PULSATION FLOW OF INCOMPRESSIBLE ELECTRICALLY CONDUCTING LIQUID WITH HEAT TRANSFER

J. Sharikadze, V. Tsutskiridze, L. Jikidze

Georgian Technical University
77 Kostava Str., Tbilisi 0175, Tbilisi, Georgia

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Abstract

It is considered pulsating flow of electro-conductive viscous incompressible liquid between two parallel walls, which is caused by drop of pulsative pressure and but pulsative motion of walls when external homogeneous magnetic field acts perpendicularly to the walls.

Key words and phrases: Viscous, fluid, pulsation, conducting liquid, heat transfer.

AMS subject classification: 85A30, 76W05.

1 Introduction

In the last years heat phenomena in pipes with action of extermal magnetic field caused special interest. In paper [1] stationary flow of electro-conductive viscous incompressible liquid between two isothermal walls with heat exchange is studied. The action of external homogeneous magnetic fild on heat exchange during constant drop of pressure and constant loss of liquid is shown. The essential relationship between the temperature of liquid flow, Nuselt number of heat exchange, the value of external magnetic field and electric- conductivity of the walls of flat pipe is revealed. In paper [2], in difference of [1], nonstationary flow of viscous incompressible liquid flow between two parallel walls with thermal exchanges is studied. The flow of velocity distribution has parabolic character. In works [3, 6–9] analytical solutions of the equations of Nave-Stoks nonstationary motion and heat exchange are obtained. The flow of liquid is considered between two parallel walls and circular form of pipes under action of external homogeneous magnetic field.

2 Basic part

In the paper, in difference of works [1, 2, 3–9], pulsating flow of electro-conductive viscous incompressible liquid in flat pipe taking into accont Jouls

$\left(M^2\left(\frac{\partial h}{\partial \xi}\right)^2\right)$ and friction heat $\left(\left(\frac{\partial u}{\partial \xi}\right)^2\right)$ is studied. The liquid flow is initiated by walls pulsating motion and pulsating drop of presure given by the foloving law: $-\frac{1}{\rho}\frac{\partial p}{\partial z} = Ae^{-i\omega t}$. The change of temperature in flat pipe is accomplished in pulsating manner. The exact solutions of Navie-Stoks and heat exchange equations are obtained. The obtained critera of similarity characterice oscillation motion of liquid caused by forces of viscosity with action of external homogeneous magnetic field.

Consider the motion of electro-conductive viscous incompressible liquid between two parallel walls, which is caused by pulsating drop of pressure $\left(-\frac{1}{\rho}\frac{\partial p}{\partial z} = Ae^{-i\omega t}\right)$ and by pulsating motion of walls, when external homogeneous magnetic field H_0 acts perpendicularly to the walls.

If we direct axis oz in the direction of liquid movement and axis ox perpendicularly to the walls, then for seeking values we have:

$$\vec{V}(0, 0, \nu_z(x,t)), \quad \vec{H}(H_0, 0, H_z(x,t)), \quad T = T(x,t).$$

Taking into account the above the equations of inductions, motion and heat exchange in dimensionless values will have the form [1–5]:

$$\left.\begin{aligned} \frac{\partial u}{\partial \tau} &= De^{-i\alpha\tau} + \frac{\partial^2 u}{\partial \xi^2} + M^2\frac{\partial h}{\partial \xi} \\ \frac{\partial h}{\partial \tau} &= \frac{R}{R_m}\frac{\partial^2 h}{\partial \xi^2} + \frac{R}{R_m}\frac{\partial u}{\partial \xi} \\ S\frac{\partial \theta}{\partial \tau} &= \frac{\partial^2 \theta}{\partial \xi^2} + \left(\frac{\partial u}{\partial \xi}\right)^2 + M^2\left(\frac{\partial h}{\partial \xi}\right)^2 \end{aligned}\right\} \tag{1}$$

where $\xi = \frac{x}{L}$, $\tau = \frac{\nu}{L^2}t$, $u = \frac{V}{V_0}$, $h = \frac{H}{H_0 R_m}$, $\alpha = \frac{\omega L^2}{\nu}$, $D = \frac{AL^2}{\nu V_0}$, $\theta = \frac{k}{\eta V_0^2}T$ - are dimensionless values, while V_0 and L are characteristic velocity and length, respectively. $M = H_0 L\sqrt{\frac{\sigma}{\eta}}$ is Hartmann number, $S = \frac{\eta C_V}{K}$ is Prandtle number, $R = \frac{V_0 L}{\nu}$ and $R_m = \frac{V_0 L}{\nu_m}$ are Reinolds ordinary and magnetic numbers of viscosity, η is dynamic coefficient of viscosity, ν is kinematic coefficient of viscosity, C_V is capacity for constant volume, K is coefficient of heat exchange, σ is coefficient of conductivity, ν_m is magnetic coefficient of viscosity, ω is frequence.

When walls are not conductive and move in pulsating manner, and at the same time heat exchange on the walls is accomplished by pulsating law, for system (1) we receive the following boundary conditions:

$$u(\xi,\tau)\big|_{\xi=\pm 1} = A_0 e^{-i\alpha\tau}, \quad h(\xi,\tau)\big|_{\xi=\pm 1} = 0, \quad \theta(\xi,\tau)\big|_{\xi=\pm 1} = B_0 e^{-2i\alpha\tau}. \tag{2}$$

Consider the flaw for $R = R_m$.

Let us search the solution of problem (1)–(2) in the following from:

$$u(\xi,\tau) = f(\xi)e^{-i\alpha\tau}, \quad h(\xi,\tau) = \varphi(\xi)e^{-i\alpha\tau}, \quad \theta(\xi,\tau) = q(\xi)e^{-2i\alpha\tau}. \tag{3}$$

Then for motion equation (1) and boundary conditions (2) we obtain:

$$\left.\begin{array}{l} f''(\xi) + M^2\varphi'(\xi) + i\alpha f(\xi) = -D \\ \varphi''(\xi) + f'(\xi) + i\alpha\varphi(\xi) = 0 \\ q''(\xi) + 2i\alpha S q(\xi) = -\big[f'^{\,2}(\xi) + M^2\varphi'^{\,2}(\xi)\big]^2 \end{array}\right\} \tag{4}$$

Then solutions of problem (4)-(5) for velocity, inductive field and het exchange will have the form:

$$\begin{aligned} u(\xi,\tau) = \frac{e^{-i\alpha\tau}}{i\alpha}\Bigg\{ & \frac{D + i\alpha A_0}{\mathrm{sh}\sqrt{M^2-4i\alpha}}\bigg(\mathrm{ch}M\frac{1+\xi}{2}\mathrm{sh}\sqrt{M^2-4i\alpha}\frac{1-\xi}{2} + \\ & + \mathrm{ch}M\frac{1-\xi}{2}\mathrm{sh}\sqrt{M^2-4i\alpha\frac{1-\xi}{2}}\bigg) - D\Bigg\}, \end{aligned} \tag{5}$$

$$\begin{aligned} h(\xi,\tau) = \frac{(D + i\alpha A_0)e^{-i\alpha\tau}}{i\alpha M\mathrm{sh}\sqrt{M^2-4i\alpha}} & \bigg(\mathrm{sh}M\frac{1-\xi}{2}\mathrm{sh}\sqrt{M^2-4i\alpha}\frac{1+\xi}{2} - \\ & - \mathrm{sh}M\frac{1+\xi}{2}\mathrm{sh}\sqrt{M^2-4i\alpha}\frac{1-\xi}{2}\bigg), \end{aligned} \tag{6}$$

$$\begin{aligned} \theta(\xi,\tau) = \Bigg\{ & a_1\mathrm{ch}(M - \sqrt{M^2-4i\alpha})\bigg(\frac{\mathrm{ch}(M-\sqrt{M^2-4i\alpha})\xi}{\mathrm{ch}(M-\sqrt{M^2-4i\alpha})} - \\ & - \frac{\mathrm{ch}\sqrt{\alpha S}(1-i)\xi}{\mathrm{ch}\sqrt{\alpha S}(1-i)}\bigg) + a_2\mathrm{ch}(M+\sqrt{M^2-4i\alpha})\times \\ & \times\bigg(\frac{\mathrm{ch}(M+\sqrt{M^2-4i\alpha})\xi}{\mathrm{ch}(M+\sqrt{M^2-4i\alpha})} - \frac{\mathrm{ch}\sqrt{\alpha S}(1-i)\xi}{\mathrm{ch}\sqrt{\alpha S}(1-i)}\bigg) + \\ & + a_3\mathrm{ch}M\bigg(\frac{\mathrm{ch}M\xi}{\mathrm{ch}M} - \frac{\mathrm{ch}\sqrt{\alpha S}(1-i)\xi}{\mathrm{ch}\sqrt{\alpha S}(1-i)}\bigg) + B_0\frac{\mathrm{ch}\sqrt{\alpha S}(1-i)\xi}{\mathrm{ch}\sqrt{\alpha S}(1-i)}\Bigg\}e^{2i\alpha\tau}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} a_1 &= \bigg(\frac{D+i\alpha A_0}{2\mathrm{sh}\sqrt{M^2-4i\alpha}}\mathrm{sh}\frac{M+\sqrt{M^2-4i\alpha}}{2}\bigg)^2\times \\ &\times\frac{(M-\sqrt{M^2-4i\alpha})^2}{\alpha^2[(M-\sqrt{M^2-4i\alpha})^2+2i\alpha S]}, \\ a_2 &= \bigg(\frac{D+i\alpha A_0}{2\mathrm{sh}\sqrt{M^2-4i\alpha}}\mathrm{sh}\frac{M-\sqrt{M^2-4i\alpha}}{2}\bigg)^2\times \\ &\times\frac{(M+\sqrt{M^2-4i\alpha})^2}{\alpha^2[(M+\sqrt{M^2-4i\alpha})^2+2i\alpha S]}, \\ a_3 &= \bigg(\frac{D+i\alpha A_0}{\mathrm{sh}\sqrt{M^2-4i\alpha}}\bigg)^2\frac{\mathrm{ch}M-\mathrm{ch}\sqrt{M^2-4i\alpha}}{ia(M^2+2i\alpha S)}. \end{aligned} \tag{8}$$

If we single out real and imaginary parts in formulas (6), (7), for the real part we get:

$$
\begin{aligned}
u(\xi,\tau)=&\frac{2}{\alpha}\Bigg\{\Big(\frac{\mathrm{ch}\frac{M}{2}(1+\xi)\mathrm{sh}\frac{a}{2}(1-\xi)\cos\frac{\alpha}{a}(1-\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}+\\
&+\frac{\mathrm{ch}\frac{M}{2}(1-\xi)\mathrm{sh}\frac{a}{2}(1+\xi)\cos\frac{\alpha}{a}(1+\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}\Big)\times\\
&\times\left[\left(\alpha A_0\mathrm{sh}a\cos\frac{2\alpha}{a}+D\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\cos\alpha\tau-\right.\\
&\left.-\left(D\mathrm{sh}a\cos\frac{2\alpha}{a}-\alpha A_0\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\sin\alpha\tau\right]-\\
&-\Big(\frac{\mathrm{ch}\frac{M}{2}(1+\xi)\mathrm{ch}\frac{a}{2}(1-\xi)\sin\frac{\alpha}{a}(1-\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}+\\
&+\frac{\mathrm{ch}\frac{M}{2}(1-\xi)\mathrm{ch}\frac{a}{2}(1+\xi)\sin\frac{\alpha}{a}(1+\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}\Big)\times\\
&\times\left[\left(\alpha A_0\mathrm{sh}a\cos\frac{2\alpha}{a}+D\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\sin\alpha\tau+\right.\\
&\left.+\left(D\mathrm{sh}a\cos\frac{2\alpha}{a}-\alpha A_0\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\cos\alpha\tau\right]+\frac{D}{2}\sin\alpha\tau\Bigg\},
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
h(\xi,\tau)=&\frac{2}{\alpha M}\Bigg\{\Big(\frac{\mathrm{sh}\frac{M}{2}(1-\xi)\mathrm{sh}\frac{a}{2}(1+\xi)\cos\frac{\alpha}{a}(1+\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}-\\
&-\frac{\mathrm{sh}\frac{M}{2}(1+\xi)\mathrm{sh}\frac{a}{2}(1-\xi)\cos\frac{\alpha}{a}(1-\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}\Big)\times\\
&\times\left[\left(\alpha A_0\mathrm{sh}a\cos\frac{2\alpha}{a}+D\mathrm{ch}a\cos\frac{2\alpha}{a}\right)\cos\alpha\tau+\right.\\
&\left.+\left(D\mathrm{sh}a\cos\frac{2\alpha}{a}-\alpha A_0\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\sin\alpha\tau\right]-
\end{aligned}
$$

$$
\begin{aligned}
&-\Bigg(\frac{\mathrm{sh}\frac{M}{2}(1-\xi)\mathrm{ch}\frac{a}{2}(1+\xi)\sin\frac{\alpha}{a}(1+\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}-\\
&-\frac{\mathrm{sh}\frac{M}{2}(1+\xi)\mathrm{ch}\frac{a}{2}(1-\xi)\sin\frac{\alpha}{a}(1-\xi)}{\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}}\Bigg)\times\\
&\quad\times\Bigg[\left(\alpha A_0\mathrm{sh}a\cos\frac{2\alpha}{a}+D\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\sin\alpha\tau+\\
&\quad+\left(D\mathrm{sh}a\cos\frac{2\alpha}{a}-\alpha A_0\mathrm{ch}a\sin\frac{2\alpha}{a}\right)\cos\alpha\tau\Bigg]\Bigg\},
\end{aligned}
\tag{10}
$$

where

$$a=\pm\sqrt{\frac{M^2}{2}+\frac{1}{2}\sqrt{M^2+16\alpha^2}}.$$

For the solution (8) of heat exchange equation let us consider some particular cases:

1) Consider the case when Prandtle number tends to zero ($S=\frac{\eta C_V}{K}\to 0$) and ($\alpha=\frac{\omega L^2}{\nu}$) is a bounded value, then in formulas (8), singling out real and imaginary parts, for the real part we have:

$$
\begin{aligned}
&\theta(\xi,\tau)-B_0\cos2\alpha\tau=\\
=&(P_1\cos\alpha\tau+q_1\sin2\alpha\tau)\big[f_1(\xi)+P_2(\mathrm{ch}M\xi-\mathrm{ch}M)\big]-\\
-&(q_1\cos2\alpha\tau+P_1\sin2\alpha\tau)\big[g_1(\xi)+q_2(\mathrm{ch}M\xi-\mathrm{ch}M)\big],
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
P_1&=\frac{(D^2-\alpha^2A_0^2)\left(\mathrm{ch}2a\cos\frac{4\alpha}{a}-1\right)-2\alpha DA_0\mathrm{sh}2a\sin\frac{4\alpha}{a}}{2\alpha^2\left(\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}\right)^2},\\
q_1&=\frac{2\alpha DA_0\left(\mathrm{ch}2a\cos\frac{4\alpha}{a}-1\right)+(D^2-\alpha^2A_0^2)\mathrm{sh}2a\sin\frac{4\alpha}{a}}{2\alpha^2\left(\mathrm{ch}2\alpha-\cos\frac{4\alpha}{a}\right)^2},\\
P_2&=\frac{4\alpha}{M^2}\left(\mathrm{sh}a\sin\frac{2\alpha}{a}-\mathrm{ch}M\right),\quad p_2=\frac{4\alpha}{M^2}\mathrm{ch}a\cos\frac{2\alpha}{M},\\
f_1(\xi)&=\frac{1}{2}\Big[\mathrm{ch}M(1+\xi)\mathrm{ch}a(1-\xi)\cos\frac{2\alpha}{a}(1-\xi)+\\
&+\mathrm{ch}M(1-\xi)\mathrm{ch}a(1+\xi)\cos\frac{2\alpha}{a}(1+\xi)-\\
&-2\mathrm{ch}M\xi\mathrm{ch}\xi\cos\frac{2\alpha}{a}\xi+2\mathrm{ch}M\mathrm{ch}a\cos\frac{2\alpha}{a}-\mathrm{ch}2a\cos\frac{4\alpha}{a}-\mathrm{ch}2M\Big],
\end{aligned}
$$

$$g_1(\xi) = -\frac{1}{2}\Big[\mathrm{ch}M(1+\xi)\mathrm{sh}a(1-\xi)\sin\frac{2\alpha}{a}(1-\xi)+$$
$$+\mathrm{ch}M(1-\xi)\mathrm{sh}a(1+\xi)\sin\frac{2\alpha}{a}(1+\xi)-$$
$$-2\mathrm{ch}M\xi\mathrm{sh}\xi\sin\frac{2\alpha}{a}\xi+2\mathrm{ch}M\mathrm{sh}a\sin\frac{2\alpha}{a}-\mathrm{sh}2a\cos\frac{4\alpha}{a}\Big].$$

2) Consider then the case when every partical of liquid oscillates with the same phase and amplitude ($\alpha \to 0$), then passing to the limit in formula for $\alpha \to 0$.

$$\frac{\theta(\xi,\tau)-B_0}{D^2} = \frac{1}{M^2}\left[\frac{1-\xi^2}{2}+2\frac{\mathrm{ch}M\xi-\mathrm{ch}M}{\mathrm{sh}M}-\frac{\mathrm{ch}2M\xi-\mathrm{ch}2M}{4\mathrm{sh}^2M}\right]. \quad (12)$$

For friction force on the walls we have

$$F\big|_{\xi=\pm1} = \frac{\alpha A_0\mathrm{sh}a\cos\frac{2\alpha}{a}+D\mathrm{ch}a\sin\frac{2\alpha}{a}}{\alpha\Big(\mathrm{ch}2a-\cos\frac{4\alpha}{a}\Big)}\times$$
$$\times\left[\Big(a\cos\alpha\tau-\frac{2\alpha}{a}\sin\alpha\tau\Big)\Big(\pm\mathrm{ch}a\cos\frac{2\alpha}{a}\pm\mathrm{ch}M\Big)\pm\right.$$
$$\left.\pm\Big(a\sin\alpha\tau-\frac{2\alpha}{a}\cos\alpha\tau\Big)\mathrm{sh}a\sin\frac{2\alpha}{a}\right]-$$
$$-\frac{D\mathrm{sh}a\cos\frac{2\alpha}{a}-A_0\mathrm{ch}a\sin\frac{2\alpha}{a}}{\alpha\Big(\mathrm{ch}2a-\cos\frac{4\alpha}{a}\Big)}\times$$
$$\times\left[\Big(\frac{2\alpha}{a}\cos\alpha\tau+a\sin\alpha\tau\Big)\Big(\pm\mathrm{ch}a\cos\frac{2\alpha}{a}\pm\mathrm{ch}M\Big)\pm\right.$$
$$\left.\pm\Big(a\cos\alpha\tau+\frac{2\alpha}{a}\sin\alpha\tau\Big)\mathrm{sh}a\sin\frac{2\alpha}{a}\right],$$

while for liquid lose we have:

$$Q = \frac{2D\mathrm{sh}a\cos\frac{2\alpha}{a}-\alpha A_0\mathrm{ch}a\sin\frac{2\alpha}{a}}{\alpha^2\Big(\mathrm{ch}2a-\cos\frac{4\alpha}{a}\Big)}\left[\Big(M\mathrm{ch}a\cos\frac{2\alpha}{a}-a\mathrm{ch}M\Big)\cos\alpha\tau+\right.$$
$$\left.+\Big(\frac{2\alpha}{a}\mathrm{ch}M-M\mathrm{sh}a\sin\frac{2\alpha}{a}\Big)\sin\alpha\tau\right]+$$
$$+\frac{2\Big(\alpha A_0\mathrm{sh}a\cos\frac{2\alpha}{a}+D\mathrm{ch}a\sin\frac{2\alpha}{a}\Big)}{\alpha^2\Big(\mathrm{ch}2a-\cos\frac{4\alpha}{a}\Big)}\left[\Big(\frac{2\alpha}{a}\mathrm{ch}M-M\mathrm{sh}a\sin\frac{2\alpha}{a}\Big)\times\right.$$

$$\times \cos\alpha\tau + + \Big(M \text{ch} a \cos\frac{2\alpha}{a} - a\text{ch}M\Big)\sin\frac{\alpha}{\tau}\Big] + \frac{2D}{a}\sin\alpha\tau.$$

Calculation show that the action of megnetic field on pulsating motion of liquid causes halting of the pulsating flow of liquid. Increase of the frequence of pulsating motion and of external magnetic field causes the decrease of velocity of electro-conductive liquid flow.

When each particle of liquid oscillates with the same amplitude in the same phase ($\alpha \to 0$), then small pulsating motion of the walls in own plan does not influence on heat exch-ange in electro-conductive liquid, while the pulsating drop of pressure plays an important role in heat exchange. Particularly, if we increase pulsating drop and decrease external magnetic field acting on liquid, then the difference in temperature between initial pulsating distribution and final pulsating distribution decreases, while pulsating drop of pressure decreases and external magnitic field increases, then the difference in temperature increases.

When pulsating flow of electro-conductive liquid is caused by pulsating drop of pressure, then in period $2\alpha\tau = 0^0$, by increase of the criterium α on the axis of flat pipe and magnetic field (when $S \to 0$ and α is finite number), the difference in temperature between initial pulsating distribution and final pulsating distribution decreases, while the difference in temperature increases with decrease of α and M.

In period $2\alpha\tau = 0^0$ and $2\alpha\tau = \frac{\pi}{4}$, by increase of external magnetic field and α the friction on wall increases, while leakage of liquid decreases.

When $A_0 = B_0 = 0$, $D = 1$, $\alpha \to 0$ and S is finite value, then obtained results consides with those given in paper [1].

3 Conclusions

In general, pulsating motion caused by pulsating drop of pressure causes decrease of temperature of electro-conductive liquid.

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