_2(z,\bar z) - \frac{2\lambda h}{3\lambda+2\mu}\left(\varphi'(z) + \overline{\varphi'(z)}\right)$$
$$= \begin{cases} \overset{(k)}{G}{}'_3, & |z| = R_1, \\ \overset{(k)}{G}{}''_3, & |z| = R_2, \end{cases} \tag{5}$$

$$\overset{k}{v}_+ = i\frac{\partial\chi_4(z,\bar z)}{\partial\bar z} - \frac{\lambda}{10(\lambda+\mu)}\frac{\partial\chi_5(z,\bar z)}{\partial\bar z} + \frac{16(\lambda+\mu)}{3(\lambda+2\mu)}\overline{f''(z)}$$
$$-\frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\bar\zeta)f'(\zeta)d\xi d\eta}{\bar\zeta-\bar z} - \left(\frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\bar\zeta)d\xi d\eta}{\bar\zeta-\bar z}\right)\overline{f'(z)} - 2\overline{g'(z)} \tag{6}$$
$$= \begin{cases} \overset{(k)}{Q}{}'_1, & |z| = R_1, \\ \overset{(k)}{Q}{}''_1, & |z| = R_2, \end{cases}$$

$$\overset{k}{u}_3 = \frac{\lambda}{20(\lambda+\mu)}\chi_5(z,\bar z) + g(z) + \overline{g(z)} - \frac{1}{\pi}\int\int_D \Lambda(\zeta,\bar\zeta)$$

$$\times\left[f'(z)+\overline{f'(z)}\right]\ln|\zeta - z|d\xi d\eta = \begin{cases} \overset{(k)}{Q}{}'_2, & |z|=R_1, \\ \overset{(k)}{Q}{}''_2, & |z|=R_2, \end{cases} \tag{7}$$

$$\overset{k}{w}_3 = \chi_5(z,\bar z) - \frac{2\lambda}{3(\lambda+2\mu)}\left(f'(z)+\overline{f'(z)}\right) = \begin{cases} \overset{(k)}{Q}{}'_3, & |z|=R_1, \\ \overset{(k)}{Q}{}''_3, & |z|=R_2, \end{cases} \tag{8}$$

where $\overset{(k)}{G}{}'_1$, $\overset{(k)}{G}{}''_1$, $\overset{(k)}{G}{}'_2$, $\overset{(k)}{G}{}''_2$, $\overset{(k)}{G}{}'_3$, $\overset{(k)}{G}{}''_3$, $\overset{(k)}{Q}{}'_1$, $\overset{(k)}{Q}{}''_1$, $\overset{(k)}{Q}{}'_2$, $\overset{(k)}{Q}{}''_2$, $\overset{(k)}{Q}{}'_3$ and $\overset{(k)}{Q}{}''_3$ are the known values.

Next $\varphi'(z)$ and $\psi(z)$ are expanded in power series of the type

$$\begin{gathered} \varphi'(z) = \sum_{-\infty}^{\infty} a_n z^n, \quad \psi(z) = \sum_{-\infty}^{\infty} b_n z^n, \\ \chi_1(z,\bar z) = \sum_{-\infty}^{\infty}\left(\alpha_{1n}I_n(\gamma_1 r) + \beta_{1n}K_n(\gamma_1 r)\right)e^{in\vartheta}, \\ \chi_2(z,\bar z) = \sum_{-\infty}^{\infty}\left(\alpha_{2n}I_n(\gamma_2 r) + \beta_{2n}K_n(\gamma_2 r)\right)e^{in\vartheta}, \\ \chi_3(z,\bar z) = \sum_{-\infty}^{\infty}\left(\alpha_{3n}I_n(\sqrt{15}r) + \beta_{3n}K_n(\sqrt{15}r)\right)e^{in\vartheta}, \end{gathered} \tag{9}$$

where $I_n(\kappa r)$ and $K_n(\kappa r)$ are Bessel's modificed functions, the expression $\overset{(k)}{G}{}'_1$, $\overset{(k)}{G}{}''_1$, $\overset{(k)}{G}{}'_2$, $\overset{(k)}{G}{}''_2$, $\overset{(k)}{G}{}'_3$ and $\overset{(k)}{G}{}''_3$ in the form of a complex Fourier series

$$\begin{gathered} \overset{(k)}{G}{}'_1 = \sum_{-\infty}^{\infty} A'_{1n}e^{in\vartheta}, \quad \overset{(k)}{G}{}'_2 = \sum_{-\infty}^{\infty} A'_{2n}e^{in\vartheta}, \quad \overset{(k)}{G}{}'_3 = \sum_{-\infty}^{\infty} A'_{3n}e^{in\vartheta}, \\ \overset{(k)}{Q}{}'_1 = \sum_{-\infty}^{\infty} A'_{4n}e^{in\vartheta}, \quad \overset{(k)}{Q}{}'_2 = \sum_{-\infty}^{\infty} A'_{5n}e^{in\vartheta}, \quad \overset{(k)}{Q}{}'_3 = \sum_{-\infty}^{\infty} A'_{6n}e^{in\vartheta}. \end{gathered} \tag{10}$$

By substituting (9) and (10) into (3), (4) and (5) we obtain:

$$\begin{gathered} -\frac{5\lambda+6\mu}{3\lambda+2\mu}\sum_{0}^{\infty} R_1^{n-1}\varepsilon_{-n}a_{-n}e^{-i(n-1)\vartheta} - \sum_{0}^{\infty} R_1^n \bar b_n e^{-in\vartheta} \\ -\frac{\lambda}{\lambda+2\mu}\sum_{-\infty}^{\infty}\Big[\left(\alpha_{1n}I_{n+1}(\gamma_1 R_1) - \beta_{1n}K_{n+1}(\gamma_1 R_1)\right)e^{i(n+1)\vartheta} \\ \left(\alpha_{2n}I_{n+1}(\gamma_2 R_1) - \beta_{2n}K_{n+1}(\gamma_2 R_1)\right)e^{i(n+1)\vartheta}\Big] = \sum_{-\infty}^{\infty} A'_{1n}e^{in\vartheta}, \end{gathered} \tag{11}$$

$$\frac{5\lambda+6\mu}{3\lambda+2\mu}\sum_{0}^{\infty}\frac{\varepsilon_n a_n}{R_2^{n+1}}e^{i(n+1)\vartheta}-\sum_{0}^{\infty}R_2^n\bar{b}_n e^{-in\vartheta}-2\varepsilon_0\sum_{0}^{\infty}R_2^{n-2}a_{n-1}$$
$$\times e^{in\vartheta}-\frac{\lambda}{\lambda+2\mu}\sum_{-\infty}^{\infty}\Big[\left(\alpha_{1n}I_{n+1}(\gamma_1R_2)-\beta_{1n}K_{n+1}(\gamma_1R_2)\right)e^{i(n+1)\vartheta} \tag{12}$$
$$+\left(\alpha_{2n}I_{n+1}(\gamma_2R_2)-\beta_{2n}K_{n+1}(\gamma_2R_2)\right)e^{i(n+1)\vartheta}\Big]=\sum_{-\infty}^{\infty}A''_{1n}e^{in\vartheta},$$

$$\sum_{-\infty}^{\infty}\left(\alpha_{1n}I_n(\gamma_1R_1)+\beta_{1n}K_n(\gamma_1R_1)\right)e^{in\vartheta}$$
$$+\sum_{-\infty}^{\infty}\left(\alpha_{2n}I_n(\gamma_2R_1)+\beta_{2n}K_n(\gamma_2R_1)\right)e^{in\vartheta} \tag{13}$$
$$-\frac{2\lambda h}{3\lambda+2\mu}\sum_{-\infty}^{\infty}(a_ne^{in\vartheta}+\bar{a}_ne^{-in\vartheta})R_1^n=\sum_{-\infty}^{\infty}A'_{2n}e^{in\vartheta},$$

$$\sum_{-\infty}^{\infty}\left(\alpha_{1n}I_n(\gamma_1R_2)+\beta_{1n}K_n(\gamma_1R_2)\right)e^{in\vartheta}$$
$$+\sum_{-\infty}^{\infty}\left(\alpha_{2n}I_n(\gamma_2R_2)+\beta_{2n}K_n(\gamma_2R_2)\right)e^{in\vartheta} \tag{14}$$
$$-\frac{2\lambda h}{3\lambda+2\mu}\sum_{-\infty}^{\infty}(a_ne^{in\vartheta}+\bar{a}_ne^{-in\vartheta})R_2^n=\sum_{-\infty}^{\infty}A''_{2n}e^{in\vartheta},$$

$$\frac{2}{3}\sum_{-\infty}^{\infty}\Big[\frac{\gamma_2}{2}\left(\alpha_{1n}I_{n+1}(\gamma_1R_1)-\beta_{1n}K_{n+1}(\gamma_1R_1)\right)$$
$$+\frac{\gamma_1}{2}\left(\alpha_{2n}I_{n+1}(\gamma_2R_1)-\beta_{2n}K_{n+1}(\gamma_2R_1)\right)$$
$$+\frac{i\sqrt{15}}{2}\left(\alpha_{3n}I_{n+1}(\sqrt{15}R_1)-\beta_{3n}K_{n+1}(\sqrt{15}R_1)\right)\Big]e^{i(n+1)\vartheta} \tag{15}$$
$$+\frac{4\lambda}{3(3\lambda+2\mu)}\sum_{-\infty}^{\infty}nR_1^{n-1}\bar{a}_ne^{-i(n-1)\vartheta}=\sum_{-\infty}^{\infty}A'_{3n}e^{in\vartheta},$$

$$\frac{2}{3}\sum_{-\infty}^{\infty}\Big[\frac{\gamma_2}{2}\left(\alpha_{1n}I_{n+1}(\gamma_1R_2)-\beta_{1n}K_{n+1}(\gamma_1R_2)\right)$$
$$+\frac{\gamma_1}{2}\left(\alpha_{2n}I_{n+1}(\gamma_2R_2)-\beta_{2n}K_{n+1}(\gamma_2R_2)\right)$$
$$+\frac{i\sqrt{15}}{2}\left(\alpha_{3n}I_{n+1}(\sqrt{15}R_2)-\beta_{3n}K_{n+1}(\sqrt{15}R_2)\right)\Big]e^{i(n+1)\vartheta} \tag{16}$$
$$+\frac{4\lambda}{3(3\lambda+2\mu)}\sum_{-\infty}^{\infty}nR_2^{n-1}\bar{a}_ne^{-i(n-1)\vartheta}=\sum_{-\infty}^{\infty}A''_{3n}e^{in\vartheta},$$

where $\varepsilon_n = 2\int\limits_{R_1}^{R_2}\Lambda(\rho)\rho^{2n+1}d\rho$.

Compare the coefficients at identical degrees (11)-(16). We obtain the following system of equations

$$\begin{aligned}
&-\frac{\lambda}{\lambda+2\mu}\left(I_{n-1}(\gamma_1R_1)\alpha_{1n}-K_{n-1}(\gamma_1R_1)\beta_{1n}\right.\\
&\left.-I_{n-1}(\gamma_2R_1)\alpha_{2n}+K_{n-1}(\gamma_2R_1)\beta_{2n}\right)-\frac{5\lambda+6\mu}{3\lambda+2\mu}R_1^{n-1}\varepsilon_{-n}\bar{a}_{-n}\\
&-R_1^{n-1}b_{n-1}=\bar{A}'_{1\;-n+1},\quad n>0,\\
&-\frac{\lambda}{\lambda+2\mu}\left(I_{n-1}(\gamma_1R_2)\alpha_{1n}-K_{n-1}(\gamma_1R_2)\beta_{1n}\right.\\
&\left.-I_{n-1}(\gamma_2R_2)\alpha_{2n}+K_{n-1}(\gamma_2R_2)\beta_{2n}\right)-2R_2^{-n-1}\varepsilon_0\bar{a}_{-n}\\
&-R_2^{n-1}b_{n-1}=\bar{A}''_{1\;-n+1},\quad n\geq 0,\\
&-\frac{\lambda}{\lambda+2\mu}\left(I_{n}(\gamma_1R_1)\alpha_{1\;n-1}-K_{n}(\gamma_1R_1)\beta_{1\;n-1}\right.\\
&\left.-I_{n}(\gamma_2R_1)\alpha_{2\;n-1}+K_{n}(\gamma_2R_1)\beta_{2\;n-1}\right)\\
&+R_1^{-n}b_{-n}=\bar{A}'_{1\;n},\quad n>0,\\
&-\frac{\lambda}{\lambda+2\mu}\left(I_{n+1}(\gamma_1R_2)\alpha_{1n}-K_{n+1}(\gamma_1R_2)\beta_{1n}\right.\\
&\left.-I_{n+1}(\gamma_2R_2)\alpha_{2n}+K_{n+1}(\gamma_2R_2)\beta_{2n}\right)-R_2^{-n-1}\bar{b}_{-n-1}\\
&\left(\frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{\varepsilon_n}{R_2^{n+1}}-2R_2^{n-1}\varepsilon_0\right)a_n=\bar{A}''_{1\;n+1},\quad n\geq 0,
\end{aligned}\tag{17}$$

$$\begin{aligned}
&I_n(\gamma_1R_1)\alpha_{1n}+K_n(\gamma_1R_1)\beta_{1n}+I_n(\gamma_2R_1)\alpha_{2n}+K_n(\gamma_2R_1)\beta_{2n}\\
&-\frac{2\lambda h}{3\lambda+2\mu}(R_1^na_n+R_1^{-n}\bar{a}_{-n})=A'_{2n},\\
&I_n(\gamma_1R_2)\alpha_{1n}+K_n(\gamma_1R_2)\beta_{1n}+I_n(\gamma_2R_2)\alpha_{2n}+K_n(\gamma_2R_2)\beta_{2n}\\
&-\frac{2\lambda h}{3\lambda+2\mu}(R_2^na_n+R_2^{-n}\bar{a}_{-n})=A''_{2n},
\end{aligned}\tag{18}$$

$$\begin{aligned}
&\frac{\gamma_2}{3}\left(I_{n+1}(\gamma_1R_1)\alpha_{1n}-K_{n+1}(\gamma_1R_1)\beta_{1n}\right)\\
&+\frac{\gamma_1}{3}\left(I_{n+1}(\gamma_2R_1)\alpha_{2n}-K_{n+1}(\gamma_2R_1)\beta_{2n}\right)\\
&+\frac{i\sqrt{15}}{3}\left(I_{n+1}(\sqrt{15}R_1)\alpha_{3n}-K_{n+1}(\sqrt{15}R_1)\beta_{3n}\right)\\
&-\frac{4\lambda n}{3(3\lambda+2\mu)}R_1^{-n-1}\bar{a}_{-n}=A'_{3\;n+1},
\end{aligned}\tag{19}$$

$$\begin{aligned}
&\frac{\gamma_2}{3}\left(I_{n+1}(\gamma_1 R_2)\alpha_{1n} - K_{n+1}(\gamma_1 R_2)\beta_{1n}\right) \\
&+\frac{\gamma_1}{3}\left(I_{n+1}(\gamma_2 R_2)\alpha_{2n} - K_{n+1}(\gamma_2 R_2)\beta_{2n}\right) \\
&+\frac{i\sqrt{15}}{3}\left(I_{n+1}(\sqrt{15}R_2)\alpha_{3n} - K_{n+1}(\sqrt{15}R_2)\beta_{3n}\right) \\
&-\frac{4\lambda n}{3(3\lambda+2\mu)}R_2^{-n-1}\bar{a}_{-n} = A'_{3\,n+1}.
\end{aligned} \tag{20}$$

The coefficients a_n b_n, α_{1n}, α_{2n}, α_{3n}, β_{1n}, β_{2n}, β_{3n} are found by solving (17)-(20).

Let us introduce the functions $f'(z)$, $g(z)$and $\omega(z,\bar{z})$, $\overset{(k)}{Q}{}'_{+}$, $\overset{(k)}{Q}{}''_{+}$, $\overset{(k)}{Q}{}'_{3}$, $\overset{(k)}{Q}{}''_{3}$ by the series

$$\begin{aligned}
&f'(z) = \sum_{-\infty}^{\infty} c_n z^n, \quad g(z) = \sum_{-\infty}^{\infty} d_n z^n, \\
&\chi_4(z,\bar{z}) = \sum_{-\infty}^{\infty}\left(\alpha_{4n} I_n(\sqrt{3}r) + \beta_{4n} K_n(\sqrt{3}r)\right)e^{in\vartheta}, \\
&\chi_5(z,\bar{z}) = \sum_{-\infty}^{\infty}\left(\alpha_{5n} I_n(\gamma r) + \beta_{5n} K_n(\gamma r)\right)e^{in\vartheta}, \\
&\overset{k}{Q}{}'_1 = \sum_{-\infty}^{\infty} A'_{4n}e^{in\vartheta}, \quad \overset{k}{Q}{}'_2 = \sum_{-\infty}^{\infty} A'_{5n}e^{in\vartheta}, \quad \overset{k}{Q}{}'_3 = \sum_{-\infty}^{\infty} A'_{6n}e^{in\vartheta}, \\
&\overset{k}{Q}{}''_1 = \sum_{-\infty}^{\infty} A''_{4n}e^{in\vartheta}, \quad \overset{k}{Q}{}''_2 = \sum_{-\infty}^{\infty} A''_{5n}e^{in\vartheta}, \quad \overset{k}{Q}{}''_3 = \sum_{-\infty}^{\infty} A''_{6n}e^{in\vartheta}.
\end{aligned} \tag{21}$$

By substituting (21) into (6-8) we now find the coefficients c_n, d_n, α_{4n}, α_{5n}, α_{6n}, β_{4n}, β_{5n} and α_{6n} from following system of algebraic equations:

$$\begin{aligned}
&\frac{i\sqrt{3}}{2}\left(I_{n+1}(\sqrt{3}R_1)\alpha_{4n} - K_{n+1}(\sqrt{3}R_1)\beta_{4n}\right) \\
&-\frac{\lambda\gamma}{20(\lambda+\mu)}\left(I_{n+1}(\gamma R_1)\alpha_{5n} - K_{n+1}(\gamma R_1)\beta_{5n}\right) \\
&-\frac{16(\lambda+\mu)n}{3(\lambda+2\mu)R_1^{n+1}}\bar{c}_{-n} + \frac{2n}{R_1^{n+1}}\bar{d}_{-n} = A'_{4\,n+1}, \;\; n \geq 0, \\
&\frac{i\sqrt{3}}{2}\left(I_{n+1}(\sqrt{3}R_2)\alpha_{4n} - K_{n+1}(\sqrt{3}R_2)\beta_{4n}\right) + \frac{n}{R_2^{n+1}}\bar{d}_{-n} \\
&-\frac{\lambda\gamma}{20(\lambda+\mu)}\left(I_{n+1}(\gamma R_2)\alpha_{5n} - K_{n+1}(\gamma R_2)\beta_{5n}\right) \\
&-\frac{16(\lambda+\mu)n}{3(\lambda+2\mu)R_2^{n+1}}\bar{c}_{-n} - \frac{\varepsilon_n}{R_2^{n+1}}c_n + 2R_2^{n-1}\varepsilon_0 c_n = A''_{4\,n+1}, \;\; n \geq 0,
\end{aligned} \tag{22}$$

$$
\begin{aligned}
&\frac{i\sqrt{3}}{2}\left(I_{n-1}(\sqrt{3}R_1)\alpha_{4n}-K_{n-1}(\sqrt{3}R_1)\beta_{4n}\right)-\varepsilon_{-n}R_1^{n-1}c_{-n}\\
&-\frac{\lambda\gamma}{20(\lambda+\mu)}\left(I_{n-1}(\gamma R_1)\alpha_{5n}-K_{n-1}(\gamma R_1)\beta_{5n}\right)\\
&+\frac{16(\lambda+\mu)nR_1^{n-1}}{3(\lambda+2\mu)}\bar{c}_n-2nR_1^{n-1}\bar{d}_{-n}=A'_{4\ -n+1},\ \ n\geq 1,\\
&\frac{i\sqrt{3}}{2}\left(I_{n-1}(\sqrt{3}R_2)\alpha_{4n}-K_{n-1}(\sqrt{3}R_2)\beta_{4n}\right)+2\varepsilon_0R_2^{-n-1}c_{-n}\\
&-\frac{\lambda\gamma}{20(\lambda+\mu)}\left(I_{n-1}(\gamma R_2)\alpha_{5n}-K_{n-1}(\gamma R_2)\beta_{5n}\right)\\
&+\frac{16(\lambda+\mu)nR_2^{n-1}}{3(\lambda+2\mu)}\bar{c}_n-2nR_2^{n-1}\bar{d}_{-n}=A''_{4\ -n+1},\ \ n\geq 1,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&\frac{\lambda}{20(\lambda+\mu)}\left(I_n(\gamma R_1)\alpha_{5n}+K_n(\gamma R_1)\beta_{5n}\right)\\
&+R_1^n d_n+R_1^{-n}\bar{d}_{-n}-\frac{R_1^n}{n}\varepsilon_0c_n-\frac{R_1^n}{n}\varepsilon_{-n}\bar{c}_{-n}=A'_{5n},\\
&\frac{\lambda}{20(\lambda+\mu)}\left(I_n(\gamma R_2)\alpha_{5n}+K_n(\gamma R_2)\beta_{5n}\right)\\
&+R_2^n d_n+R_2^{-n}\bar{d}_{-n}-\frac{\varepsilon_n}{nR_2^n}c_n-\frac{\varepsilon_0}{nR_2^n}\bar{c}_{-n}=A''_{5n},
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&\frac{\lambda}{20(\lambda+\mu)}\left(I_0(\gamma R_1)\alpha_{50}+K_0(\gamma R_1)\beta_{50}\right)\\
&+d_0+\bar{d}_0-2\varepsilon_0(c_0+\bar{c}_0)=A'_{50},\\
&\frac{\lambda}{20(\lambda+\mu)}\left(I_0(\gamma R_2)\alpha_{50}+K_0(\gamma R_2)\beta_{50}\right)\\
&+d_0+\bar{d}_0-2\varepsilon_0\ln R_2(c_0+\bar{c}_0)=A''_{50},
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&I_n(\gamma R_1)\alpha_{5n}+K_n(\gamma R_1)\beta_{5n}-\frac{2\lambda}{3(\lambda+2\mu)}R_1^nc_n=A'_{6n},\\
&I_n(\gamma R_2)\alpha_{5n}+K_n(\gamma R_2)\beta_{5n}-\frac{2\lambda}{3(\lambda+2\mu)}R_2^nc_n=A'_{6n}.
\end{aligned}
\tag{26}
$$

The coefficients c_n, d_n, α_{4n}, α_{5n}, α_{6n}, β_{4n}, β_{5n} and α_{6n} are found by solving (22)-(26).

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