ABOUT METHODS OF APPROXIMATE SOLUTIONS FOR COMPOSITE BODIES WEAKENED BY CRACKS IN THE CASE OF ANTIPLANE PROBLEMS OF ELASTICITY THEORY

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Abstract

These problems lead to a system of singular integral equations with immovable singularity with respect to leap of the tangent stress. The problems of behavior of solutions at the boundary are studied. In the present work questions of the approached decision of one system (pair) of the singular integral equations are investigated. The study of boundary value problems for the composite bodies weakened by cracks has a great practical significance. The system of singular integral equations is solved by a collocation method, in particular, a discrete singular method in cases both uniform, and non-uniformly located knots.

Key words and phrases: singular integral equations, method of integral equations, antiplane problems, cracks, collocation method.

AMS subject classification: 65R20, 45E05, 45F15, 65M70.

1 Statement of the problem

Let the elastic body Ω occupy the complex variable plane $z = x + iy$ which is cut on the line $L = [-1; +1]$. The plane consists of two orthotropic homogeneous semiplanes

$$\Omega_1 = \{z : Rez \geq 0,\, x \notin L_1 = [0,1]\}$$

and

$$\Omega_z = \{z : Rez \leq 0,\, x \notin L_2 = [-1,0]\},$$

which are welded on the axis y. Define by index $k, k = 1,2$ values and functions connected with Ω_k.

The problem is to find the function $w_k(x,y)$, which satisfies the differential equation:

$$\frac{\partial^2 w_k(x,y)}{\partial x^2} + \lambda_k^2 \frac{\partial^2 w_k(x,y)}{\partial y^2} = 0, \quad (x,y) \in \Omega_k, \tag{1.1}$$

and the following boundary conditions:
a) on the boundary of the crack tangent stresses are given:

$$b_{44}^{(k)} \frac{\partial w_k(x, \pm 0)}{\partial y} = q_k^{(\pm)}(x), \quad x \in L_k, \tag{1.2}$$

b) on the axis y the condition of continuity is fulfilled:

$$w_1(0; y) = w_2(0; y), \; y \in (-\infty; \infty), \; y \neq 0, \tag{1.3}$$

$$b_{55}^{(1)} \frac{\partial w_1(0; y)}{\partial x} = b_{55}^{(2)} \frac{\partial w_2(0; y)}{\partial x}, \tag{1.4}$$

where $\lambda_k^2 = \dfrac{b_{44}^{(k)}}{b_{55}^{(k)}}$, $b_{44}^{(k)}$, $b_{55}^{(k)}$ are elastic constants, $q_k(x)$ is a function of Hölder's class, $k = 1, 2$; In particular, if we, have an isotropic case $b_{44}^{(k)} = b_{55}^{(k)} = \mu_k$, $\lambda_k = 1$, where μ_k is the module of displacement, $k = 1, 2$.

By using the theory of analytical functions, also boundary value Sokhotski-Plemel formula of Cauchy type integral ([1], §31.16) problem (1.1) with the account of boundary conditions (1.2)-(1.4) has been led to system of singular integral equations with respect to leaps $\rho_k(x)$ (see [2], [3]).

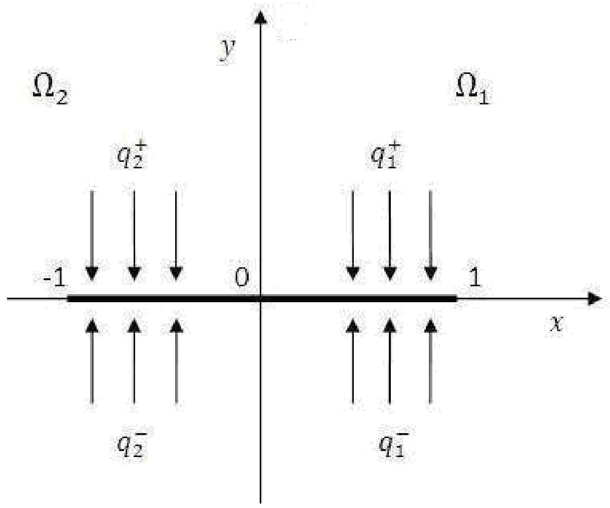

Fig 1.

Let's consider the system of singular integral equations containing an immovable singularity (see [3])

$$\begin{aligned} &\int_0^1 \left[\frac{1}{t-x} - \frac{a_1}{t+x}\right] \rho_1(t)dt + b_1 \int_{-1}^0 \frac{\rho_2(t)dt}{t-x} = 2\pi f_1(x), \quad x \in (0;1), \\ &b_2 \int_0^1 \frac{\rho_1(t)dt}{t-x} + \int_{-1}^0 \left[\frac{1}{t-x} - \frac{a_2}{t+x}\right] \rho_2(t)dt = 2\pi f_2(x), \quad x \in (-1;0), \end{aligned} \tag{1.5}$$

where $\rho_k(x)$, $f_k(x)$ are unknown and given real functions, respectively, a_k, b_k constants, $a_k = \dfrac{1-\gamma_k}{1+\gamma_k}$, $b_k = \dfrac{2}{1+\gamma_k}$, $\gamma_1 = 1/\gamma_2$, $\gamma_2 = \dfrac{b_{55}^{(2)}}{b_{55}^{(1)}}$, $f_k(x) = \dfrac{\lambda_k}{b_{44}^{(k)}} q_k(x)$, $f_k(x) \in H$, $\rho_k(x) \in H^*$, $k = 1, 2$.

2 Collocation method

The system (1.5) of the singular integral equations is solved by a collocation method, in particular, a discrete singular method (see[4]) in cases both uniform, and non-uniformly located knots.

A. Algorithm of uniform division.

Solutions of equations system (1.5) has the form $\rho_1(t) = \dfrac{\rho_1^*(t)}{\sqrt{1-t}}$, $\rho_2(t) = \dfrac{\rho_2^*(t)}{\sqrt{1+t}}$, (see [3]).

Let's enter such distribution of knots for variables of integration and account points accordingly

$$t1_i = 0 + ih, \quad t2_i = -1 + ih, \quad i = 1, 2, \cdots, n;$$
$$x1_j = t1_j - h/2, \quad x2_j = t2_j + h/2, \quad j = 1, 2, \cdots, n;$$
$$h = \frac{1}{n+1}.$$

The pair of the equations (1.5) can be presented as follows with the help quadrature formulas (see [4])

$$\begin{gathered}
\sum_{i=1}^{n} \left(\frac{h}{t1_i - x1_j} - \frac{a_1 h}{t1_i + x1_j} \right) \rho_1(t1_i) \\
+ b_1 \sum_{i=1}^{n} \left(\frac{h}{t2_i - x1_j} \right) \rho_2(t2_i) = 2\pi f_1(x1_j), \quad j = 1, 2, \cdots, n; \\
b_2 \sum_{i=1}^{n} \left(\frac{h}{t1_i - x2_j} \right) \rho_2(t1_i) + \\
+ \sum_{i=1}^{n} \left(\frac{h}{t2_i - x2_j} - \frac{a_2 h}{t2_i + x2_j} \right) \rho_2(t2_i) = 2\pi f_2(x2_j), \ \ j = 1, 2, \cdots, n.
\end{gathered} \tag{2.1}$$

Thus, we have $2n$ equations with $2n$ unknowns. The obtained system of the linear equations can be solved with the help of one of direct methods, for example, by Gauss modified method.

B. Algorithm of non-uniformly division.

All terms of system (1.5) of the singular integral equations will be transfered in a interval $(-1,1)$.

Let's apply the following transformation of variables

integration and account points accordingly

$$x=\frac{x1+1}{2},\quad x1=2x-1,\quad x\in[0;1],\quad x1\in[-1;+1];$$

$$t=\frac{t1+1}{2},\quad t1=2t-1,\quad t\in[0;1],\quad t1\in[-1;+1];$$

$$t=\frac{t2-1}{2},\quad t2=2t+1,\quad t\in[-1;0],\quad t2\in[-1;+1];$$

$$x=\frac{x2-1}{2},\quad x2=2x+1,\quad x\in[-1;0],\quad x2\in[-1;+1].$$

As a result of the above stated transformation of variables, the system of the integral equations (1.5) takes the following form

$$\begin{gathered}\int_{-1}^{1}\left[\frac{1}{t1-x1}-\frac{a_1}{t1+x1+2}\right]\rho_1\left(\frac{t1+1}{2}\right)dt1+\\ +\int_{-1}^{1}\frac{b_1}{t2-x1-2}\rho_2\left(\frac{t2-1}{2}\right)dt2=2\pi f_1\left(\frac{x1+1}{2}\right),\ \ x1\in(-1;1),\\ \int_{-1}^{1}\left[\frac{1}{t2-x2}-\frac{a_2}{t2+x2-2}\right]\rho_2\left(\frac{t2-1}{2}\right)dt2+\\ +\int_{-1}^{1}\frac{b_2}{t1-x2+2}\rho_1\left(\frac{t1+1}{2}\right)dt1=2\pi f_2\left(\frac{x2-1}{2}\right),\ \ x2\in(-1;1).\end{gathered}\tag{2.2}$$

For finding unknown functions $\rho_1\left(\frac{t1+1}{2}\right)$, $\rho_2\left(\frac{t2-1}{2}\right)$ we will apply quadrature formulas of the following form (see[4])

$$
\begin{gathered}
\sum_{i=1}^{n}\left(\frac{1}{t1_i - x1_j} - \frac{a_1}{t1_i + x1_j + 2}\right) A1_i \rho_1\left(\frac{t1_i + 1}{2}\right) + \\
+\sum_{i=1}^{n}\left(\frac{b_1}{t2_i - x1_j - 2}\right) A2_i \rho_2\left(\frac{t2_i - 1}{2}\right) = 2\pi f_1\left(\frac{x1_j + 1}{2}\right), \\
j = 1, 2, \cdots, n; \\
\sum_{i=1}^{n}\left(\frac{b_2}{t1_i - x2_j + 2}\right) A1_i \rho_1\left(\frac{t1_i + 1}{2}\right) + \\
+\sum_{i=1}^{n}\left(\frac{1}{t2_i - x2_j} - \frac{a_2}{t2_i + x2_j - 2}\right) A2_i \rho_2\left(\frac{t2_i - 1}{2}\right) = \\
= 2\pi f_2\left(\frac{x2_j - 1}{2}\right), \quad j = 1, 2, \cdots, n;
\end{gathered}
\tag{2.3}
$$

where

$$
\begin{aligned}
t1_i &= \cos\frac{2i-1}{2n+1}\pi, && i = 1, 2, \cdots, n; \\
t2_i &= \cos\frac{2i}{2n+1}\pi, && i = 1, 2, \cdots, n; \\
x1_j &= \cos\frac{2j}{2n+1}\pi, && j = 1, 2, \cdots, n; \\
x2_j &= \cos\frac{2j-1}{2n+1}\pi, && j = 1, 2, \cdots, n; \\
A1_i &= \frac{4\pi}{2n+1}\sin^2\frac{i}{2n+1}\pi, && i = 1, 2, \cdots, n; \\
A2_i &= \frac{4\pi}{2n+1}\sin^2\frac{n+i}{2n+1}\pi, && i = 1, 2, \cdots, n.
\end{aligned}
$$

Thus we have $2n$ equations with $2n$ unknowns as well as at uniform division.

3 Numerical experiments

As have noted above, for the definition of values of unknown functions in knots we receive system of the equations of order $2n$ with $2n$ unknown variable. (2.1), (2.3) problems are solved by Mathcad. To solve the system of the linear algebraic equations procedure "lsolve" is applied. Corresponding graphics of the approached decisions of various specific problems are constructed.

On some co-ordinate plane four graphics are given in case of different loadings.

$gr1: \quad f(i) = 2\pi, \quad f(n+i) = 2\pi$, solid
$gr2: \quad f(i) = 2\pi, \quad f(n+i) = 4\pi$, dots
$gr3: \quad f(i) = 4\pi, \quad f(n+i) = 2\pi$, dashdot

A. Algorithm of uniform division.

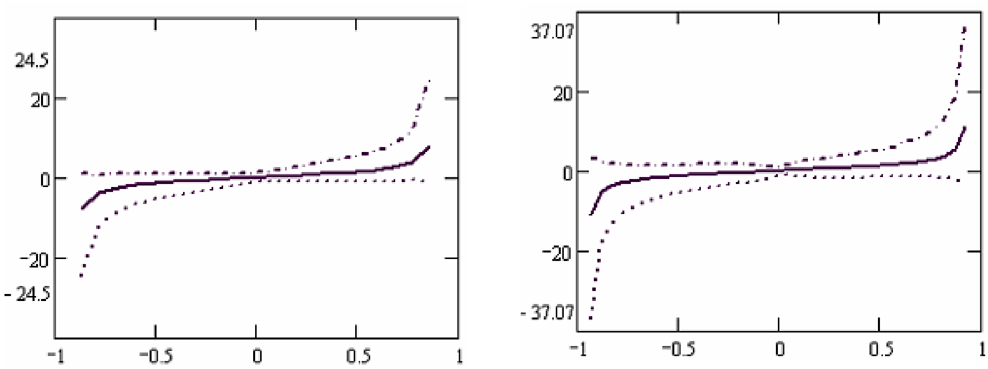

Fig. 2

B. Algorithm of non-uniform division.

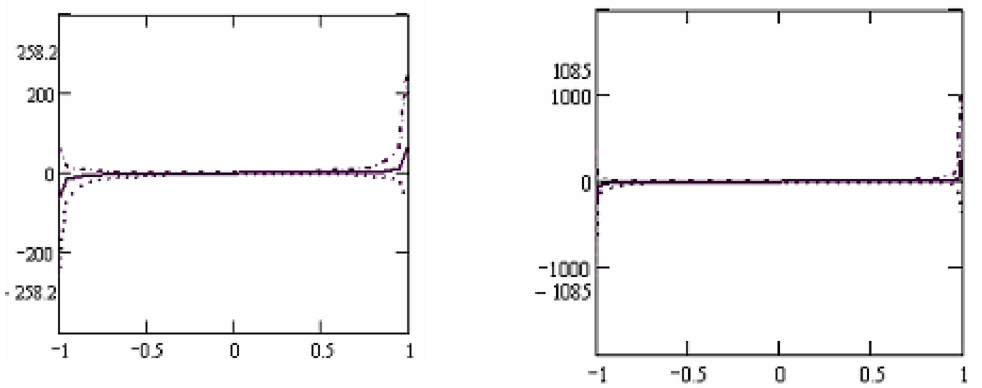

Fig. 3

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