ON SOLVING HEAT-CONDUCTION PROBLEMS UNDER VARIABLE TRANSFER COEFFICIENTS

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Abstract

In the paper various boundary value problems are considered for the heat-conduction equation with variable coefficients. Approximate solutions of these problems are constructed.

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Let us consider a heat-conduction boundary value problem [1] in inhomogeneous medium Ω under variable thermophysical parameters dependent on coordinates. Having imposed $(0 \le x \le l)$, $c\rho = const$, $\lambda = \lambda_0 e^{-kx}$ and $q = 0$ we reduce the energy equation to the form [1]

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_0 e^{-kt} \frac{\partial T}{\partial x}\right).$$

Approximate solution of this equation under uniqueness conditions

$$T(0,t) = 0, \quad T(\ell,t) = \varphi(t), \quad T(x,0) = 0$$

in Laplace transforms $T^*(M,S) = \int\limits_0^\infty T(M,t)\exp(-St)\,dt$ (where $T^*(M,S)$ is Laplace integral transform of temperature $T(M,t)$; S is a parameter of Laplace transform; $M(x,y,z)$ is a moving point at the domain Ω) is found in the family of functions

$$T_n^*(x,S) = \frac{e^{kx}-1}{e^{kl}-1}\varphi^*(S) + \sum_{i=1}^{n} a_i^*(S)\left(1-\frac{x}{l}\right)\left(\frac{x}{l}\right)^i.$$

Under constant boundary conditions $\varphi(t) = T_c = const$, in the first approximation this solution is reduced to the form

$$\frac{T(x,F_0)}{T_c} = \frac{e^{kx}-1}{e^{kl}-1} + \left(1-\frac{x}{l}\right)\frac{x}{l}\exp\left[-B(\alpha)F_0\right], \tag{1}$$

where

$$B(\alpha) = \frac{30\left[\alpha^2 - 4\alpha + 8\right] - e^{-\alpha}\left(\alpha^2 - 4\alpha + 8\right)}{\alpha^3}$$

$$\alpha = kl, \quad F_0 = \frac{a_0 t}{l^2}, \quad a_0 = \frac{\lambda_0}{\rho c}.$$

Having calculated the limit by L'Hospital's rule we can write

$$\lim_{\alpha \to 0} B(\alpha) = 10, \quad \lim_{k \to 0} \frac{e^{kx} - 1}{e^{kl} - 1} = \frac{x}{l}.$$

Then the solution (1) in the limit as $k \to 0$ will be

$$\frac{T(x, F_0)}{T_c} = \frac{x}{l} + \left(1 - \frac{x}{l}\right)\frac{x}{l}\exp\left(-10F_0\right).$$

Hence it follows that as $\alpha \to 0\,(kl \leq 0,1)$ the approximate solution (1) in actual fact coincides with the known solution for a plate under constant transfer coefficients [5]. Furthermore, solution (1) as $F_0 \to \infty$ tends to exact solution of appropriate stationary problem [4].

For an unbounded plate $\Omega\left(-R \leq x \leq R\right)$ under symmetric boundary conditions we write the problem in the form

$$\frac{\partial T}{\partial F_0} = \frac{\partial}{\partial \xi}\left(e^{-\omega|\xi|}\frac{\partial T}{\partial \xi}\right), \quad \xi = \frac{x}{R}, \quad \omega = kR,$$

$$T(\xi, 0) = T_0, \quad \left[T(\xi, F_0)\right]_{\xi = \pm 1} = \varphi(F_0). \tag{2}$$

Without loss of generality, we assume $\varphi(F_0) = T_0(1 + PdF_0)$, where $Pd = \frac{bR^2}{T_0 a_0}$, then in the transforms

$$\frac{d}{d\xi}\left(e^{-\omega\xi}\frac{dT^*}{d\xi}\right) - ST^*(\xi, S) + T_0 = 0$$

$$\left[T^*(\xi, S)\right]_{\xi=1} = \varphi^*(S) = T_0\left(\frac{1}{S} + \frac{Pd}{S^2}\right), \quad \left(\frac{dT^*}{d\xi}\right)_{\xi=0} = 0.$$

We look for the approximate solution of the boundary value problem in the family of linear combination

$$T_0^*(\xi, S) = \varphi^*(S) + a_1^*(S)\left[e^{\omega}\left(1 - \frac{1}{\omega}\right) - \frac{e^{\omega\xi}(\omega\xi - 1)}{\omega}\right]$$

$$+\sum_{i=2}^{n} a_i^*(S)\left(1 - \xi^2\right)\xi^{2(i-1)}. \tag{3}$$

The first coordinate function in the solution (3) was chosen so that to within constant factor it equals the solution of the equation

$$\frac{\partial}{\partial \xi}\left(e^{-\omega\xi}\frac{\partial T}{\partial \xi}\right) = T_0 Pd.$$

Relative excess temperature in the first approximation is written in the form

$$\theta(\xi, F_0, Pd, \omega) = \frac{T(\xi, F_0) - T_0}{T_0}$$

$$= Pd\left\{F_0 - \left[1 - \exp\left(-A(\omega)F_0\right)\right]\left[\frac{e^{\omega}}{\omega}\left(1 - \frac{1}{\omega}\right) - \frac{e^{\omega\xi}}{\omega}\left(\xi - \frac{1}{\omega}\right)\right]\right\} \quad (4)$$

where

$$A(\omega) = \frac{4\left[e^{\omega}\left(\omega^4 - 2\omega^3 + 2\omega^2\right) - 2\omega^2\right]}{e^{2\omega}\left(4\omega^3 - 14\omega^2 + 22\omega - 11\right) - 16e^{\omega}(\omega - 1) - 5}.$$

For quasi-stationary condition ($\exp(-A(\omega)F_0) \approx 0$), from the solution of (4) we get [1]

$$\frac{\theta(\xi, F_0, \omega)}{Pd} = F_0 - \left[\frac{e^{\omega}}{\omega}\left(1 - \frac{1}{\omega}\right) - \frac{e^{\omega\xi}}{\omega}\left(\xi - \frac{1}{\omega}\right)\right], \quad F_0 \geq F_{0_1},$$

that coincides with the exact solution.

After opening L'Hospital indeterminacy, we get of indeterminate terms

$$\lim_{\omega\to 0} A(\omega) = 2,5,$$

$$\lim_{\omega\to 0}\left[\frac{e^{\omega}}{\omega}\left(1 - \frac{1}{\omega}\right) - \frac{e^{\omega\xi}}{\omega}\left(\xi - \frac{1}{\omega}\right)\right] = \frac{1}{2}\left(1 - \xi^2\right).$$

Expression (4) as $\omega \to 0$ is written in the form

$$\theta(\xi, F_0, Pd, 0) = Pd\left\{F_0 - \frac{1}{2}\left(1 - \xi^2\right)\left[1 - \exp(-2,5F_0)\right]\right\}. \quad (5)$$

In thin plates ($\omega = kR \leq 0,2$) temperature may be calculated by formula (5) that is the known solution under constant heat conduction coefficient [2].

For an unbounded cylinder $\Omega\left(x^2 + y^2 \leq R^2\right)$ we put $\lambda = \lambda_0 e^{kr}$, $\rho c = const$,
$\rho^2 = (r/R)^2 = \xi^2 + \eta^2 \leq 1$, then the problem is written as

$$\frac{\partial T}{\partial t} = a_0\left[\frac{\partial}{\partial x}\left(e^{k\sqrt{x^2+y^2}}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(e^{k\sqrt{x^2+y^2}}\frac{\partial T}{\partial y}\right)\right]. \quad (6)$$

In cylindrical coordinates under symmetric boundary conditions

$$T(\rho,0)=T_0,\quad [T(\rho,F_0)]_{\rho=1}=\varphi(F_0) \tag{7}$$

equation (6) is reduced to the form

$$\frac{\partial T}{\partial F_0}=e^{\omega\rho}\left[\frac{\partial^2 T}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial T}{\partial\rho}+\frac{\omega\partial T}{\partial\rho}\right]. \tag{8}$$

Assume

$$\lim_{F_0\to\infty}\varphi(F_0)=T_c=const, \tag{9}$$

then stationary condition onsets in the cylinder, and $\frac{\partial T}{\partial F_0}=0$. Integrating the equation

$$e^{\omega\rho}\left(\frac{\partial^2 T}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial T}{\partial\rho}+\frac{\omega\partial T}{\partial\rho}\right)=0,$$

we get

$$\rho\frac{\partial T}{\partial\rho}=Ce^{-\omega\rho}.$$

As $\left(\rho\frac{\partial T}{\partial\rho}\right)_{\rho=0}=0$, then $C=0$. Therefore, for $(T)_{\rho=1}=T_c=const$ the solution of the equation will be $T(\rho)=T_0$, i.e. under stationary condition, as in the case of homogeneous cylinder, uniform temperature is set up in the entire inhomogeneous cylinder. Subject to condition (9), we look for the solution of problem (8), (7) in the area of transform [1]

$$T_n^*(\rho,S)=\varphi^*(S)+\sum_{i=1}^{n}a_i^*(S)\left(1-\rho^2\right)\rho^{2(i-1)}.$$

Let us compose discrepancy at $T^*(\rho,S)=T_1^*(\rho,S)$ for the equation

$$e^{\omega\rho}\left[\frac{d}{d\rho}\left(\rho\frac{dT^*}{d\rho}\right)+\omega\rho\frac{dT^*}{d\rho}\right]-S\rho T^*(\rho,S)+T_0\rho=0$$

and require orthogonality of the obtained expression to the function $\psi_1(\rho)=\left(1-\rho^2\right)$, then we have

$$\int_0^1\left\{2e^{\omega\rho}\left(2\rho+\omega\rho^2\right)\left(1-\rho^2\right)+S\left(1-\rho^2\right)^2\rho\right\}d\rho$$

$$-\left[T_0-S\varphi^*(S)\right]\int_0^1\left(1-\rho^2\right)\rho d\rho=0,$$

whence

$$a_1^*(S) = \frac{3}{2}\left[T_0 - S\varphi^*(S)\right]\left[S + A(\omega)\right]^{-1},$$

where

$$A(\omega) = \frac{12\left[e^{\omega}\left(2\omega^3 - 6\omega^2 + 12\omega - 12\right) + 12\right]}{\omega^4}.$$

For $\varphi(F_0) = T_c = const$ the relative excess temperature in the first approximation

$$\theta(\rho, F_0, \omega) = \frac{T(\rho, F_0) - T_c}{T_0 - T_c} = \frac{3}{2}\left(1 - \rho^2\right)\exp\left[-A(\omega)F_0\right].$$

Relative excess temperature interior to the cylinder under exponential temperature fall of the wall

$$\varphi(F_0) = T_c + (T_0 - T_c)\exp(-PdF_0)$$

in the first approximation is written by the dependence

$$\theta(\rho, F_0, Pd, \omega) = \frac{T(\rho, F_0) - T_c}{T_0 - T_c}$$

$$= \exp(-PdF_0) - \frac{3}{2}\left\{\exp\left[-A(\omega)F_0\right]\right.$$

$$\left. - \frac{1}{\left[Pd - A(\omega)\right]}\left[Pd\exp(-PdF_0) - A(\omega)\exp(-A(\omega)F_0)\right]\right\}\left(1 - \rho^2\right).$$

When condition (9) is violated, for improving the convergence of the solution the system of coordinate functions should be chosen as follows. As a first coordinate function to within constant factor, the solution of equation (8) for the period of quasi-stationary condition is taken. For example, at linear temperature rise at the boundary (8) into equation $[\varphi(F_0) = T_0(1 + PdF_0)]$ we put (8) $\frac{\partial T}{\partial F_0} = PdT_0$. Then for quasi-stationary condition, the solution of boundary value problem (8), (7) is written in the form

$$T(\rho, F_0, Pd, \omega) = T_0(1 - PdF_0)$$

$$-\frac{PdT_0}{2}\left[e^{-\omega}\left(\frac{1}{\omega^2} + \frac{1}{\omega}\right) - \left(\frac{\rho}{\omega} + \frac{1}{\omega^2}\right)e^{-\omega\rho}\right]. \quad (10)$$

Thus, we look for temperature field interior to the cylinder under linear temperature rise of the surface, in the family of functions of the form

$$T_n^*(\rho, S, \omega) = \varphi^*(S) + a_1^*(S)\left[e^{-\omega}\left(\frac{1}{\omega^2} + \frac{1}{\omega}\right) - \left(\frac{\rho}{\omega} + \frac{1}{\omega^2}\right)e^{-\omega\rho}\right]$$

$$+\sum_{i=2}^{n} a_i^*(S)\left(1-\rho^{2i}\right). \tag{11}$$

For simplicity, we are restricted with definition of the solution in the first approximation. Then we have:

$$a_1^*(S)\left[\frac{e^{-\omega}\left(\omega^3+2\omega^2+6\omega+6\right)-6}{\omega^4}\right.$$

$$\left.+S\frac{e^{-2\omega}\left(4\omega^4+20\omega^3+54\omega^2+78\omega+39\right)-48(\omega+1)e^{-\omega}+9}{8\omega^6}\right]$$

$$=\frac{T_0 Pd}{S}\left[\frac{6-e^{-\omega}\left(\omega^3+2\omega^2+6\omega+6\right)}{\omega^4}\right].$$

Whence

$$a_1^*(S)=\frac{PdT_0}{2}\left[\frac{1}{S}+\frac{1}{S+D(\omega)}\right]$$

where

$$D(\omega)$$

$$=\frac{8\omega^2\left[e^{-\omega}\left(\omega^3+3\omega^2+6\omega\right)-6\right]}{e^{-2\omega}\left(4\omega^4+20\omega^3+54\omega^2+78\omega+39\right)-48(\omega+1)e^{-\omega}+9}.$$

The relative excess temperature is written by the dependence

$$\theta(\rho,F_0,Pd,\omega)=\frac{T_0(\rho,F_0)-T_0}{T_0}$$

$$=Pd\left\{F_0-\frac{1}{2}\left[1-\exp(-D(\omega))F_0\right]\left[e^{-\omega}\left(\frac{1}{\omega^2}+\frac{1}{\omega}\right)-\left(\frac{\rho}{\omega}+\frac{1}{\omega^2}\right)e^{-\omega\rho}\right]\right\}. \tag{12}$$

For rather great F_0, this solution coincides with formula (10). Calculate the limits by L'Hospital's rule, then

$$\lim_{\omega\to 0} D(\omega)=6,\quad \lim_{\omega\to 0}\left[e^{-\omega}\left(\frac{1+\omega}{\omega^2}\right)-e^{-\omega\rho}\left(\frac{\rho\omega+1}{\omega^2}\right)\right]=\frac{1}{2}\left(1-\rho^2\right).$$

From (12) in the limit as $\omega\to 0$ we get appropriate solution for a cylinder with constant heat conduction coefficient $\left(\lim\limits_{\omega\to 0}\lambda_0 e^{m\rho}=\lambda_0\right)$:

$$\theta(\rho,F_0,Pd)=\left\{F_0-\frac{1}{4}\left(1-\rho^2\right)\left[1-\exp(-6F_0)\right]\right\}Pd.$$

In relation (11), for any $n \geq 2$ the following limit properties

$$\lim_{S \to 0} S a_1^* (S) = \lim_{F_0 \to \infty} a_1 (F_0) = -\frac{T_0 P d}{2},$$

$$\lim_{S \to 0} S a_i^* (S) = \lim_{F_0 \to \infty} a_i (F_0) = 0 \, (i \geq 2)$$

are fulfilled, i.e. in the second, third and in the next approximations, as $F_0 \to \infty$ the solution has an asymptotic coinciding with exact solution of quasi-stationary condition.

To compare the approximate solution obtained by the above mentioned methods with exact solution, we give the results of temperature calculation interior to the homogeneous ball under first kind boundary conditions.

Relative excess temperature in the n-th approximation is given by the formula [1]

$$\theta_n (\rho, F_0) = \frac{T (\rho, F_0) - T_C}{T_0 - T_C} = \sum_{i=1}^{n} (-1)^{i+1} f_i^{(n)} \exp\left(-S_i^{(n)} F_0\right), \quad (13)$$

where $f_i^{(n)} (\rho)$ are fourth degree polynomials of order $2n$. The results of calculations of eigenvalues to the fifth approximation and the functions $f_i^{(n)} (\rho)$ for $n = 4$ are given in table 1. Good convergence of eigenvalues to exact ones is seen from table 1. The polynomial $f_1^{(4)} (\rho)$ in actual fact coincides with the first eigenfunction $(2 \sin \pi\rho) / \pi\rho$ in the exact solution [3].

Table 1.

Approximate eigenvalues of functions

n						
	$S_i^{(n)}$					exact values
i	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	
1	2	3	4	5	6	7
$i=1$	$10,5$	$9,8751$	$9,8696$	$9,8696$	$9,8696$	$9,8696$
$i=2$		$50,1246$	$39,9978$	$39,4893$	$39,4784$	$39,4784$
$i=3$			$142,6322$	$94,1187$	$88,8825$	$88,8264$
$i=4$				$324,6256$	$194,4327$	$157,9137$
$i=5$					$515,2573$	$246,7401$
$f_1^{(4)} = 1,9999 - 3,2888\rho^2 + 1,6178\rho^4 - 0,3691\rho^6 + 0,0402\rho^8$						
$f_2^{(4)} = 1,9661 - 12,5072\rho^2 + 22,4466\rho^4 - 16,2848\rho^6 + 1,3799\rho^8$						
$f_4^{(4)} = 1,6381 - 19,6264\rho^2 + 60,7957\rho^4 - 68,5992\rho^6 + 25,791\rho^8$						
$f_1^{(4)} = 1,1642 - 18,9394\rho^2 + 78,3586\rho^4 - 114,8404\rho^6 + 54,2570\rho^8$						

Multiply (13) by $3\rho^2$ and integrate from 0 to 1, then

$$\overline{\theta}_4 (F_0) = 0,6079 \exp(-9,8696 F_0) + 0,1523 \exp(-39,4893 F_0)$$

$$+0,0855\exp\left(-94,1187F_0\right)+0,0999\exp\left(-157,9137F_0\right). \qquad (14)$$

Integration of four summands in the exact solution gives

$$\overline{\theta}_4\left(F_0\right)=0,6079\exp\left(-9,8696F_0\right)+0,1520\exp\left(-39,4784F_0\right)$$

$$+0,0675\exp\left(-88,8264F_0\right)+0,0038\exp\left(-157,9137F_0\right). \qquad (15)$$

Obviously, the exact solution $\overline{\theta}\left(0\right)=1$. From (14), (15) we have $\overline{\theta}_4\left(0\right)=0,9454$, $\overline{\theta}_4=0,8388$. Maximum errors in solutions (14), (15) are attained for $F_0=0$ and they equal $5,56$; $15,88\%$. On the whole, solution (14) gives best convergence than (15).

Comparison of $\theta_4\left(F_0\right)$ with the graph of exact solution is given in fig 1. The results for other bodies were obtained in the paper [5].

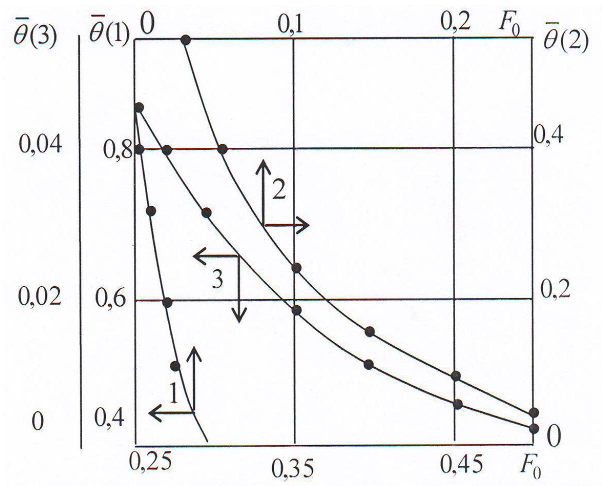

Figure 0.1:

Fig. 1. Dependence between average relative temperature $\overline{\theta}$ and the number F_0 for bodies, the points-calculation

Under symmetric power law of change of heat capacity and heat conductivity coefficients $c\rho=c_0\rho_0\left(1+\omega\left|\xi\right|\right)^m$, $\lambda=\lambda_0\left(1+\omega\left|\xi\right|\right)^n$, $-1\leq\xi=\frac{x}{R}\leq1$ and symmetric boundary conditions (2) the temperature field interior to $\Omega\left\{-1\leq\xi\leq1\right\}$ will be an even function of variable ξ. Therefore, for an unbounded plate, the heat-conduction equation is reduced to the form

$$\left(1+\xi\omega\right)^m\frac{\partial T}{\partial F_0}=\frac{\partial}{\partial\xi}\left[\left(1+\xi\omega\right)^n\frac{\partial T}{\partial\xi}\right],\quad 0\leq\xi\leq1. \qquad (16)$$

As the derivative of the function $\lambda = \lambda_0 (1 + \omega |\xi|)^2$ bears discontinuity at the point $\xi = 0$, then even under symmetric distribution of temperature along the plate's thickness the derivative $\partial T/\partial \xi$ in the middle of the plate ($\xi = 0$) is a discontinuous function and the case $(\partial T/\partial \xi)_{\xi=0} \neq 0$ is possible.Therefore, consideration of the problem in the interval $0 \leq \xi \leq 1$ takes the discontinuity point $\xi = 0$ to the end of the interval at the inside of which temperature and its derivative are already continuous functions. Herewith the condition $(\partial T/\partial \xi)_{\xi=0} = 0$ that is necessary in the case of constant heat conduction coefficient need not hold.

Assume that for rather great F_0 the temperature of the wall is equivalent to the linear function, i.e.

$$\lim_{F_0 \to \infty} \frac{\varphi (F_0)}{F_0} = const, \tag{17}$$

then temperature interior to the plate for the period of quasi-stationary condition has asymptotic solution.

In equation (16) put $\partial T/\partial F_0 = T_0 Pd$, then the solution under zero boundary conditions at the point $\xi = 1$ will be

$$T (\xi, \omega, m, n, Pd) = \frac{PdT_0}{\omega^2 (m+1)} \left\{ \left[\frac{(1+\omega\xi)^{m-n+2}}{(m-n+2)} - \frac{(1+\omega\xi)^{1-n}}{(1-n)} \right] - \left[\frac{(1+\omega)^{m-n+2}}{(m-n+2)} - \frac{(1+\omega)^{1-n}}{(1-n)} \right] \right\}.$$

Subject to condition (17) we look for the solution of problem (16), (2) in the area of Laplace transform, in the form

$$T_n^* (\xi, S, Pd) = \varphi^* (S) + a_1^* (S) \left\{ \left[\frac{(1+\omega\xi)^{m-n+2}}{(m-n+2)} - \frac{(1+\omega\xi)^{1-n}}{(1-n)} \right] - \left[\frac{(1+\omega)^{m-n+2}}{(m-n+2)} - \frac{(1+\omega)^{1-n}}{(1-n)} \right] \right\} + \sum_{i=2}^{n} a_i^* (S) \left(1 - \xi^2\right) \xi^{2(i-1)}. \tag{18}$$

Having determined the coefficients $a_i^* (S)$ and passing to the domain of pre-images, we find the solution of the input problem. Under the shown selection of coordinates of functions, the following limit equalities are valid:

$$\lim_{S \to 0} S a_1^* (S) = \lim_{F_0 \to \infty} a_1 (F_0) = \frac{PdT_0}{\omega^2 (m+1)},$$

$$\lim_{S \to 0} S a_i^* (S) = \lim_{F_0 \to \infty} a_i (F_0) = 0 \, (i \leq 2) .$$

If the wall's temperature satisfies the condition $\lim\limits_{F_0\to\infty} a_i\left(F_0\right) = T_0$, then from (18) in the area of pre-images for the quasi-stationary condition $(F_0 > F_{0_1})$ we get the exact solution [1]

$$T\left(\xi, F_0, Pd, \omega\right) = T_0\left(1 + PdF_0\right)\frac{PdT_0}{\omega^2\left(m+1\right)}$$
$$\times\left\{\left[\frac{\left(1+\omega\xi\right)^{m-n+2}}{(m-n+2)} - \frac{\left(1+\omega\xi\right)^{1-n}}{(1-n)}\right] - \left[\frac{\left(1+\omega\right)^{m-n+2}}{(m-n+2)} - \frac{\left(1+\omega\right)^{1-n}}{(1-n)}\right]\right\}.$$

note that

$$\lim_{\omega\to 0}\frac{1}{\omega^2}\left\{\left[\frac{\left(1+\omega\xi\right)^{m-n+2}}{(m-n+2)} - \frac{\left(1+\omega\xi\right)^{1-n}}{(1-n)}\right]\right.$$
$$\left. - \left[\frac{\left(1+\omega\right)^{m-n+2}}{(m-n+2)} - \frac{\left(1+\omega\right)^{1-n}}{(1-n)}\right]\right\} = -\frac{(m+1)}{2}\left(1-\xi^2\right)$$

and in the first approximation the temperature field (18) in the are of pre-images coincides as $\omega \to 0$ with solution (5).

Thus, the stated result allows to study temperature field in one-dimensional and many-dimensional bodies under variable transfer coefficients

If in the heat-conduction equation instead of variable F_0 we put $X = \frac{1}{Pe}\frac{z}{R}$, then by the given method we can get effective solutions for internal problems of convective heat exchange under turbulent flow of medium in pipes and canals.

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