ABOUT SOME EXACT SOLUTIONS OF NONLINEAR SYSTEM OF THE DIFFERENTIAL EQUATIONS DESCRIBING THREE-PARTY ELECTIONS

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Abstract

In the paper nonlinear mathematical model describing dynamics of voters pro-governmental and two opposition parties (three selective subjects, the coalitions) is offered. In model four objects: governmental and administrative structures influencing by means of administrative resources citizens (first of all in opposition adjusted voters) for the purpose of their attraction on the party of pro-governmental party are considered; citizens with the selective voice, at present supporting the first opposition party; citizens with the selective voice, at present supporting the second opposition party; citizens with the selective voice, at present supporting pro-governmental party. In some special cases of constancy or variability of coefficients of model, exact analytical solutions are found and theorems which show conditions under which the pro-government party will lose the next elections are proved.

Key words and phrases: Nonlinear mathematical model; pro-governmental party; administrative resources; two opposition parties; elections; exact analytical solutions; theorems.

AMS subject classification: 45K05, 65M06.

1 Introduction

Mathematical modeling and computing experiment in the last decades gained comprehensive recognition in science as the new methodology which is roughly developing and widely introduced not only in natural-science and technological spheres, but also in economy, sociology, history, political science and other public disciplines [1, 2].

In [3, 4] quantities of information streams by means of new mathematical models of information warfare are studied. By information warfare the authors mean an antagonism by means of mass media (an electronic and printing press, the Internet) between the two states or the two associations of states, or the economic structures (consortiums) conducting purposeful misinformation, propagation against each other. It was shown that in case of high aggression of the contradictory countries, not preventive image the operating peacekeeping organizations won't be able to extinguish the expanding information warfare.

In [5] linear and nonlinear mathematical models of information warfare, and also optimizing problems are considered.

The article [6] concerns of Chilker task is entered refers to the boundary value problem for a system of ordinary differential equations and optimal control problem. In Chilker tasks right boundary conditions are set in different, uncommitted time points for different coordinates of the unknown vector – functions. Proposed methods solutions of Chilker tasks.

In [7] the new nonlinear mathematical and computer model of information warfare with participation of interstate authoritative institutes is offered. The model is described by Cauchy's problem for nonlinear nonhomogeneous system of the differential equations. Confronting sides in extend of provocative statements, the third side (the peacekeeping international organizations) extends of soothing statements, interstate authoritative institutes the peacekeeping statements call the sides for the termination of information warfare. In that specific case, modes of information warfare "aggressor-victim", for the third peacekeeping side are received exact analytical solutions, and functions defining number of the provocative statements distributed by the antagonistic sides satisfy to Cauchy's problems for Riccati certain equations which are solved by a numerical method. For the general model computer modeling is carried out and shown that irrespective of high aggression of confronting sides, interstate authoritative institutes will be able to extinguish information warfare and when for this purpose efforts of only the international organizations insufficiently.

In works [8 - 10] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

These papers [11] present the nonlinear mathematical model of the public or the administrative management (or the macro and micro model). The cases of both constant and variable pressure forces on freethinking people were analyzed. Exact analytical decisions which determine dynamics of a spirit both free-thinking people, and operated (conformists) of people by time are received. During this analysis various governance systems were considered: a liberal, democratic, semi dictatorial and dictatorial.

In [12] consider the nonlinear mathematical model of bilateral assimilation without demographic factor. It was shown that the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people.

In [13] mathematical modeling of nonlinear process of assimilation taking into account demographic factor is offered. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed.

In [14] mathematical modeling of nonlinear process of the assimilation taking into account positive demographic factor which underwent bilateral assimilation of the side and zero demographic factor of the assimilating sides is considered.

Considerable interest represents creation of the mathematical model, allowing to define dynamics of voters of political subjects. Elections can be divided into two parts: the two-party and multi-party elections.

These works [15 - 19] considered a two or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections when coefficients are constant. The assumption was made that the number of voters remain the same between two consecutive elections (zero demographic factor of voters). The exact analytical solutions were received. The conditions under which opposition party can win the upcoming elections were established.

These works [20, 21] considered a two-party (pro-government and opposition parties) nonlinear mathematical model of elections with variable coefficients.

In [22] the nonlinear mathematical model with variable coefficients was proposed in the case of three-party elections, that describes the dynamics of the quantitative change of the votes of the pro-government and two opposition parties from election to election.

The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account. The model considered the cases with variable coefficients. In the particular case exact analytical solutions was obtained. The conditions have been identified under which the opposition can win the forthcoming elections, and in some cases, the pro-government party can stay in power. In general the Cauchy problem numerically using the MATLAB software package was solved.

In this publication the nonlinear mathematical model with variable coefficients in case of three-party elections which describes dynamics of quantitative change of votes of pro-government and two oppositional parties is presented. The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account.

In model four objects are considered:

1. The state and administrative structures that utilize state resources in order to have an influence on the pro-oppositions voters with the aim to gain their support for the pro-government party.

2. Voters who support first opposition party.

3. Voters who support second opposition party.

4. Voters who support the pro-government party.

In model it is possible to consider various indicators of a turnout of voters in the election day, and also a possibility of falsification in favor of pro-government party.

2 System of the equations and initial conditions

For dynamics description between elections of voters pro-governmental and two opposition parties (three selective subjects) we offer the following nonlinear mathematical model:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_2(t))N_1(t)\,N_2(t) \\ +(\alpha_1(t) - \alpha_3(t))N_1(t)\,N_3(t) - f_1(t, N_1(t)), \\ \\ \dfrac{dN_2(t)}{dt} = (\alpha_2(t) - \alpha_1(t))N_1(t)\,N_2(t) \\ +(\alpha_2(t) - \alpha_3(t))N_2(t)\,N_3(t) - f_2(t, N_2(t)), \\ \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t) - \alpha_1(t))N_1(t)\,N_3(t) \\ +(\alpha_3(t) - \alpha_2(t))N_2(t)\,N_3(t) + f_3(t, N_1(t), N_2(t)), \end{cases} \tag{2.1}$$

$$N_1(0) = N_{10}, \quad N_2(0) = N_{20}, \quad N_3(0) = N_{30}, \quad N_{10} + N_{20} < N_{30}. \tag{2.2}$$

where $N_1(t), N_2(t)$ - according to number of the voters supporting the first and second opposition parties in a time t, and $N_3(t)$ - number of the voters supporting pro-governmental party in a time t, and $t \in [0, T]$, $t = 0$ - time of the last elections owing to which the party won elections and became pro-governmental ($N_{10} + N_{20} < N_{30}$); $t = T$ - the moment of the following, for example, parliamentary elections (as a rule $T = 4$ years or 1460 days if t changes on days); $\alpha_1(t)$, $\alpha_2(t)$, $\alpha_3(t)$ - respectively coefficients of attraction of votes of the first and the second oppositional, and also pro-governmental parties in the time t connected with the action program, financial and information opportunities (PR technology) of these parties; $\alpha_1(t)$, $\alpha_2(t)$, $\alpha_3(t) \in C[0, T]$; $f_i(t, N_i(t))$, $i = 1, 2$, $f_3(t, N_1(t), N_2(t))$ - the continuous, positive functions of the arguments characterizing use of the state and administrative resources, the opposition parties directed on voters, for the purpose of their attraction on the party and power preservation that is the purpose of any authorities in power.

In model (2.1), (2.2) it is supposed that total of voters (from elections to elections doesn't change ($N_{10} + N_{20} + N_{30} = N_0$, $f_3(t, N_1(t), N_2(t)) = f_1(t, N_1(t)) + f_2(t, N_2(t))$ (often, in many countries, their change is insignificant in comparison with a total number of voters). Thus, we consider that in a period between elections ($t \in [0, T]$) the number of the died voters and the voters who for the first time have acquired a vote are equal in many countries of 18 years.

In model it is supposed that k_1, k_2, k_3 parts of the voters supporting by the time of elections ($t = T$) the relevant parties came to the election day to polling stations. As a rule, depending on elections and the countries $k_i \in (0,2;\, 0,8)$, $i = \overline{1,\, 3}.$ In model also falsifications of the received votes of opposition parties can be considered ($0 < k_j < 1$; $j = 4; 5$). It is clear, that the falsification indicators are higher, the elections are less democratic and the state authorities are illegitimate.

Two cases are considered:

1. $\alpha_1(t) = \alpha_1 = const > 0$, $\alpha_2(t) = \alpha_2 = const > 0$, $\alpha_3(t) = \alpha_2 = const > 0$, $f_1(t, N_1(t)) = d_1 N_1(t)$, $f_2(t, N_2(t)) = d_2 N_2(t)$, $f_3(t, N_1(t), N_2(t)) = d_1 N_1(t) + d_2 N_2(t)$, $d_1, d_2 > 0, d_1 \neq d_2$ - various nature of influence of the government and administrative institutions on voters of opposition parties (in proportion to a number of voters of opposition parties).

2. $\alpha_1(t) > 0$, $\alpha_2(t) > 0$, $\alpha_3(t) > 0$, $f_1(t, N_1(t)) = d_1(t) N_1(t)$, $f_2(t, N_2(t)) = d_2(t) N_2(t)$, $f_3(t, N_1(t), N_2(t)) = d_1(t) N_1(t) + d_2(t) N_2(t), d_1(t) > 0$, $d_2(t) > 0$, $t \in [0, T]$ - the general case with variable coefficients of model.

2 Various nature of influence of the government and administrative institutions on voters of opposition parties

In the first case the system of equations (2.1) has an appearance

$$\begin{cases} \dfrac{dN_1\,(t)}{dt} = (\alpha_1-\alpha_2)N_1\,(t)\,N_2\,(t) + (\alpha_1-\alpha_3)N_1\,(t)\,N_3\,(t) \\ -d_1N_1(t), \\ \\ \dfrac{dN_2\,(t)}{dt} = (\alpha_2-\alpha_1)N_1\,(t)\,N_2\,(t) + (\alpha_2-\alpha_3)N_2\,(t)\,N_3\,(t) \\ -d_2N_2(t), \\ \\ \dfrac{dN_3\,(t)}{dt} = (\alpha_3-\alpha_1)N_1\,(t)\,N_3\,(t) + (\alpha_3-\alpha_2)N_2\,(t)\,N_3\,(t) \\ +d_1N_1(t) + d_2N_2(t). \end{cases} \tag{2.1}$$

Putting the equations of system (2.1) it is easy to obtain its first integral

$$N_1(t) + N_2(t) + N_3(t) = N_{10} + N_{20} + N_{30} = N_0, \quad t \in [0,T]. \tag{2.2}$$

We will enter designations

$$\alpha_1-\alpha_2 = a, \quad \alpha_1-\alpha_3 = b, \quad \alpha_2-\alpha_3 = c. \tag{2.3}$$

Then (2.1), taking into account (2.2), (2.3) will correspond in the following look:

$$\begin{cases} \dfrac{dN_1\,(t)}{dt} = N_1\,(t)\,[-bN_1\,(t) - cN_2(t) + bN_0 - d_1], \\ \dfrac{dN_2\,(t)}{dt} = N_2\,(t)\,[-bN_1\,(t) - cN_2(t) + cN_0 - d_2], \\ \dfrac{dN_3\,(t)}{dt} = -bN_1\,(t)\,N_3\,(t) - cN_2\,(t)\,N_3\,(t) + d_1N_1(t) + d_2N_2(t). \end{cases} \tag{2.4}$$

We will consider a special case

$$bN_0 - d_1 = cN_0 - d_2, \tag{2.5}$$

or it agrees (2.3)

$$N_0 = d_1 - d_2. \tag{2.6}$$

Then from system (2.4) it is easy to obtain its second first integral

$$N_1(t) = qN_2(t), \quad q = \frac{N_{10}}{N_{20}} > 0. \tag{2.7}$$

From (2.2), (2.7) we will have

$$N_2(t)=\frac{N_0-N_3(t)}{q+1},\quad N_1(t)=\frac{q}{q+1}(N_0-N_3(t)). \tag{2.8}$$

Then from the third equation of system (2.4), taking into account (2.8), it is easy to receive

$$\frac{dN_3\left(t\right)}{dt}=\frac{1}{q+1}(N_0-N_3(t))\left[d_1q+d_2-(bq+c)N_3(t)\right]. \tag{2.9}$$

We will consider various cases

2.1. $b\le 0,\quad c\le 0$. Then it agrees (2.9)

$$\frac{dN_3(t)}{dt}>0,\quad t\in[0,T]$$

and the pro-governmental party will win elections.

2.2. $bc<0,\quad \left\{\begin{array}{l} b>0\\ c<0\\ a>0\end{array}\right. \quad or \quad \left\{\begin{array}{l} b<0\\ c>0\\ a<0\end{array}\right.$

2.2.1. $bq+c=0$.

Then (2.9) has an appearance

$$\frac{dN_3\left(t\right)}{dt}=\frac{d_1q+d_2}{q+1}(N_0-N_3(t)). \tag{2.10}$$

The exact decision (2.10), taking into account an entry condition (2.2) will have in the following form:

$$N_3(t)=N_0-(N_0-N_{30})e^{-\frac{d_1q+d_2}{q+1}t}. \tag{2.11}$$

From (2.10) it is easy to notice that

$$\frac{dN_3(t)}{dt}>0$$

and the pro-governmental party will win elections.

2.2.2. $bq+c\neq 0$.

Then (2.9) it is possible to copy in a look

$$\frac{dN_3\left(t\right)}{dt}=\frac{bq+c}{q+1}(N_0-N_3(t))(p-N_3(t)), \tag{2.12}$$

$$p\equiv\frac{d_1q+d_2}{bq+c}.$$

2.2.3 $0 < p = N_0, \quad bq + c > 0,$

$$N_3(t) = \frac{N_0(bq+c)(N_0 - N_{30})t + (q+1)N_{30}}{(q+1) + (bq+c)(N_0 - N_{30})t},$$

then from (2.9) we will obtain $\frac{dN_3(t)}{dt} > 0$, as the pro-governmental party will win elections.

2.2.4. $p \neq N_0$.

The exact decision (2.9) has the form

$$N_3(t) = \frac{p(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})N_0}{(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30}.)} \tag{2.13}$$

2.2.5. $p \geq N_0$, then from (2.9) we will receive $\frac{dN_3(t)}{dt} > 0$, as the pro-governmental party will win elections.

2.2.6. $0 < p < N_0, \quad p \equiv \frac{d_1 q + d_2}{bq+c}, \quad bq + c > 0.$

The exact decision (2.9) has the form

$$N_3(t) = \frac{p(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})N_0}{(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})}. \tag{2.14}$$

2.3. $b > 0, c > 0, \quad bq + c > 0,$

$$0 < p < N_0.$$

The exact decision (2.9) has the form

$$N_3(t) = \frac{p(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})N_0}{(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})}, \tag{2.15}$$

$$N_3(t) = \frac{p(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})N_0}{(N_0 - N_{30}) \exp\left[\frac{(N_0-p)(bq+c)}{q+1}t\right] - (p - N_{30})} \leq \frac{N_0}{2}. \tag{2.16}$$

Theorem 2.1. *If inequalities are executed*

$$bq + c > 0, \quad 0 < p < N_0/2,$$

$$t_1 = \frac{q+1}{(N_0 - p)(bq+c)} \ln \frac{(N_{30} - p)N_0}{(N_0 - N_{30})(N_0 - 2p)} < T$$

then the inequality takes place

$$t_1 < t \leq T \quad N_3(t) < N_0/2.$$

Proof. We will enter designation

$$f(t) \equiv (N_0 - N_{30}) \exp\left[\frac{(N_0 - p)(bq + c)}{q + 1} t\right] - (p - N_{30})$$

It is clear, that

$$f(0) \equiv N_0 - p > 0,$$

$$\frac{df}{dt} = (N_0 - N_{30}) \frac{(N_0 - p)(bq + c)}{q + 1} \exp\left[\frac{(N_0 - p)(bq + c)}{q + 1} t\right] > 0,$$

$$f(t) \in C[0, T],$$

therefore,

$$(N_0 - N_{30}) \exp\left[\frac{(N_0 - p)(bq + c)}{q + 1} t\right] - (p - N_{30}) > 0, \forall t \in [0, T].$$

And the solution of an inequality (2.16) has the form

$$0 \le t \le t_1 = \frac{q + 1}{(N_0 - p)(bq + c)} \ln \frac{(N_{30} - p)N_0}{(N_0 - N_{30})(N_0 - 2p)},$$
$$N_3(t) \ge N_0/2, \tag{2.17}$$
$$t_1 < t \le T \quad N_3(t) < N_0/2$$

and by that the Theorem 2.1 is proved.

Thus it is proved that at

$$bq + c > 0, \quad 0 < p < N_0/2$$

then the pro-governmental party will lose elections ($N_3(T) < N_1(T) + N_2(T)$; $k_i = 1, i = \overline{1, 3}$).

2.3.2. $p = \frac{N_0}{2}$.

Then inequality (2.16) has no decision.

2.3.3. $\frac{N_0}{2} < p < N_{30}$ then pro-governmental party will win elections.

2.3.4. $N_{30} \le p < N_0$.

$N_3(t) = N_{30}, \quad \forall t \in [0, T)$ and the pro-governmental party will win elections.

3 The general case with variable coefficients of model.

We will consider the general case when all coefficients of system of the differential equations (2.1) are various and are continuous functions of time.

Thus, we will enter designations:

$$\begin{aligned} &\alpha_1(t) - \alpha_2(t) \equiv a(t), \quad \alpha_1(t) - \alpha_3(t) \equiv b(t), \\ &\alpha_2(t) - \alpha_3(t) \equiv c(t), \\ &b(t) = a(t) + c(t) \quad \forall t \in [0, T], \\ &d_1(t) > 0, d_2(t) > 0, \quad \forall t \in [0, T]. \end{aligned} \tag{3.1}$$

Taking into account (3.1) system of the equations (2.1) will correspond in the following look:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = N_1(t)\left[-b(t)N_1(t) - c(t)N_2(t) + b(t)N_0 - d_1(t)\right], \\ \dfrac{dN_2(t)}{dt} = N_2(t)\left[-b(t)N_1(t) - c(t)N_2(t) + c(t)N_0 - d_2(t)\right], \\ \dfrac{dN_3(t)}{dt} = -b(t)N_1(t)N_3(t) - c(t)N_2(t)N_3(t) + d_1(t)N_1(t) \\ +d_2(t)N_2(t). \end{cases} \tag{3.2}$$

We will consider a special case

$$b(t)N_0 - d_1(t) = c(t)N_0 - d_2(t)$$

or

$$a(t)N_0 = d_1(t) - d_2(t), \quad \forall t \in [0, T]. \tag{3.3}$$

Then the two first integrals of system of the differential equations (3.2), also have an appearance (2.2), (2.7), ratios (2.8) are respectively right.

Then from the third equation of system (3.2), taking into account (2.8), it is easy to receive Cauchy's task:

$$\begin{aligned} &\frac{dN_3(t)}{dt} = \frac{N_0 - N_3(t)}{q+1}\left[q d_1(t) + d_2(t) - (q b(t) + c(t))N_3(t)\right], \\ &N_3(0) = N_{30}. \end{aligned} \tag{3.4}$$

We will consider cases:

a.1 $qb(t) + c(t) = 0, \quad \forall t \in [0, T]$

$$a(t) \neq 0, \quad sgn(b(t)) = -sgn(c(t)) \tag{3.5}$$

i.e. in each time t the coefficient of attraction of votes of pro-governmental party is more than greatest attraction of votes of the first and second opposition parties smallest less coefficients.

$$\min(\alpha_1(t), \alpha_2(t)) < \alpha_3(t) < \max(\alpha_1(t), \alpha_2(t)), \quad t \in [0, T]. \tag{3.6}$$

Then the only solution of a task (3.4) has the form:

$$N_3(t) = N_0 - (N_0 - N_{30})e^{-\frac{\int_0^t (qd_1(\tau)+d_2(\tau))d\tau}{q+1}}. \tag{3.7}$$

It is clear, that from (3.4) in a case a.1 it is easy to notice that

$$\frac{dN_3(t)}{dt} > 0, \quad \forall t \in [0, T),$$

therefore, the pro-governmental party will win elections.
b.1 $qb(t) + c(t) \neq 0, \quad \forall t \in [0, T]$
b.1.1 $qb(t) + c(t) < 0, \quad \forall t \in [0, T]$
It is clear, that from (3.4) in case of b.1 it is easy to notice that

$$\frac{dN_3(t)}{dt} > 0, \quad \forall t \in [0, T],$$

therefore, the pro-governmental party will win elections.
b.1.2 $qb(t) + c(t) > 0, \forall t \in [0, T]$
We will assume that equality takes place

$$qd_1(t) + d_2(t) = p_1[qb(t) + c(t)], \quad p_1 = const > 0. \tag{3.8}$$

Then from (3.4), (3.8) we will receive

$$\frac{dN_3(t)}{dt} = \frac{N_0 - N_3(t)}{q+1}[qb(t) + c(t)][p_1 - N_3(t)], N_3(0) = N_{30}, \tag{3.9}$$

$$sgn\frac{dN_3(t)}{dt} = sgn[p_1 - N_3(t)], \quad \forall t \in [0, T]. \tag{3.10}$$

b.1.2.1 $p_1 \geq N_0$.
Then from (3.10) it is easy to receive

$$\frac{dN_3(t)}{dt} > 0, \quad \forall t \in [0, T]$$

the pro-governmental party will win elections.
b1.2.2 $0 < p_1 < N_0$ then the decision (3.9) has the form

$$N_3(t) = \frac{p_1(N_0 - N_{30}) \exp\left[\frac{(N_0-p_1)\int\limits_0^t (qb(\tau)+c(\tau))d\tau}{q+1}\right] - (p_1 - N_{30})N_0}{(N_0 - N_{30}) \exp\left[\frac{(N_0-p_1)\int\limits_0^t (qb(\tau)+c(\tau))d\tau}{q+1}\right] - (p_1 - N_{30})} \tag{3.11}$$

Theorem 3.1. *If inequalities are executed*

$$b(t)q + c(t) > 0, \quad \forall t \in [0, T], \quad 0 < p_1 < N_0/2$$

and in an interval $t_2 \in (0, T)$ there is a solution of the equation

$$\int\limits_0^{t_2} (qb(\tau) + c(\tau))d\tau = \frac{q+1}{(N_0 - p_1)} \ln \frac{(N_{30} - p_1)N_0}{(N_0 - N_{30})(N_0 - 2p_1)},$$

and also inequalities are carried out

$$0 < \int\limits_0^t (qb(\tau) + c(\tau))d\tau \le \frac{q+1}{(N_0 - p_1)} \ln \frac{(N_{30} - p_1)N_0}{(N_0 - N_{30})(N_0 - 2p_1)}, \quad t \in (0, t_2),$$

$$\int\limits_0^t (qb(\tau) + c(\tau))d\tau \ge \frac{q+1}{(N_0 - p_1)} \ln \frac{(N_{30} - p_1)N_0}{(N_0 - N_{30})(N_0 - 2p_1)}, \quad t \in (t_2, T]$$

that the inequality is carried out

$$t_2 < t \le T \quad N_3(t) < N_0/2.$$

Proof. We will introduce the notation

$$f_1(t) \equiv (N_0 - N_{30}) \exp\left[\frac{(N_0 - p_1)\int\limits_0^t (qb(\tau) + c(\tau))d\tau}{q+1}\right] - (p_1 - N_{30}).$$

It is clear, that

$$f_1(0) \equiv N_0 - p_1 > 0,$$

$$\frac{df_1(t)}{dt} = (N_0 - N_{30}) \frac{(N_0 - p_1)(qb(t) + c(t))}{q+1}$$
$$\times \exp \left[\frac{(N_0 - p_1) \int\limits_0^t (qb(\tau) + c(\tau))d\tau}{q+1} \right] > 0, \quad \forall t \in [0, T],$$
$$f_1(t) \in C[0, T].$$

Therefore,

$$(N_0 - N_{30}) \exp \left[\frac{(N_0 - p_1) \int\limits_0^t (qb(\tau) + c(\tau))d\tau}{q+1} \right] - (p_1 - N_{30}) > 0, \quad \forall t \in [0, T].$$

And the solution of an inequality

$$N_3(t) = \frac{p_1(N_0 - N_{30}) \exp \left[\frac{(N_0 - p_1) \int\limits_0^t (qb(\tau) + c(\tau))d\tau}{q+1} \right] - (p_1 - N_{30})N_0}{(N_0 - N_{30}) \exp \left[\frac{(N_0 - p_1) \int\limits_0^t (qb(\tau) + c(\tau))d\tau}{q+1} \right] - (p_1 - N_{30})} \leq \frac{N_0}{2}$$

has an appearance

$$0 < \int\limits_0^t (qb(\tau) + c(\tau))d\tau \leq \frac{q+1}{(N_0 - p_1)} \ln \frac{(N_{30} - p_1)N_0}{(N_0 - N_{30})(N_0 - 2p_1)}, \quad t \in (0, t_2),$$
$$N_3(t) \geq N_0/2,$$
$$\int\limits_0^t (qb(\tau) + c(\tau))d\tau \geq \frac{q+1}{(N_0 - p_1)} \ln \frac{(N_{30} - p_1)N_0}{(N_0 - N_{30})(N_0 - 2p_1)}, \quad t \in (t_2, T],$$
$$N_3(t) < N_0/2$$

and by that the Theorem 3.1 is proved.

It is clear, that similar theorems can be proved also at the accounting of incomplete appearances at elections of voters ($0 < k_i < 1$; $i = \overline{1, 3}$) and a certain falsification ($0 < k_j < 1$; $j = 4; 5$).

The mathematical model except theoretical interest has also important practical value, as all three parties (government institutions in together with pro-governmental party; opposition parties) can use results according to the purposes. It allows the parties, according to the chosen strategy, to select parameters of action and to achieve desirable results for them.

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