

MATHEMATICAL MODELING OF NATURAL GAS LEAK DETECTION AND LOCALIZATION IN THE BRANCHED PIPELINES

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Abstract

In this article for detection of gas accidental escape localization in the branched gas pipelines two mathematical models are suggested. The first model is indented for leak detection and localization in the horizontal branched pipeline and second one for an inclined section of the main gas pipeline. The algorithm of leak localization in the branched pipeline is not demand on knowledge of corresponding initial hydraulic parameters at entrance and ending points of each sections of pipeline. For detection of the damaged section and then leak localization in this section special functions and equations are constructed. Some results of calculations for horizontal pipelines having two, four and five sections are presented. Also a method and formula for the leak localization in the inclined section of the main gas pipeline are suggested. Some results of numerical calculations for the inclined pipeline are presented too.

Key words and phrases: Branched and inclined gas pipelines, leak detection, mathematical modelling.

AMS subject classification: 35Q80, 86A30.

1 Introduction

For the last decades natural gas consumption in many countries of the world supercharged (reached top level, became very intensive). It is expected that petroleum gas production (natural gas expenditure) increase in the nearest future. For example it is expected that natural consumption will increase on 50% during the next 20 years in the US [1]- [3]. Nowadays pipelines become the main practical means for natural gas transportation worldwide [2], [4], but it should be noted that at the same time the gas delivery infrastructure is rapidly aging [3]. The main fault of the outdated pipelines is leak and as a consequence deterioration of surrounding environment [2], [5], [6]. For instance methane emissions from leaking pipelines is a serious problem related to the environment as methane is one of the most principal greenhouse gases contributing to climate change [7], [8]. According to the experience of European transit countries the transit of oil and gas

causes great losses regarding the ecological situation thus counteracting the intended political and economical benefits [9]. The leaks caused by damage of pipelines are usually very dangerous. Intensive leaks can stimulate explosions, fires and environment pollution, which can lead to the ecological catastrophe. In this case there can be an enormous economical loss. Although it seems that small leaks are not so dangerous, but in practice it is important to carry out special actions preventing such kind of leaks as well, because the spilt oil or escape gas can damage the corrosion-resistant cover of pipeline and can cause the corrosion processes [10]-[13]. This may outgrow in intensive leaks with the above-mentioned results. That is why the determination of damage place in pipelines in time is the significant problem [12], [14], [15]. In pipeline networks that transport gas or oil leaks may occur at any time and location [2], therefore, timely detection of leaks can stop or minimize contamination of environment and leak detection and location is important for the safe operation of pipelines [2], [13]. Pipelines prognostic leak detection systems play a key role in minimization of the occurrence of leaks probability and their impacts on environment [16]. So elaboration of the leak detection and location system in a large scale water, oil and gas pipeline system networks is an urgent and sensitive issue of nowadays [5], [9], [15], [16]. That is why a great attention is paid to finding out the new methods and techniques for the leak fast detection and location in the pressurized pipeline and fortunately this process for the last three decades has been in progress [1]-[16]. There are many different approaches and techniques for leak detection and location. A spectrum of the methods for detection and location of the leaks in the natural gas pipelines is wide from using trained dogs to advanced satellite based hyper spectral imaging [13], [17]. The methods of leak measurement differ from each other by several features [11]: the determination of the minimum leak value; the precision (fixation) of the leak place; the working regimes (constant, periodic or episodic). At the same time leak controlling methods can be divided into two types: contact method (directly on pipeline) and remote control method (which implies moving controlling device across the pipeline trace) [5], [9], [12], [13], [15] [18]-[25]. According to the above mentioned classification we can list the following methods: visual method of leak detection - detection of oil and gas on ground surface using painting gases; radioactive isotopes, etc.; hydrodynamic method - launching the special material in pipeline; acoustic method and mathematical modelling. There are also fundamental theoretical and technical methods for leak detection and location in natural gas pipelines and the various methods can be classified into non-optical and optical methods [11]. From existing methods the mathematical modelling with hydrodynamic method is more acceptable as it is very cheap and reliable and has high sensitive and operative features [6], [12], [13], [27].

It is also significant that in this case the defined leak detection placement is possible to control by remote satellite and the earth surface appliances.) This article provides the proper methodology for leak detection in horizontal branched (with several sections and branches) pipeline and for leak detection in inclined section of the main gas pipeline. The methodology does not required knowledge of corresponding initial hydraulic parameters at entrance and ending points of each sections of pipeline. The methodology is based on an algorithm describing gas stationary movement in the simple and branched gas pipelines and some results of calculations are presented.

2 Problem formulation for leak localization in branched pipeline

In case of the main gas pipeline's damage first of all the gas expenditure and pressure are altered by gas leakage and initially gas flow has non-stationary character. After some time from the gas leakage (under some conditions), a new stationary state of gas flow is formed in the pipeline. So when there is gas accidental escape from the main gas pipeline it is necessary to study as a non-stationary stage [9], [11] as well a stationary stage of gas flow in the pipeline [5], [13], [16], [18], [28]. We study only large-scale gas leakage problem from the main gas pipeline and we consider this question as a reverse task of hydraulic calculation problem. Analytical methods of a large-scale gas escape location determination in the simple section of main gas pipeline are in use when data of the gas pressures and expenditure at the entrance and ending of the gas pipeline are known [12], [29]. But these methods cannot be used for main gas pipelines having several sections and branches and for inclined pipes. The methods offered by us gives possibility first of all detect the placement of the section having accidental gas escape using minimal information (data of the gas pressures and expenditure at the main gas pipeline's entrance and ending points before and after gas escape) then leak localization in the determined section of main pipeline and for leak detection in inclined section of the main gas pipeline.

Thus suppose that there is a complex main gas pipeline having n off-shots, with expenses q_k ($k = \overline{1, n-1}$) and the pipeline is divided by off-shots on n simple sections with length L_k ($k = \overline{1, n}$). If at the entrance of pipeline gas expands in unit of time is M_0 , then at the entrance of the per simple sections the gas expands are calculated in the following way [13],

$$M_1 = M_0$$

$$M_k = M_{k-1} - q_{k-1}, \quad k = \overline{2, n}$$

where numbering is performed from the beginning of the pipeline to the ending. As it is known in case of gas stationary movement in the horizontal gas pipeline we have the following equality [12], [13], [29]:

$$P_1^2 - P_2^2 = \sum_{k=1}^n M_k^2 \beta_k L_k, \quad (1)$$

where $\beta_k = \frac{\lambda_k Z R T}{F_k^2 D_k}$, P_1 and P_2 are values of the pressures at the entrance and at the ending of the main gas pipe-line, respectively; M_k - are expenses of gas in the unit area of pipe-line for unit time in the branches; L_k - are lengths of simple section k of the main pipe-line; Z is a coefficient expressing deviation of natural gas from ideal gas; λ_k is a hydraulic resistance of a gas; T is an absolute temperature; R is a gas constant; D_k are diameters of pipelines; F_k are areas of branches profile sections.

Suppose that at the entrance of the main gas pipeline in the unit of time through pipe passes M_0 mass of gas, and at the ending of pipeline instant of gas mass M_n expenditure of gas is $M_n - Q$, which indicates that gas with mass Q is loosen, although the consumers are getting the same mass of gas q_k ($k = \overline{1, n-1}$) which is conditioned by gas distributive stations (service management). Let us suppose that gas leakage is placed on the section i and gas escape is located on the distance x ($0 \leq x \leq L_1$) from the entrance of the section i . Also we suppose that accidental gas escape represents additional ramification of the main gas pipeline with expenditure Q (see Fig.1).

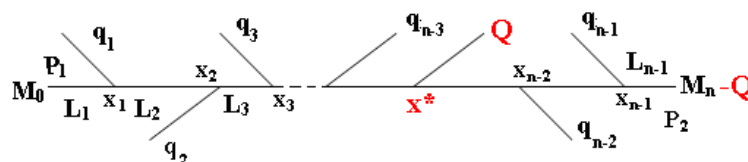


Fig. 1 Schematic map of a branched main gas pipeline

Fig.1 represents a general scheme of a complex main branched gas pipeline where x^* is a location of gas escape in the pipeline. It is evidence that expenditure of gas is remained the same in the ramifications located before the section i but after the section i instead of expenditure M_k it will be $M_k - Q$ ($k = \overline{1, n}$). Analogously of the right side of the equation (1) let us initiate the following functions $f_i(x)$, $i = \{1, 2, \dots, n\}$ [13]:

$$f_1(x) = \sum_{k=1}^n [M_k - Q]^2 \beta_k L_k + Q [2M_1 - Q] \beta_1 x, \quad (0 \leq x \leq L_1),$$

$$f_i(x) = \sum_{k=1}^{i-1} M_k^2 \beta_k L_k + \sum_{k=1}^n [M_k - Q]^2 \beta_k L_k + Q [2M_1 - Q] \beta_i x,$$

$$i \in (2, 3, \dots, n - 1), \quad (0 \leq x \leq L_1),$$

$$f_n(x) = \sum_{k=1}^{n-1} M_k^2 \beta_k L_k + [M_n - Q]^2 \beta_n L_n + Q [2M_n - Q] \beta_n x, \quad (0 < x \leq L_n).$$

Let us assume that after gas escape \bar{P}_1^2 and \bar{P}_2^2 are values of the gas pressures, at the entrance and ending of main pipeline, respectively (which are obtained by the measuring instruments). Therefore, analogously of the equation (1) we have [13]:

$$\bar{P}_1^2 - \bar{P}_2^2 = f_i(x). \tag{2}$$

So for detection of the section of accidental gas escape and the point of gas escape in this section we have the following mathematical model (algorithm): first of all it is required to search such kind of value i_0 from the sequence $i = \{1, 2, \dots, n\}$ and then the value of the x from the interval $[0, L_{i_0}]$ which will satisfy equation (2).

For convenience here and further we are defining some properties of the above mentioned function $f_i(x)$:

1. Every function $f_i(x)$ ($i = \overline{1, n}$) represents linear increasing functions of x forasmuch as $Q [2M_1 - Q] \beta_1 > 0$, ($i = \overline{1, n}$)
2. The following equalities are correctness:

$$f_{i-1}(L_{i-1}) = f_i(0), \quad (i = \overline{2, n})$$

Indeed, let us consider the cases when $i = 1, 2$ separately. We will get:

$$f_1(L_1) = \sum_{k=1}^n (M_k - Q)^2 \beta_k L_k + Q [2M_1 - Q] \beta_1 L_1,$$

$$f_2(0) = M_1^2 \beta_1 L_1 + \sum_{k=1}^n (M_k - Q)^2 \beta_k L_k = \sum_{k=1}^n [M_k - Q]^2 \beta_k L_k + M_1^2 \beta_1 L_1 -$$

$$(M_1 - Q)^2 \beta_1 L_1 = \sum_{k=1}^n (M_k - Q)^2 \beta_k L_k + Q (2M_1 - Q) \beta_1 L_1 = f_1(L_1),$$

when $i = (3, 4, \dots, n - 1)$, then

$$f_{i-1}(L_{i-1}) = \sum_{k=1}^{i-2} M_k^2 \beta_k L_k + \sum_{k=1}^n (M_k - Q)^2 \beta_k L_k + Q (2M_{i-1} - Q) \beta_{i-1} L_{i-1} =$$

$$\sum_{k=1}^{i-1} M_k^2 \beta_k L_k + \sum_{k=1}^{n1} (M_k - Q)^2 \beta_k L_k - M_{i-1}^2 \beta_{i-1} L_{i-1} + (M_{i-1} - Q)^2 \beta_{i-1} L_{i-1} +$$

$$Q(2M_1 - Q) \beta_{i-1} L_{i-1} = \sum_{k=1}^{i-1} M_k^2 \beta_k L_k + \sum_{k=1}^{n1} (M_k - Q)^2 \beta_k L_k = f_1(0).$$

when $i = n$ we have

$$f_{n-1}(L_{i-1}) = \sum_{k=1}^{i-2} M_k^2 \beta_k L_k + \sum_{k=i-1}^n (M_k - Q)^2 \beta_k L_k + Q(2M_{i-1} - Q) \beta_{n-1} L_{n-1} =$$

$$\sum_{k=1}^{i-1} M_k^2 \beta_k L_k - M_{n-1}^2 \beta_{n-1} L_{n-1} + (M_{n-1} - Q)^2 \beta_{n-1} L_{n-1} + (M_n - Q)^2 \beta_n L_n +$$

$$Q(2M_{n-1} - Q) \beta_{n-1} L_{n-1} = \sum_{k=1}^{n-1} M_k^2 \beta_k L_k + (M_n - Q)^2 \beta_n L_n = f_n(0).$$

The last fully proofs equality 2.

Using the above proved properties of the function $f_i(x)$ we have constructed the numerical algorithm for definition of a placement of the damage section and then the placement of the gas escape in the damaged section which was found by the following equations [13]. Afterwards it is determined the location (number of the section i_0) of the gas accidental escape the appropriate distance x can be defined by solution of equation (2) which has the following form [9]: if we have $i_0 = 1$ then

$$x = \frac{\bar{P}_1^2 - \bar{P}_2^2 - \sum_{k=i-1}^n (M_k - Q)^2 \beta_k L_k}{Q(2M_1 - Q) \beta_1}, \quad (3)$$

If fulfilled the following inequality $2 \leq i_0 \leq n - 1$ then

$$x = \frac{\bar{P}_1^2 - \bar{P}_2^2 - \sum_{k=1}^{i_0-1} M_k^2 \beta_k L_k - \sum_{k=1}^n (M_k - Q)^2 \beta_k L_k}{Q(2M_{i_0} - Q) \beta_1}, \quad (4)$$

And at last if $i_0 = n$, then

$$x = \frac{\bar{P}_1^2 - \bar{P}_2^2 - \sum_{k=1}^{i_0-1} M_k^2 \beta_k L_k - (M_n - Q)^2 \beta_k L_k}{Q(2M_n - Q) \beta_n}, \quad (5)$$

On the basis of the numerical algorithm for definition of a placement of the damage section [9] and then formulas (3)-(5) for a placement of the

gas escape in the damaged section we have investigated the pipes with the length 30 km. diameter $D = 0,7m$, cross section $F = 0,38m^2$, having one, two three, four, five and six ramifications and we supposed that gas escape had happened at varies points of sections. It is important to say that we have investigated the pipelines with sections having radically different length and location of gas escape in the pipeline. Numerical simulations have performed on the basis of a reverse task of hydraulic calculation problem when damaged sections leak location was known beforehand. Below some results of calculations for a horizontal branched main pipelines are presented. In this experiment there were two branches. Gas escape was located in the third section and distance from the beginning of the third section was equal to 3000m. (see Fig.2) The parameters have accepted the following values: $R = 500Nm(kg^0k)$, $Z = 0,95$, $T = 280K$, $M_0 = 15kg/s$, $q_1 = 2g/s$, $q_2 = 2g/s$, $q_3 = 2g/s$, $L_1 = 10 * 10^3m$; $L_2 = 10 * 10^3m$, $L_3 = 10 * 10^3m$, $M_1 = 15kg/s$, $M_2 = 13kg/s$, $M_3 = 10kg/s$, $x^* = 3000m$, $Q = 1kg/s$, $\omega = 2570$.

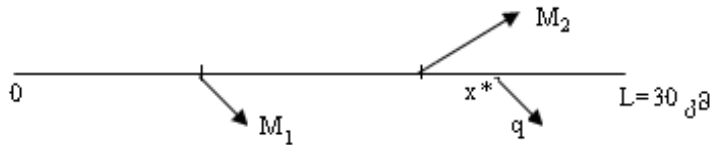


Fig.2 Gas escape is located in the third section of the pipeline having two branches.

The algorithm have defined the number of section correctly and Calculations have shown that distance from the beginning of the third section was equal to 3070m and the error between location of gas known in advance and calculated value was 70m.

In this experiment there were four branches. Gas escape was located in the third section and distance from the beginning of the third section was 3500m (see Fig 3). The length of sections differ from each other. The parameters have accepted the following values: $M_0 = 15kg/s$, $q_1 = 2g/s$, $q_2 = 2g/s$, $q_3 = 2g/s$, $q_4 = 2g/s$, $L_1 = 5 * 10^3m$; $L_2 = 5 * 10^3m$, $L_3 = 4 * 10^3m$, $L_4 = 10 * 10^3m$, $L_5 = 6 * 10^3m$ $M_1 = 19kg/s$, $M_2 = 17kg/s$, $M_3 = 15kg/s$, $M_4 = 12kg/s$, $x^* = 3500m$, $Q = 1kg/s$.

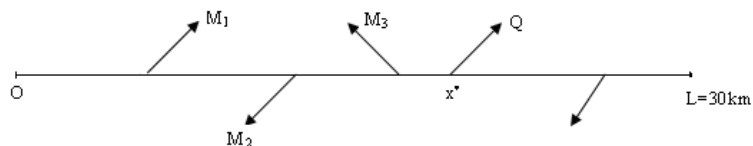


Fig.3 Gas escape is located in the fourth section of the pipeline having four branches.

The algorithm has defined the number of section correctly and calculations have shown that distance from the beginning of the fourth section was $3500m$ an error was $120m$.

In this experiment there were five branches. Gas escape was located in the fourth section and distance from the beginning of the fourth section was $2000m$ (see Fig. 4) The length of sections differ from each other. The parameters have accepted the following values:

$M_0 = 15kg/s$, $q_1 = 2g/s$, $q_2 = 2g/s$, $q_3 = 2g/s$, $q_4 = 2g/s$, $q_5 = 2g/s$, $L_1 = 4 * 10^3m$; $L_2 = 6 * 10^3m$, $L_3 = 5 * 10^3m$, $L_4 = 5 * 10^3m$, $L_5 = 4 * 10^3m$, $L_6 = 6 * 10^3m$, $M_1 = 15kg/s$, $M_2 = 13kg/s$, $M_3 = 11kg/s$, $M_4 = 7kg/s$, $M_5 = 5kg/s$, $x^* = 200m$, $Q = 1kg/s$.

The algorithm has defined the number of section correctly and calculations have shown that distance from the beginning of the fourth section was $2090m$. The error was $-90m$.

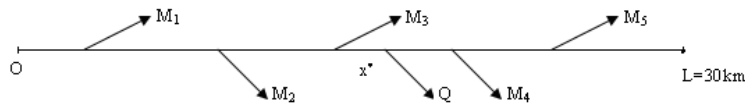


Fig.4 Location of the gas escape is located in the fourth section of the pipeline having five branches.

Results of calculations have shown that in all cases the number of damaged section was defined correctly and the maximum of error between calculated and previously known of gas escape point did not exceed $200m$ in the pipeline with length $30km$.

3 Leak localization in inclined pipeline

On the territory of Georgia (Georgia lies between the Main Caucasian Ridge and the minor Caucasus mountains and territory of Georgia is characterized by the compound orography as Mountain Range forming about 85% of the total land area) there are functioning 4 gas pipelines [30]. The gas pipelines were constructed in the conditions of compound relief, so they are inclined to the horizontal plane. For example North Caucasus - (Transcaucasia) gas pipeline- Grozni-Tbilisi-Yerevan passes from Russia to Armenia through main Caucasus mountain range with the maximum altitude $2400m$. South Caucasus pipeline- Baku-Tbilisi-Erzurum (BTC) gas pipeline transports gas from Azerbaijan to Turkey through the territory of Georgia. The BTC pipeline on the territory of Azerbaijan is located almost on the horizontal surface but from the territory of Georgia to Erzurum the orography is compound. It is also important to emphasize that the territories of Georgia and Turkey are located in the seismic active zone and on the territory of

Caucases there are frequently such kind of phenomenon such are floods, landslides with huge stones and as a consequence with high probability of damage of gas pipelines. So investigation of the question of gas escape localization in the inclined pipes is an urgent problem for the Caucasus region and for Georgia too.

For the simple inclined main gas pipeline there is the following formula[29]:

$$P_H^2 - P_K^2 \cdot e^{a\Delta z} = M^2 \cdot b \cdot L \cdot \frac{e^{a\Delta z} - 1}{a\Delta z}, \tag{6}$$

where P_H and P_k are values of the pressures at the entrance and at the ending of the main gas pipeline, respectively, M is expenses of gas in the unit area of pipeline for unit time, L is length of a simple pipeline, Δz is difference of heights between endings of the sloping pipeline, a and b are known constants[8, 31].

$$a = \frac{2g}{ZRT}, \quad b = \frac{16ZRT}{\pi^2 D^5} \cdot \lambda, \quad \lambda = \frac{0.03817}{D0.2},$$

where g is gravitational acceleration. For performing numerical calculations the parameters have accepted the following values: $Z = 0.9m$, $D = 0.7m^2$, $R = 287.04N.m/kg.K$, $T = 276K$.

Suppose that an accidental gas escape with intensity q , is located on the distance x from the begging of the pipeline and the gas accidental escape point B represents an additional ramification of the pipeline OA (see Fig.5).

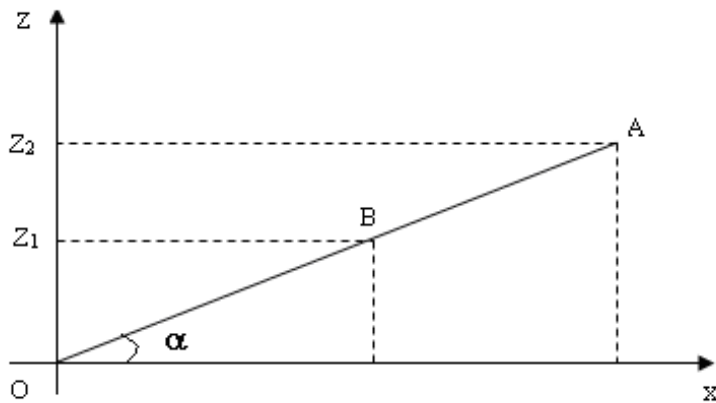


Fig.5 represents inclined by angle α pipeline OA to the horizontal line OX , with length L . In figure O and A are begging and ending points of the pipeline, B is accidental gas escape point, $|OA| = L$; $|OB| = L$; . Suppose that an accidental gas escape with the intensity q , is located at the distance x from the begging of the pipeline at the point B . We assume that the gas

accidental escape point B is a ramification and a simple pipeline is divided by the point B into two simple sections. Then in analogues of the equation (6) for the sections OB and BA it is possible to write the following system of equations:

$$P_H^2 - P_x^2 \cdot e^{\omega_1} = M^2 \cdot b \cdot x \cdot \frac{e^{\omega_1} - 1}{az_1}, \quad (7)$$

$$P_x^2 - P_K^2 \cdot e^{a(z_2 - z_1)} = (M - q)^2 \cdot b \cdot (L - x) \cdot \frac{e^{a(z_2 - z_1)} - 1}{a(z_2 - z_1)}, \quad (8)$$

where we assume that $z_0 = 0$, z_1 and z_2 are vertical coordinates of the entrance, gas escape and ending points of the pipeline, respectively.

From Fig.5 it is evidence that $z_1 = \frac{z_2}{L} \cdot x = x \cdot \sin(\alpha)$, $z_2 - z_1 = (L - x) \cdot \sin(\alpha)$. Taking into consideration the appropriate equality

$$e^\beta - 1 \approx \beta, \quad \text{when } 0 < \beta < 1$$

Then equations (7) and (8) can be rewritten in the following form

$$P_H^2 - P_x^2 \cdot (1 + az_1) = M^2 \cdot b \cdot x. \quad (9)$$

For the second section we have

$$P_x^2 - P_K^2 \cdot [1 + a(z_2 - z_1)] = (M - q)^2 \cdot b \cdot (L - x), \quad (10)$$

Equations (9) and (10) compose the system of equations for defining unknown variables x and P_x . For defining unknown variable x we get the following quadratic equation

$$\left[\frac{a^2}{L^2} Z_2^2 P_K^2 + (M - q)^2 \cdot ba \frac{z_2}{L} \right] x^2 + \left[P_K^2 a^2 \frac{Z_2^2}{L^2} - (M - q)^2 ba z_2 + (M - q)^2 b - M^2 b \right] x - (M - q)^2 b L - P_K^2 (1 + az_2) + P_H^2 = 0. \quad (11)$$

We have investigated the discriminant of (11) and for the reasonable values of parameters it was greater-than zero. So (11) has two solutions. In all of tested experiments the absolute value of one root always was greater-than the length of pipeline L , but the other one was less than the inclined pipe's length L , which was solution of (11). In table 1 some values of the roots (11) and its dependence on the values of gas pressure at the ending point of pipeline are given.

Table I

Influence of the pressure on the quadratic equation roots

P_K	x_1	x_2
$1.43 * 10^7$	$1.62945 * 10^5$	$1.3460702 * 10^8$
$1.44 * 10^7$	$1.22808 * 10^5$	$1.3458845 * 10^8$
$1.45 * 10^7$	82416.06	$1.34569702 * 10^8$
$1.46 * 10^7$	41768.79	$1.34551015 * 10^8$
$1.47 * 10^7$	866.99	$1.34532134 * 10^8$

Table I shows that values of the root x_2 of quadratic equation are always greater than the length of the pipeline. The other one is the solution of the problem.

Table II

Precise and calculated values (inaccuracies) of leak localization for varies rates of gas expenses, pressure at entrance and ending points and gas escape

M kg	P_H H/M^2 (atm.)	P_K H/M^2 (atm.)	q kg	precise point of escape (km)	calculated (km)	drift rate (km)
2.1	$1.18 * 10^6$	$7.73 * 10^5$	0.1	150	152	2
2.1	$1.18 * 10^6$	$7.9 * 10^5$	0.1	100	102	2
2.1	$1.18 * 10^6$	$8.04 * 10^5$	0.1	50	47	3
12	$1.5 * 10^7$	$1.5 * 10^7$	4	100	117	17
12	$1.5 * 10^7$	$1.5 * 10^7$	4	150	145	5
12	$1.5 * 10^7$	$1.5 * 10^7$	4	50	58	8

Table II shows some results of calculations using for all the experiments the following common values $L=200km.$ and $z_2= 300m.$ There were performed mainly two type of experiments with gas expenditure $0.13 kg/s$ and $4 kg/s.$ and the values of the pressure at the entrance and ending points of pipeline were changed. We had known exactly solution of the task when escape was at the pointa 50, 100 and 150 km a long way off the entrance point of pipe. The calculations had shown that maximal inaccuracy with the gas expenditure $0.13 kg/s$ was 3 km. and with the gas expenditure $4 kg/s$ maximal inaccuracy was equal to 7 km.

If the appropriate equality

$$e^\beta - 1 \approx \beta,$$

is not true then equations (7) and (8) can be rewritten in the following form

$$P_H^2 - P_x^2 \cdot y = M^2 \cdot b \cdot (y - 1)/A. \tag{12}$$

$$P_x^2 \cdot y \cdot A - P_K^2 \cdot e^{AL} = (M - q)^2 \cdot b \cdot (e^{A(L-1)} - 1), \tag{13}$$

where $A = a \cdot \sin(\alpha)$, $y = e^{Ax}$.

Equations (12) and (13) compose the system of equations for definition of the unknown variables y and P_x and after some simple transformation from the (12) and (13) we have :

$$y = \frac{A(P_H^2 - P_K^2 \cdot e^{AL}) + M^2b - (M - q)^2 \cdot b \cdot e^{AL}}{q \cdot b(2M - q)},$$

Taking into account denotation $y=eAx$ the last equation can be rewritten in the following form:

$$x = 1/A \ln [A(P_H^2 - P_K^2 \cdot e^{AL}) + M^2b - (M - q)^2 \cdot b \cdot e^{AL}] / q \cdot b(2M - q). \quad (14)$$

We had investigated an applicable domain of (14) and for the reasonable values of parameters expression under sign of logarithm was greater-than zero. Namely if the following inequalities are true

$$\frac{P_H^2}{P_K^2} > e^{AL}, \quad \frac{M^2}{(M - q)^2} > e^{AL},$$

then for reasonable values of parameters there exist exact solution (14) which determines leak location in the inclined pipeline with inclination of α . We have investigated solution (14) for different values of parameters. Namely in this experiments the following values of parameters were fixed: $P_h = 1.18111 \cdot 10^6$; $M = 5, 1$; $q = 1.2$; $T = 276$; $Z = 0.9$; $R = 287, 04$.

Results of calculations show that when $P_k = 1.18111 \cdot 10^6$; $L = 30000$ and $\sin(\alpha) = 0.02$, then leak's location is $x = 6838, 7$. When $L = 25000$; $\sin(\alpha) = 0.04$ and $P_k = 1.024435066 \cdot 10^6$ then leak's location is $x = 18790, 1$. When $L = 25000$; $\sin(\alpha) = 0.12$ and $P_k = 9.5690622 \cdot 10^5$ then leak's location is $x = 18695, 1$. When $L = 30000$; $\sin(\alpha) = 0.01$ and $P_k = 1.02508942 \cdot 10^6$ then leak's location is $x = 6843, 77m$ from the entrance point of the inclined pipe.

4 Conclusion

As foreign experience with pipelines shows, the main reasons of crashes and spillages (and fires as a consequence) are destruction of the gas pipes as a result of corrosion, defects of welding and natural phenomena (earthquakes, landslides, floods etc). Besides terrorist attacks and sabotage may occur. Mathematical modelling with hydrodynamic method is a very cheap, reliable and high sensitive method for discovering placement of accidental gas escape [31]-[34]. In the present paper in case of gas stationary flow we suggest a new method for discovering placement of gas accidental escape in the main pipeline with several gas offshoots. The algorithm of leak localization in the branched pipeline represents a generalized view of a mathematical model describing gas stationary flow in the simple gas pipeline. For detection of damaged section and then leak localization special functions and equations have been constructed. Some results of numerical calculations for compound pipelines having two, four and five sections are presented. The method offered by us gives possibility first of all detect the placement of the damaged

section using minimal information (data of the gas pressures and expenditure at the main gas pipeline's at entrance and ending points before and after gas escape) and then leak localization in the determined section of main pipeline. We have investigated the pipelines with sections having radically different length, location of gas escape in the pipeline and sections which was known in advance, (previously, beforehand). The calculations had shown that maximal inaccuracy with the gas expenditure 0.13 kg/s was 3 km and with the gas expenditure 4 kg/s was 7 km. For the verification of the proposed method we created quite general test, the manner of the solution had been known in advance. Comparison of the solutions has shown the affectivity of the following method. Besides a simple mathematical model (an algorithm) for definition a placement of a gas accidental escape in a inclined section of the main gas pipeline is suggested. The algorithm required knowledge of corresponding initial hydraulic parameters at entrance and ending points of the inclined pipeline. A method and a formula for the determination a location of the gas accidental escape in simple inclined main gas pipeline are suggested. Some results of numerical calculations defining localization of gas escape for the inclined pipeline are presented. The results of calculations on the basis of observation data have shown that the performed simulations were much closer to the results of observation.

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