INTEGRAL METHOD FOR SOLVING THE PROBLEM OF A STATIONARY BOUNDARY LAYER VISCOUS CONDUCTING FLUID

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Abstract

Considered a stationary boundary layer of non-Newtonian fluid. Obtained self-similar solutions tasks of free convection a non-Newtonian fluid when variable conduction. The problem is solved by the integral method. Is shown that by choosing the parameters can be controlled surface friction.

Key words and phrases: Non-Newtonian fluid, Boundary layer, The thickness of the loss of impulse, Velocity of seepage, Surface friction.

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1 Introduction

Most gas and liquids are Newtonian. Thin suspensions, solutions clay, oil paint on their characteristics are different from Newtonian fluids.research regularities non-Newtonian fluids receives great importance in industry and technology development for the wider use of new materials, as well as for the study of various biological fluids.

In this paper we study one of the tasks of boundary layer theory. Boundary layer theory describes the mechanical processes near the surface a solid body in the case of high Reynolds number. in case of flow around the body entire flow of fluid can be divided into two areas:

1. area with small thickness near the surface of the body which is called the boundary layer, in which effect of viscosity forces is as essential as other forces;

2. The area in which the viscosity not taken into account is called outer region. Can be considered that here is a potential flow.The outer region of the liquid can be considered to be ideal.

Consider a stationary boundary layer of non-Newtonian fluid. Assume that the fluid conductivity coefficient is constant: $\sigma = \sigma_0$; External flow velocity is U_∞. External magnetic field is B. Ox axis is located on the plate and is directed along the stream. Axis oy is directed vertically above. B_0 is a component vector of magnetic induction on the oy-axis. Boundary-layer equations are:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{nk}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right)\left(\frac{\partial u}{\partial y}\right)^{n-1} - \frac{\sigma_0 B_0^2}{\rho}u, \tag{1}$$

$$U_\infty \frac{\partial U_\infty}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\sigma_\infty B_0^2}{\rho}U_\infty, \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{3}$$

2 Statement of the Problems

Assume that $U_\infty \neq$ const end this is a function of x. Find the form of Karman's integral equation. To do this, write the continuity equation as follows:

$$\frac{\partial u U_\infty}{\partial x} + \frac{\partial U_\infty v}{\partial y} = u\frac{\partial U_\infty}{\partial x}, \tag{4}$$

which together with equations of motion yields the following relation:

$$\frac{\partial}{\partial x}u\left(U_\infty - u\right) + \frac{\partial}{\partial y}v\left(U_\infty - u\right) + \frac{\partial U_\infty}{\partial x}\left(U_\infty - u\right)$$
$$= -\frac{k}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n - \frac{\sigma_0 B_0^2}{\rho}U_\infty\left(1 - \frac{u}{U_\infty}\right). \tag{5}$$

Let us take integral of (2) end will take into an account expressions thickness ousting and loss of impulse:

$$\delta^* = \int_0^{\delta,\infty}\left(1 - \frac{u}{U_\infty}\right)dy, \tag{6}$$

$$\delta^{**} = \int_0^{\delta,\infty}\frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy. \tag{7}$$

Then from (2) we get:

$$\frac{d}{dx}\left(U_\infty^2\delta^{**}\right) - U_\infty V_W + U_\infty\frac{dU_\infty}{dx}\delta^* = \frac{k}{\rho}\left(\frac{\partial u}{\partial y}\right)^n\bigg|_{y=0} - N_0\delta^* U_\infty, \tag{8}$$

where $N_0 = \sigma_0 B_0^2/\rho$, and V_W is the velocity of seepage. If we consider that σ is not constant, and we take the expression for it:

$$\sigma = \sigma_0 \left(1 - \frac{u}{U_\infty}\right). \tag{9}$$

Then equation (8) takes the form

$$U_\infty^2 \frac{d\delta^{**}}{dx} + 2U_\infty \delta^{**} \frac{dU_\infty}{dx} - U_\infty V_W + U_\infty \frac{dU_\infty}{dx}\delta^* = \frac{k}{\rho}\left(\frac{\partial u}{\partial y}\right)^n \bigg|_{y=0} - N_0 \left(a\delta^* - b\delta^{**}\right) U_\infty. \tag{10}$$

If $a = 1$ and $b = 0$, We will have a classic task model, which corresponds to the case of constant conductivity and if $a = 0$ and $b = 1$, electric conductivity will be variable. (2) is Karman's integral relation in case of any contour. When $U_\infty = \text{const}$, Then we have a flat wall. If $V_W = 0$, from (2) we have:

$$\frac{d\delta^{**}}{dx} + \frac{N_0}{U_\infty}\left(a\delta^* - b\delta^{**}\right) = \frac{k}{U_\infty^2 \rho}\left(\frac{\partial u}{\partial y}\right)^n_{y=0}. \tag{11}$$

We introduce self – similar variable $\eta = \dfrac{y}{\delta}$ and the velocity of fluid $u\,(\mathrm{y})$ we shall seek

As follows:

$$u\,(y) = U_\infty f\,(\eta)\,, \tag{12}$$

Then

$$\frac{\partial u}{\partial y} = \frac{U_\infty}{\delta} f', \tag{13}$$

$$\delta^* = \delta \int\limits_0^1 (1 - f)\, d\eta, \tag{14}$$

$$\delta^{**} = \delta \int\limits_0^1 f\,(1 - f)\, d\eta. \tag{15}$$

Introduce some notation:

$$A \equiv \int\limits_0^1 (1 - f)\, d\eta, \tag{16}$$

$$B \equiv \int\limits_0^1 f\,(1 - f)\, d\eta. \tag{17}$$

Take account (16) and (17) in (11) end we multiply (11) by δ^n, we get:

$$\frac{B}{n+1}\frac{d\delta^{n+1}}{dx}+\frac{N_0}{U_\infty}\left(aA+bB\right)\delta^{n+1}=\frac{kU_\infty^{n-2}}{\rho}\left[f'(0)\right]^n. \tag{18}$$

We solve the task in some following cases. To do this, shall use the following boundary conditions:

I. $u|_{y=0}=0 \implies f|_{\eta=0}=0.$

II. $u|_{y=\delta}=U_\infty \implies f|_{\eta=1}=1.$

III. $\left.\frac{du}{dy}\right|_{y=\delta}=0 \implies f'|_{\eta=1}=0.$

IV. The fourth condition is the equation of motion on the $y=0$.

$$-V_W\left.\frac{du}{dy}\right|_{y=0}=U_\infty {U_\infty}'+\frac{k}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n_{y=0}+N_0U_\infty a. \tag{19}$$

If not have velocity of seepage ($V_W=0$) end $U_\infty=\text{const}$, then we will have:

$$\frac{k}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n_{y=0}+N_0U_\infty a=0. \tag{20}$$

$$\frac{U_\infty^{n-1}n}{\delta^{n+1}}\frac{k}{\rho}\left(f'\right)^{n-1}f''|_{\eta=0}+\frac{\sigma_0B_0^2}{\rho}a=0, \tag{21}$$

and if $U_\infty\neq\text{const}$, we have:

$$U_\infty\frac{dU_\infty}{dx}+\frac{U_\infty^n n}{\delta^{n+1}}\frac{k}{\rho}\left(f'\right)^{n-1}f''|_{\eta=0}+\frac{\sigma_0B_0^2}{\rho}U_\infty a=0. \tag{22}$$

3 Solving Problems

We solve the task in following some particular cases:

1) $f(\eta)$ function is linear with respect to its argument:

$$f\left(\eta\right)=a_1+a_2\eta. \tag{23}$$

Where a_1 and a_2 are some constants. To find them, we use the conditions I and II. We get that $a_1=0$ and $a_2=1$, and

$$f\left(\eta\right)=\eta, \quad u=U_\infty\frac{y}{\delta}. \tag{24}$$

From (16) and (17) we get that $A = \frac{1}{2}$, $B = \frac{1}{6}$. From (19) we will have an equation for $\delta(x)$:

$$\frac{d\delta^{n+1}}{dx} + \frac{N_0(n+1)(3a+b)}{U_\infty}\delta^{n+1} = 6(n+1)U_\infty^{n-2}\frac{k}{\rho}. \tag{25}$$

If we assume that $\delta = 0$, when $x = 0$, then the solution to (30) will look like:

$$\delta = \left[\frac{6U_\infty^{n-1}k}{N_0\rho(3a+b)}\left(1 - e^{-\frac{(n+1)(3a+b)N_0}{U_\infty}x}\right)\right]^{\frac{1}{n+1}}. \tag{26}$$

For surface friction forces we have:

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{U_\infty}{\delta}f'|_{\eta=0} = \frac{U_\infty}{\delta}. \tag{27}$$

2) $f(\eta)$ is a quadratic function:

$$f(\eta) = a_1 + a_2\eta + a_3\eta^2. \tag{28}$$

Of I, II and III of the boundary conditions we obtain:

$$f(\eta) = 2\eta - \eta^2, \tag{29}$$

from which it follows that

$$u = U_\infty\left(2\frac{y}{\delta} - \frac{y^2}{\delta^2}\right). \tag{30}$$

Taking into account (16) and (17) we will have an equation for $\delta(x)$

$$\frac{d\delta^{n+1}}{dx} + \frac{N_0(5a+2b)}{2U_\infty}\delta^{n+1} = \frac{15(n+1)kU_\infty^{n-2}}{\rho}, \tag{31}$$

the solution of equation (26) taking into account the condition $\delta|_{x=o} = 0$, is as follows:

$$\delta(x) = \left(U_\infty^{n-1}\frac{30(n+1)k}{N_0(5a+2b)}\right)^{\frac{1}{n+1}}\left(1 - e^{-\frac{N_0(5a+2b)}{2U_\infty}x}\right)^{\frac{1}{n+1}}. \tag{32}$$

For surface friction forces we have:

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{U_\infty}{\delta}f'|_{\eta=0} = \frac{2U_\infty}{\delta}.$$

3) $f(\eta)$ is a cubic function:

$$f(\eta) = a_1 + a_2\eta + a_3\eta^2 + a_4\eta^3.$$

Of I, II and III the boundary conditions for the function $f(\eta)$ we obtain

$$f(\eta) = 3\eta - 3\eta^2 + \eta^3.$$

If $B_0 = 0$, i.e. there is no external magnetic field, then we get from condition IV

$$f(\eta) = \frac{3}{2}\eta - 3\eta^2.$$

Taking into account (16) and (17) we will have an equation for $\delta(x)$:

$$\frac{d\delta^{n+1}}{dx} + \frac{N_0(7a+3b)}{3U_\infty}\delta^{n+1} = \frac{k\cdot 3^n U_\infty^{n-2}}{\rho},$$

That subject to $\delta|_{x=o} = 0$, is as follows:

$$\delta(x) = \left[\frac{k\cdot 3^n U_\infty^{n-1}}{N_0(7a+3b)(n+1)}\left(1 - e^{-\frac{N_0(7a+3b)(n+1)}{3U_\infty}x}\right)\right]^{\frac{1}{n+1}}.$$

For the surface friction forces we have:

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{U_\infty}{\delta} f'|_{\eta=0} = \frac{3U_\infty}{\delta}.$$

References

1. Stanyukovich K. P. Unsteady motion of a continuous environment, *M. SPTTL.* 1995.

2. Baum F. A., Kaplan S. A., Stanyukovich K. P. Introduction to cosmic gas dynamics. *Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow* 1958.

3. Golitsyn G. S. *Journal of Experimental and Theoretical Physics*, **30** (1958), no. 2, p. 382.

4. Kaplan S. A., Stanyukovich K. P. *R. A. S. USSR*, **95** (1954).

5. Djumagulov T. Magnetic hydrodynamics 1969, no. 3, 134–137.

6. Shulman Z. P., Berkovski B. M. *Boundary layer of non-Newtonian fluids.* Minsk, 1966.