SOME QUESTIONS OF APPROXIMATE SOLUTIONS FOR COMPOSITE BODIES WEAKENED BY CRACKS IN THE CASE OF ANTIPLANAR PROBLEMS OF ELASTICITY THEORY

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Abstract

Antiplane problems of the theory of elastisity by using the theory of analytical functions are presented in the papar. These problems lead to a system of singular integral equations with immovable singularity with the respected to leap of the tangent stress. The probems of behavior of solutions at the boundary are studied. A singular integral equation containing an immovable singularity is solved by collocation and asymptotic methods. It is shown that the system of the corresponding algebraic equations is solvable for sufficiently big number of the integral division. Experimental convergence of aproximate solutions to the exact one is detected.

Key words and phrases: cracks, antiplane problems, method integral equations, singular integral equations, collocation method, asymptotic method, alternative method.

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1 Introduction

Natural phenomena, such as earthquakes and floods, may cause occurrence of cracks in the pipes of great diameter, and in gas- and petrol tanks. Increase of the number and dimensions of cracks, from its part, causes to increase the volume of leakage that will result in great ecocatastrophe (see, for example [1]-[2]). The study of boundary value problems for the composite bodies weakened by cracks has a great practical significance. More interesting cases when cracks intersect an interface or penetrate the boundary at any angle will be investigated (see, for example [3]-[14]).

2 Statement of the problem

Let elastic Ω body occups complex variable plane $z = x+iy$ which is cut on the line $L = [-1; +1]$. The plane consists of two orthotropic homogeneous semiplanes

$$\Omega_1 = \{z : Rez \geq 0,\ x \notin L_1 = [0,1]\}$$

and

$$\Omega_z = \{z : Rez \leq 0,\ x \notin L_2 = [-1,0]\},$$

which are welded on the axis y. Define by index $k, k = 1,2$ values and functions connected with Ω_k. The problem is to find the function $w_k(x,y)$, which satisfies differential equation:

$$\frac{\partial^2 w_k(x,y)}{\partial x^2} + \lambda_k^2 \frac{\partial^2 w_k(x,y)}{\partial y^2} = 0, \quad (x,y) \in \Omega_k, \tag{2.1}$$

and boundary conditions: a) on the boundary of the crack tangent stresses are given:

$$b_{44}^{(k)} \frac{\partial w_k(x,\pm 0)}{\partial y} = q_k^{(\pm)}(x), \quad x \in L_k, \tag{2.2}$$

b) on the axis y the condition of continuity is fulfilled:

$$w_1(0;y) = w_2(0;y),\ y \in (-\infty;\infty),\ y \neq 0, \tag{2.3}$$

$$b_{55}^{(1)} \frac{\partial w_1(0;y)}{\partial x} = b_{55}^{(2)} \frac{\partial w_2(0;y)}{\partial x}, \tag{2.4}$$

where $\lambda_k^2 = \dfrac{b_{44}^{(k)}}{b_{55}^{(k)}}$, $b_{44}^{(k)}$, $b_{55}^{(k)}$ are elastic constants, $q_k(x)$ is a function of Holder's class, $k = 1,2$; In particular, if we have isotropic case $b_{44}^{(k)} = b_{55}^{(k)} = \mu_k,\ \lambda_k = 1$, where μ_k is module of displacement, $k = 1,2$;

If we use affine transformation and introduce the symbols

$$w_k(\xi, \lambda_k\eta) \equiv \widetilde{w}_k(\xi,\eta), \quad \frac{\lambda_k}{b_{44}^{(k)}} q_k(\xi) \equiv f_k(\xi), \quad \frac{b_{55}^{(2)}}{b_{55}^{(1)}} \equiv \gamma_2, \quad k = 1,2$$

we get $x = \xi,\ y = \lambda_k\eta,\ \lambda_k > 0,\ k = 1,2$ and (1)-(4) boundary problems will have forms:

$$\Delta\widetilde{w}_k(\xi,\eta) = \frac{\partial^2 \widetilde{w}_k(\xi,\eta)}{\partial \xi^2} + \frac{\partial^2 \widetilde{w}_k(\xi,\eta)}{\partial \eta^2} = 0, \quad (\xi,\eta) \in \Omega_k, \tag{2.5}$$

$$\frac{\partial \widetilde{w}_k(\xi,\pm 0)}{\partial \eta} = f_k(\xi), \quad \xi \in L_k, \tag{2.6}$$

$$\widetilde{w}_1(0,\eta) = \widetilde{w}_2(0,\eta), \quad \eta \in (-\infty;\infty), \quad \eta \neq 0, \tag{2.7}$$

$$\frac{\partial \widetilde{w}_1(0,\eta)}{\partial \xi} = \frac{\partial \widetilde{w}_2(0,\eta)}{\partial \xi}. \tag{2.8}$$

3 Integral equation method

Antiplane problems of the theory of elasticity by using the theory of analytical functions are presented in the paper. By using the functions of complex variable a unknown harmonic function $\widetilde{w}_k(\xi,\eta)$ will be presented in the following form:

$$\widetilde{w}_k(\xi,\eta)=\frac{1}{2}\left(\varphi_k(z)+\overline{\varphi}_k(z)\right), \tag{3.9}$$

where $\varphi_k(z)=u_k(\xi,\eta)+iv_k(\xi,\eta), \quad z=\xi+i\eta$.

There fore

$$\widetilde{w}_k(\xi,\eta)=Re\varphi_k(z). \tag{3.10}$$

These problems lead to a system of singular equations with immovable singularity with the respected to leap of the tangent stress. The problems of behavior of solutions at the boundary are studied. With using the theory of analytical functions (in particular, we use formulas of definition of piecewise holomorpic functions for given leap), also boundary value Sokhotski-Plemel formula of Cauchy type integral ([15],) from boundary conditions (6)-(8) the system of singular integral equations with respect to leaps $\rho_k(\xi)$

$$\int_0^1\left(\frac{1}{t-\xi}-\frac{a_1}{t+\xi}\right)\rho_1(t)dt+b_1\int_{-1}^0\frac{\rho_2(t)dt}{t-\xi}=2\pi f_1(\xi),\ \ \xi\in(0;1),$$

$$b_2\int_0^1\frac{\rho_1(t)dt}{t-\xi}+\int_{-1}^0\left(\frac{1}{t-\xi}-\frac{a_2}{t+\xi}\right)\rho_2(t)dt=2\pi f_2(\xi),\ \ \xi\in(-1;0), \tag{3.11}$$

where $\rho_k(x)=\Phi_k^+(x)-\Phi_k^-(x)$ real functions, $\Phi_k(z)=\varphi_k'(z)$,

$$a_k=\frac{1-\gamma_k}{1+\gamma_k},\quad b_k=\frac{2}{1+\gamma_k},\quad \gamma_1=1/\gamma_2,\quad \rho_k(x)\in H^*,\quad k=1,2.$$

Explanation of behavior of solutions near the ends of the boundary presents a special interest. The solutions of the system (11) of the integral equations can be presented in the following way:

$$\rho_1(t)=\frac{\chi_1(t)}{t^{\alpha_1}(1-t)^{\beta_1}},\qquad \rho_2(t)=\frac{\chi_2(t)}{t^{\alpha_2}(1+t)^{\beta_2}}, \tag{3.12}$$

where α_k, β_k are unknown constants $0<\alpha_k,\ \beta_k<1$, and $\chi_k(t)$ are functions, which belong to Holder's class, $k=1,2$. In the point $t=\pm1$ we obtain correspondingly $\beta_1=\beta_2=\dfrac{1}{2}$. In the considered case there is no peculiarity in the point $t=0$.

In a partial case when one half-plane has a rectilinear cut of finite length, which is perpendicular to the boundary , and one end of which is located on the boundary. We have one singular integral equation containing an immovable singularity. In a partial case, when a crack reaches the boundary of separation, we get that an order of peculiarity in the point $t = 0$. depends on elastic constants of material and belongs to $0 < \alpha < 1$. Let, $\rho_2(x) \equiv 0,\ \rho_1(x) \neq 0$, then integral of the system (7) we have one integral equation :

$$\int_0^1 \left(\frac{1}{t-\xi} - \frac{a_1}{t+\xi} \right) \rho(t)dt = 2\pi f_1(\xi),\ \ \xi \in (0;1). \qquad (3.13)$$

We get that an order of peculiarity in the point $t = 0$ depends on elastic constants of material and belongs to interval $(0; 1)$. $\alpha = 1 - \frac{1}{\pi} arccos \left(\frac{b_{55}^{(1)} - b_{55}^{(2)}}{b_{55}^{(1)} + b_{55}^{(2)}} \right) \in (0;1),\ \beta = \frac{1}{2}$. If $b_{55}^{(1)} = b_{55}^{(2)}$, then $\alpha = \frac{1}{2}$.

4 On approximate solution of one singular integral equation containing an immovable singularity

Antiplaned problems of elasticity theory , composed for orthotropic plane, weakened with cracks, are reduced to the following integral equation containing an immovable singularity (see [11]-[12]):

$$\frac{1}{\pi} \int_0^1 \left(\frac{1}{t-x} + \frac{\varepsilon}{t+x} \right) u(t)dt = f(x),\ \ x \in [0;1], \qquad (4.14)$$

where $u(t) \in H^*([0,1]),\ \varepsilon \in [-1,1],\ f(x) \in H_\mu[0,1],\ 0 < \mu \leq 1$.

Analysis of the above-mentioned integral equation and study of their exact and approximate solving methods are accompanied with some specific complexities due to the fact, that the solution has a composite asymptotic, which can be considered only in certain cases introducing weight functions. If we have square root type singularity on both ends of the integral, then Chebishev orthogonal polynoms can be used.

The solution of equation (16) we can represent in the following form:

$$u(t) = \frac{u_0(t)}{t^\alpha \sqrt{1-t}}\ , \qquad (4.15)$$

where $u_0(t) \in H([0,1])$. α depends on material elasticity constants, $0 < \alpha < 1$.

Integral equation (14) can be solved by three approximate methods: spectral, collocation and asymptotic ones. In the work we use the collocation and asymptotic method.

5 Collocation method

In the work we use the collocation method (see, for example [16]-[19]). We consider cases as regular intervals, so nonregular located knots in relation to the integrated variable. In the first case the integral is replaced with the quadrature formula of open type, and in the second case the quadrature formula of the higher accuracy. We take roots of polynoms Chebishev of the first sort as knots. For approximate solution of the above-mentioned problem we form the program. The program is examined with testing problem. Several numerical experiments gave the satisfactory results.

As it was mentioned above, with respect to the variable t we get $t_j = \varepsilon_1 + (j-1)h_1,\ j = 1, 2, \cdots, 2n+1,\ h_1 = \dfrac{1-2\varepsilon_1}{2n},\ \varepsilon_1 \neq 0$ is a small parameter, $2n$ - even number of the interval division. We use trapezoid and Simpson's quadrature formulas. The variable x takes in turn the values: $0 < x_1 < x_2 < \cdots < x_{2n} < x_{2n+1} < 1$; to determine the values of required functions we get a linear algebraic equation system. We can consider nodes x_i of different form, such as

a)$x_i = \varepsilon_2 + (i-1)h_2,\ h_2 = \dfrac{1-2\varepsilon_2}{2n},\ \varepsilon_2 \neq \varepsilon_1,\ i = 1, 2, \cdots, 2n+1.$

b) $x_i = \sin(\varepsilon_3 + (i-1)h_3),\ h_3 = \dfrac{\pi/2 - 2\varepsilon_3}{2n},\ i = 1, 2, \cdots, 2n+1.$

For algorithm realization by the main condition is that factor that it is necessary to elimination of especial cases in underintegral functions. For definition of values of unknown function in netgrids we receive system of the linear algebraic equations of an order $2n+1$. In the second case the set integral is replaced by Gauss cvadratur formulas of the higher order. We take as knots roots of Chebishev polynoms of the first sort $t_j = \cos\dfrac{2j}{2n+1}\pi$, $j = 1, 2, \cdots, n$. We apply following Gauss formula for syngular integrals (see [19])

$$\int_{-1}^{+1} \frac{\varphi(\xi)}{\xi-\eta} \frac{d\xi}{\sqrt{1-\xi^2}} \cong \frac{\pi}{n} \sum_{j=1}^{n} \varphi(\xi_j) \frac{1 - T_{n-1}(\xi_j)U_{n-1}(\eta)}{\xi_j - \eta}, \tag{5.16}$$

$$-1 \leq \eta \leq +1,$$

which is exact for polynoms of degree no more than $2n$. We take as knots also roots of Chebishev polynom of the first sort on a variable $x_i =$

$\cos\dfrac{2i-1}{2n+1}\pi,\ i = 1,2,\cdots,n$. As well as in the previous case we receive system of the linear algebraic equations of an order n in knots of a grid for definition of values of unknown function.

6 Asymptotic method

Naturally, the small parameter enters in the integrated equation (14) in a member with motionless feature and consequently for it solve we use asymptotical method.

Perturbation theory is a method to study and solve topical problems of science and technology. It comprises mathematical methods that are used to find an approximate solution to a problem which cannot be solved exactly; among them Poincare-Lyapunov's method, known as a small parameter method, is widely applied as one of the most powerful methods of research and calculation, but its convergence is asymptotic. While using the asymptotic method we obtain a two-point recurrent system of equations. In the present paper, problems of approximate solution of some boundary value problems are studied by means of a numerical-experimental method based on an approach, alternative to the asymptotic method. The method is developed by T. Vashakmadze ([20], p. 124-127). It is based on presentation of the required vector with an orthogonal series instead of asymptotic one. In this case there is obtaining three-point recurrent system of operator equations of the special structure. A solution of this system is obtained inverting a relatively simple operator N-times and acting with the operator describing a perturbation degree over the known magnitudes N-times. The degree N of the polynomial defines exactness of the method.

Algorithms of approximate solution of a linear nonhomogeneous operator equation are considered using both asymptotic and its alternative methods in [21]-[22]. There were shown general formulation of both methods and essential differences between them. Their comparative analysis was made. In the present paper the above-mentioned methods are approved for problems of finding an approximate solution of a singular integral equations containing an immovable singularity.

Let us have a nonhomogeneous operator equation

$$Lu + \varepsilon Mu = f, \tag{6.17}$$

where L and M are linear operators in any standardized space. In addition, there exist inverse operators L^{-1} and $(L+\varepsilon M)^{-1}$, where $\varepsilon \in [-1;+1]$ is a small parameter.

Represent the solution $u(x)$ in the form of Fourier-Legandre series

$$u(x) = \gamma \sum_{k=0}^{\infty} \varepsilon^k \nu_k(x) + (1-\gamma) \sum_{k=0}^{\infty} P_k(\varepsilon) w_k(x), \tag{6.18}$$

where $\{P_k(\varepsilon)\}$ is a system of Legendre polynomials, $w_k(x)$ and $\nu_k(x)$ are unknown coefficients, γ is numerical parameter.

In case $\gamma = 1$ we have an asymptotic method (Poincare-Lyapunov's method)

$$u(x) = \sum_{k=0}^{\infty} \varepsilon^k \nu_k(x). \tag{6.19}$$

By putting series (19) into equation (17) and equating coefficients of terms with the same degrees of ε, we get a system of two-point recurrence operator equations having the following form:

$$\begin{cases} L\nu_0 = f_0, \\ L\nu_k = -M\nu_{k-1} + f_k, \quad k = \overline{1, \infty}. \end{cases} \tag{6.20}$$

When $\gamma = 0$, we have an approach alternative to the asymptotic method

$$u(x) = \sum_{k=0}^{\infty} P_k(\varepsilon) w_k(x). \tag{6.21}$$

If set series (21) into equation (17) use the main properties of Legandre polynomial and equate coefficients with equal degrees of ε, we shall get a system of three -point recurrence operator equations of the following form:

$$\begin{cases} Lw_0 + \dfrac{1}{3} M w_1 = f_0, \\ Lw_k + \dfrac{k}{2k-1} M w_{k-1} + \dfrac{k+1}{2k+3} M w_{k+1} = f_k, \quad k = 1, 2, 3, \cdots. \end{cases} \tag{6.22}$$

Suppose that the right hand side of equations f does not depend on small parameter ε therefore, $f_k = 0,\ k = 1, 2, 3, \cdots$, and even it depends on ε, this fact has no essential influence on realization of computing schemes.

Solutions of system of double-point operator equations (20) have the following simple form:

$$\nu_0 = L^{-1} f_0, \quad \nu_i = (-1)^i L^{-1} M \nu_{i-1} \quad i = 1, 2, 3, \cdots. \tag{6.23}$$

Instead of infinite system of three-point operator equations (22), let us take its finite part, in addition even number of equations ($N = 2n,\ n \in \mathbb{N}$). The obtained system of equations may be decomposed into two subsystems, We find w_k, by means of regular process (see [20], p. 124-127). For clearness, let us consider special cases, when $N = 2$ and $N = 4$. When $N = 2$, we get $w_0 = \nu_0$, $w_1 = \nu_1$, and approximate solution of equation (17) has a form:

$$u_1(x) = w_0(x) + \varepsilon w_1(x). \tag{6.24}$$

When $N = 4$ then we have $w_0 = \nu_0 + \frac{1}{3}\nu_2$, $w_2 = \frac{2}{3}\nu_2$, $w_1 = \nu_1 + \frac{3}{5}\nu_1$, $w_3 = \frac{2}{5}\nu_3$ and approximate solution of equation (17) has a form

$$u_3 = w_0(x) + \varepsilon w_1(x) + P_2(\varepsilon)w_2(x) + P_3(\varepsilon)w_3(x), \tag{6.25}$$

where $P_2(\varepsilon) = \frac{1}{2}(3\varepsilon^2 - 1)$, $P_3(\varepsilon) = \frac{1}{2}(5\varepsilon^3 - 3\varepsilon)$.

The above-mentioned method has been approved for approximate solution of singular integral equations containing an immovable singularity. In the present paper, boundary problem (16) is solved by asymptotic method and by an approach, alternative to the asymptotic one. In our case the basic operator is $Lu = \frac{1}{\pi}\int_0^1 \frac{1}{t-x}u(t)dt$ and operator describing the perturbation degree is $Mu = \int_0^1 \frac{1}{t+x}u(t)dt$.

Let us discuss a case when there is a square root type singularity on each end of the interval . To invert main operators we use well-known formula (see [15]). Analogously, in the case of asymptotic method the approximation formula of the inverted operator are constructed :

$$\nu_0(x) = \frac{\pi}{\sqrt{x(1-x)}}\int_0^1 \frac{\sqrt{t(1-t)}}{t-x}f(t)dt,$$

$$\nu_i(x) = \frac{\pi}{\sqrt{x(1-x)}}\int_0^1 \frac{\sqrt{t(1-t)}}{t-x}\int_0^1 \frac{1}{t+\xi}\nu_{i-1}(\xi)d\xi dt, \quad i = 1, 2, 3, \cdots . \tag{6.26}$$

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