THE PROBLEM OF STATICS IN THE LINEAR THEORY OF ELASTIC MIXTURES FOR INFINITE REGION WITH PARTIALLY UNKNOWN BOUNDARY

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Abstract

Using methods of the theory of analytic functions, in the paper we investigate the plane problem of statics in the linear theory of elastic mixtures for infinite region weakened by an curvilinear shape having cuts off with equidurable boundaries at the vertices.

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1 Some auxiliary formulas and operators

The homogeneous equation of statics of the linear theory of elastic mixture in the complex form is written as in [4]

$$\frac{\partial^2 U}{\partial z \partial \overline{z}} + \c{K} \, \frac{\partial^2 \overline{U}}{\partial \overline{z}^{\,2}} = 0, \tag{1}$$

where $z = x_1 + ix_2$, $\overline{z} = x_1 - ix_2$, $\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\,\frac{\partial}{\partial x_2}\right)$, $\frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\,\frac{\partial}{\partial x_2}\right)$, $U = (u_1 + iu_2, u_3 + iu_4)^\top$, $u' = (u_1, u_2)^\top$ and $u'' = (u_3, u_4)^\top$ are the partial displacements;

$$\c{K} = -\frac{1}{2}\, e m^{-1}, \quad e = \begin{bmatrix} e_4 & e_5 \\ e_5 & e_6 \end{bmatrix}, \quad m^{-1} = \frac{1}{\Delta_0}\begin{bmatrix} m_3 & -m_2 \\ -m_2 & m_1 \end{bmatrix},$$

$\Delta_0 = m_1 m_3 - m_2^2$, $m_k = e_k + \frac{1}{2}\, e_{3+k}$, $k = 1, 2, 3$, $e_1 = a_2/d_2$, $e_2 = -c/d_2$, $e_3 = a_1/d_2$, $d_2 = a_1 a_2 - c^2$, $a_1 = \mu_1 - \lambda_5$, $a_2 = \mu_2 - \lambda_5$, $c = \mu_3 + \lambda_5$, $e_1 + e_4 = b/d_1$, $e_2 + e_5 = -c_0/d_1$, $e_3 + e_6 = a/d_1$, $d_1 = ab - c_0^2$, $a = a_1 + b_1$, $b = a_2 + b_2$, $c_0 = c + d$, $bq = \mu_1 + \lambda_2 + \lambda_5 + (-1)^q \alpha_2 \rho_{3-q}/\rho$, $q = 1, 2$, $\alpha_2 = \lambda_3 - \lambda_4$, $\rho = \rho_1 + \rho_2$, $d = \mu_2 + \lambda_3 - \lambda_5 - \alpha_2 \rho_1/\rho \equiv \mu_3 + \lambda_4 - \lambda_5 + \alpha_2 \rho_1/\rho$.

Here ρ_1 and ρ_2 are partial densities ($\rho_1 > 0$, $\rho_2 > 0$) and μ_k, $k = 1,2,3$, λ_p, $p = \overline{1,5}$, are constants characterizing physical properties of the elastic mixture and satisfying certain inequalities [7].

In [2] M. Basheleishvili obtained the representations

$$U = (u_1 + iu_2, u_3 + iu_4)^\top = m\varphi(z) + \frac{1}{2}\, ez\overline{\varphi'(z)} + \overline{\psi(z)}, \tag{2}$$

$$\begin{aligned} TU &= \begin{pmatrix} (Tu)_2 - i(Tu)_1 \\ (Tu)_4 - i(Tu)_3 \end{pmatrix} \\ &= \frac{\partial}{\partial s(x)} \left[(A - 2E)\varphi(z) + Bz\overline{\varphi'(z)} + 2\mu\overline{\psi(z)} \right], \end{aligned} \tag{3}$$

where $\varphi(z) = (\varphi_1, \varphi_2)^\top$ and $\psi(z) = (\psi_1, \psi_2)^\top$ are arbitrary analytic vector-functions,

$$A = 2\mu m, \quad \mu = \begin{bmatrix} \mu_1 & \mu_3 \\ \mu_3 & \mu_2 \end{bmatrix}, \quad m = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_3 \end{bmatrix},$$

$$B = \mu m, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \frac{\partial}{\partial s(x)} = -n_2 \frac{\partial}{\partial x_1} + n_1 \frac{\partial}{\partial x_2},$$

n_1 and n_2 are the projections of the unit vector of the normal onto the axes $0x_1$ and $0x_2$, $(Tu)_p$, $p = \overline{1,4}$, are components of the stress vector [2]:

$$\begin{aligned}
(Tu)_1 &= r'_{11}n_1 + r'_{21}n_2, \quad r'_{11} = a\theta' + c_0\theta'' - 2\frac{\partial}{\partial x_2}(\mu_1 u_2 + \mu_3 u_4), \\
& r'_{21} = -a_1\omega' - c\,\omega'' + 2\frac{\partial}{\partial x_1}(\mu_1 u_2 + \mu_3 u_4), \\
(Tu)_2 &= r'_{12}n_1 + r'_{22}n_2, \quad r'_{12} = a_1\omega' + c\omega'' + 2\frac{\partial}{\partial x_2}(\mu_1 u_1 + \mu_3 u_3), \\
& r'_{22} = a\theta' + c_0\theta'' - 2\frac{\partial}{\partial x_1}(\mu_1 u_1 + \mu_3 u_3), \\
(Tu)_3 &= r''_{11}n_1 + r''_{21}n_2, \quad r''_{11} = c_0\theta' + b\theta'' - 2\frac{\partial}{\partial x_2}(\mu_3 u_2 + \mu_2 u_4), \\
& r''_{21} = -c\omega' - a_2\omega'' + 2\frac{\partial}{\partial x_1}(\mu_3 u_2 + \mu_2 u_4), \\
(Tu)_4 &= r''_{12}n_1 + r''_{22}n_2, \quad r''_{12} = c\,\omega' + a_2\omega'' + 2\frac{\partial}{\partial x_2}(\mu_3 u_1 + \mu_2 u_3), \\
& r''_{22} = c_0\theta' + b\theta'' - 2\frac{\partial}{\partial x_1}(\mu_3 u_1 + \mu_2 u_3), \\
& \theta' = \operatorname{div} u' = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}, \quad \theta'' = \operatorname{div} u'' = \frac{\partial u_3}{\partial x_1} + \frac{\partial u_4}{\partial x_2}, \\
& \omega' = \operatorname{rot} u' = \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}, \quad \omega'' = \operatorname{rot} u'' = \frac{\partial u_4}{\partial x_1} - \frac{\partial u_3}{\partial x_2}.
\end{aligned}$$

Let us now consider the vectors

$$\overset{(1)}{\tau} = (r'_{11}, r''_{11})^\top, \quad \overset{(2)}{\tau} = (r'_{22}, r''_{22}), \quad \tau = \overset{(1)}{\tau} + \overset{(2)}{\tau}, \tag{4}$$

$$\overset{(1)}{\eta} = (r'_{21}, r''_{21})^\top, \quad \overset{(2)}{\eta} = (r'_{12}, r''_{12}), \quad \eta = \overset{(1)}{\eta} + \overset{(2)}{\eta}, \quad \varepsilon^* = \overset{(1)}{\eta} - \overset{(2)}{\eta}. \tag{5}$$

After lengthy but elementary calculations we obtain

$$\tau = \overset{(1)}{\tau} + \overset{(2)}{\tau} = 2(2E - A - B)\,\mathrm{Re}\,\Phi(z), \tag{6}$$

$$\varepsilon^* = \overset{(1)}{\eta} - \overset{(2)}{\eta} = 2(A - B - 2E)\,\mathrm{Im}\,\Phi(z), \tag{7}$$

$$\overset{(1)}{\tau} - \overset{(2)}{\tau} - i\eta = 2(B\overline{z}\Phi'(z) + 2\mu\Psi(z)); \tag{8}$$

here $\Phi(z) = \varphi'(z)$, $\Psi(z) = \psi'(z)$, $\det(2E - A - B) > 0$ [3].

Let L be a smooth curve and $O_1 \in L$. Consider the right orthogonal coordinate system $(\boldsymbol{n}O_1\boldsymbol{s})$. By n we denote the outer normal vector to L at the point O_1 and by $\boldsymbol{s}$ the tangent vector. Suppose that $\boldsymbol{n} = (n_1, n_2)^\top = (\cos\alpha, \sin\alpha)^\top$ and $\boldsymbol{s} = (-n_2, n_1)^\top = (-\sin\alpha, \cos\alpha)^\top$, where $\alpha = \alpha(t)$ is the angle between the outer normal $\boldsymbol{n}$ to the contour L at the point O_1 and the $0x_1$ axis.

Next we construct the vectors

$$\sigma_n = \begin{pmatrix} (Tu)_1 n_1 + (Tu)_2 n_2 \\ (Tu)_3 n_1 + (Tu)_4 n_2 \end{pmatrix} = \overset{(1)}{\tau}\cos^2\alpha + \overset{(2)}{\tau}\sin^2\alpha + \eta\sin\alpha\cos\alpha, \tag{9}$$

$$\sigma_s = \begin{pmatrix} (Tu)_2 - i(Tu)_1 \\ (Tu)_4 - i(Tu)_3 \end{pmatrix} = \frac{1}{2}(\overset{(2)}{\tau} - \overset{(1)}{\tau})\sin 2\alpha + \frac{1}{2}\eta\cos 2\alpha - \frac{1}{2}\varepsilon^*, \tag{10}$$

$$\sigma_s^* = \sigma_s + \frac{1}{2}\varepsilon^*, \tag{11}$$

$$\begin{aligned} \sigma_t &= \begin{pmatrix} [r'_{21}n_2 - r'_{11}n_2, r'_{22}n_1 - r'_{12}n_2]^\top s \\ [r''_{21}n_1 - r''_{11}n_2, r''_{22}n_1 - r''_{12}n_2]^\top s \end{pmatrix} \\ &= \overset{(1)}{\tau}\sin^2\alpha + \overset{(2)}{\tau}\cos^2\alpha - \eta\sin\alpha\cos\alpha. \end{aligned} \tag{12}$$

From (9)–(12) and (6)–(8) we obtain

$$\sigma_n + \sigma_t = \tau = \overset{(1)}{\tau} + \overset{(2)}{\tau} = 2(2E - A - B)\,\mathrm{Re}\,\Phi(t), \tag{13}$$

$$\sigma_n - i\sigma_s = (2E - A)\overline{\Phi(t)} - B\Phi(t) + \big(B\overline{t}\Phi(t) + 2\mu\Psi(t)\big)\,e^{2i\alpha(t)}, \tag{14}$$

on L and with elementary calculation we get

$$\sigma_n + 2\mu\left(\frac{\partial U_s}{\partial s} + \frac{U_n}{\rho_0}\right) + i\left[\sigma_s - 2\mu\left(\frac{\partial U_n}{\partial s} - \frac{U_s}{\rho_0}\right)\right] = 2\Phi(t), \tag{15}$$

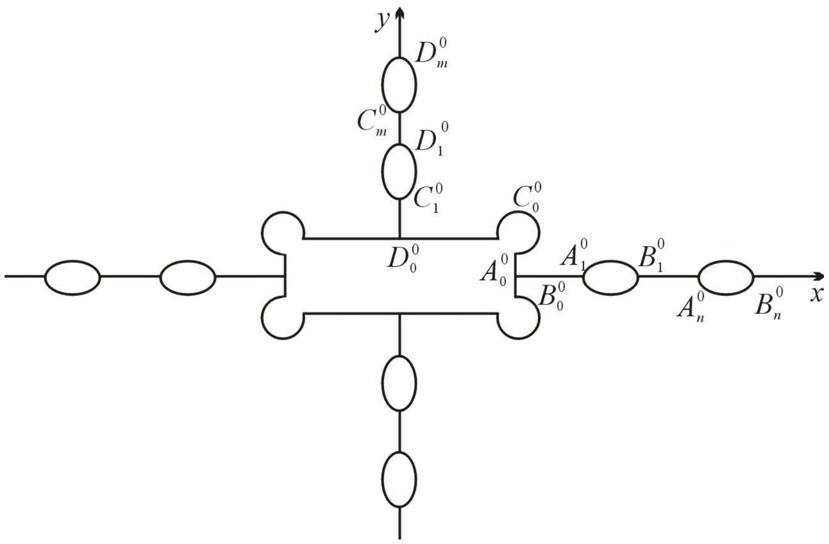

Figure 1:

on L where $1/\rho_0$ is the curvature of the curve at the point $t = t_1 + it_2$,

$$U_n = \begin{pmatrix} u_1 n_1 + u_2 n_2 \\ u_3 n_1 + u_4 n_2 \end{pmatrix}, \quad U_s = \begin{pmatrix} u_2 n_1 - u_1 n_2 \\ u_4 n_1 - u_3 n_2 \end{pmatrix}. \tag{16}$$

(2), (3), (6), (8) and (13)–(15) represent analogues to Kolosov–Muskhelishvili formulas in the theory of elastic mixtures.

2 Statement of the problem and the method of it solve

Let an infinite elastic plate occupy on the $z = x_1 + ix_2$ the exterior of the rectangle with cuts off at the vertices and with holes located along the coordinate axes. Suppose that unknown contours of the holes and cuts off are smooth and the region occupied by the plate is symmetric with respect to the $0x_1$ and $0x_2$ axis (Figure 1).

Suppose that the contours of the cuts off and holes are free from external forces and, moreover, vectors σ_s and U_n on the sides of the rectangle equal to zero. The constant field of contracting stresses acts at infinite

$$\overset{(1)}{\tau}{}^{\infty} = p, \quad \overset{(2)}{\tau}{}^{\infty} = q, \quad \eta^{\infty} = 0, \tag{17}$$

where $p = (p_1, p_2)^\top$ and $q = (q_1, q_2)^\top$ are real known constant vectors and satisfying the conditions

$$(p_1 - q_1)(p_1 + q_1)^{-1} = (p_2 - q_2)(p_2 + q_2)^{-1} = \gamma, \quad \gamma \neq 1. \tag{18}$$

We are seeking both the state of stress in the plate and shape of the contours of the holes and cuts off such that the vector (12) (σ_t) on them would take the constant value $\sigma_t = k^0$, $k^0 = (k_1^0, k_2^0) = const$ will be defined later in the course of solving the problem.

Such kind of problem for equations of statics in the plane theory of elasticity has been considered in [6].

By the phisical and geometric symmetry of the problem we consider only the first quadrant $x_1 > 0$, $x_2 > 0$, which we denote by $\mathcal{D}$ and its boundary by $L = L_0 + L_1 + L_2$. L_0 denotes a set of unknown arcs, L_1 and L_2 are the sets of rectangular parts of the boundary which are parallel to the $0x_1$ and $0x_2$ axis respectively:

$$L_0 = A^0C^0 + \bigcup_{j=1}^{n} A_j^0 B_j^0 + \bigcup_{j=1}^{m} C_j^0 D_j^0, \quad L_1 = D^0C^0 + \bigcup_{j=0}^{n} B_j^0 A_j^0,$$

$$A_{n+1}^0 = \infty, \quad L_2 = B^0A^0 + \bigcup_{j=0}^{m} D_j^0 C_j^0, \quad C_{m+1}^0 = \infty.$$

Here $2n$ and $2m$ are numbers of holes along the $0x_1$ and $0x_2$ axis, respectively. The points of the plane affixes of these points are denoted by one and the same symbols.

From the conditions of the problem it follows that

$$\sigma_n = 0, \quad \sigma_s = 0, \quad \sigma_t = k^0 \qquad \text{on } L_0, \tag{19}$$

$$U_n = 0, \quad \sigma_s = 0 \qquad \text{on } L_1 + L_2. \tag{20}$$

By virtue of (13)–(15) and the conditions (17), (19) and (20) the stressed state of the body $\mathcal{D}$ is described by two analytic vector-functions $\Phi(z)$ and $\Psi(z)$.

In our case

$$\alpha(t) = \frac{\pi}{2}(3j - 4), \quad t \in L_j, \quad j = 1, 2, \tag{21}$$

but the angle $\alpha(t)$ on the contour L_0 is unknown because contour L_0 itself is unknown.

The vector-functions $\Phi(z) = (\phi_1, \phi_2)^\top$ and $\Psi(z) = (\Psi_1, \Psi_2)^\top$ for large $|z|$ can be represented in the form (see (6), (8) and (17))

$$\Phi(z) = M + O(z^{-2}), \quad M = \frac{1}{2}(2E - A - B)^{-1}(p + q), \tag{22}$$

$$\Psi(z) = h + O(z^{-2}), \quad h = \frac{1}{4}\,\mu^{-1}(p-q), \tag{23}$$

and the boundaries in the neighborhood of angular points satisfy the conditions

$$|\phi_j(z)|,\ |\psi_j(z)| < M_j^0 |z-c|^{-\delta}, \quad 0 \le \delta < 1, \quad j = 1,2. \tag{24}$$

By condition (20) from (15) we obtain $\operatorname{Im}\Phi(t) = 0$, $t \in L_1 + L_2$. Equation (13) yields (see (19))

$$\operatorname{Re}\Phi(t) = \frac{1}{2}\,(2E - A - B)^{-1} k^0, \quad t \in L_0.$$

Relying on the above results we obtain [1]

$$\Phi(z) = M, \quad k^0 = p + q. \tag{25}$$

Substituting (25) into (14) and taking into account (13) and (20), for boundary value of $\Psi(z)$ we have

$$e^{2i\alpha(t)}\Psi(t) = H, \quad t \in L_0, \tag{26}$$

$$\operatorname{Im}\Psi(t) = 0, \quad t \in L_1 + L_2, \tag{27}$$

where

$$H = -\frac{1}{4}\,\mu^{-1}k^0 = -\frac{1}{4}\,\mu^{-1}(p+q). \tag{28}$$

It can easily be shown that

$$\operatorname{Re} e^{-i\alpha(t)} t = A^*(t), \quad t \in L_1 + L_2, \tag{29}$$

where $A^*(t)$ is the piecewise constant function.

Let the function $z = \omega(\zeta)$ map conformally the half-plane $\operatorname{Im}\zeta > 0$ ($\zeta = \xi_1 + i\xi_2$) onto the region $\mathcal{D}$ in such a way that $\omega(\infty) = \infty$, $\omega(a_j) = A_j^0$, $\omega(b_j) = B_j^0$, $j = \overline{0,n}$; $\omega(c_j) = C_j^0$, $\omega(d_j) = D_j^0$, $j = \overline{0,m}$, where a_j, b_j, c_j and d_j are points of the real axis of the half-plane $\operatorname{Im}\zeta > 0$,

$$-\infty < d_m < c_m < \cdots < d_0 < c_0^0 < 0 < a_0 < b_0 < \cdots < a_n < b_n < +\infty.$$

Here we can fix three points and the remaining ones we have to define.

Denote by ℓ_0, ℓ_1 and ℓ_2 the images of the lines L_0, L_1 and L_2, respectively. We have

$$\ell_0 = \bigcup_{j=1}^{m}(d_j, c_j) + (c_0, a_0) + \bigcup_{j=1}^{n}(a_j, b_j),$$

$$\ell_1 = (d_0, c_0) + \bigcup_{j=0}^{n} (b_j, a_{j+1}), \quad a_{n+1} = \infty,$$

$$\ell_2 = (a_0, b_0) + \bigcup_{j=1}^{m} (d_{j+1}, c_j), \quad d_{m+1} = -\infty.$$

By the change of variable $z = \omega(\zeta)$ conditions (26)–(29) take the form

$$\begin{aligned} &\Psi_0(\xi_1)\omega'(\xi_1) + H\overline{\omega'(\xi_1)} = 0, \quad \xi_1 \in \ell_0, \\ &\operatorname{Im}\Psi_0(\xi_1) = 0, \quad \xi_1 \in \ell_1 + \ell_2, \\ &\operatorname{Re}\omega(\xi_1) = A_0^*(\xi_1), \quad \xi_1 \in \ell_2, \\ &\operatorname{Im}\omega(\xi_1) = B^*(\xi_1), \quad \xi_1 \in \ell_1, \end{aligned} \tag{30}$$

where $A_0^*(\xi_1)$ and $B^*(\xi_1)$ are piecewise constant functions, the vector function $\Psi_0(\zeta) = \Psi(\omega(\zeta))$ is analytic in the region $\operatorname{Im}\zeta > 0$ which for large $|\zeta|$ can be represented in the form

$$\Psi_0(\zeta) = h + O(\zeta^{-1}),$$

$\omega(\zeta)$ in the neighborhood of the points a_j, b_j, c_j, d_j the function $\omega(\zeta)$is represented as

$$\omega(\zeta) = (\zeta - c)^{\delta}\omega_0(\zeta) + \omega(c), \quad 0 < \delta \le \frac{1}{2},$$

where $\omega_0(\zeta)$ is the holomorphic function in the neighbohood of the point c and $\omega_0(c) \ne 0$. For large $|\zeta|$ the function $\omega(\zeta)$ behaves $\zeta^{\frac{1}{2}}$.

In the region $\operatorname{Im}\zeta > 0$ we introduce analytic vector-functions $W(\zeta)$ and $\Omega(\zeta)$ which are defined as follows:

$$W(\zeta) = \frac{1}{2}\,\omega'(\zeta)(\Psi_0(\zeta) + H), \quad \Omega(\zeta) = \frac{1}{2}\,\omega'(\zeta)(\Psi_0(\zeta) - H). \tag{31}$$

It follows from (24) and (30) that the above vector-functions have at infinity zero of order $\frac{1}{2}$ and at the endpoints of the lines ℓ_0, ℓ_1 and ℓ_2 they have singularity of order less than unity.

Differentiating the last two formulas (31), for determination of the vector-functions $W(\zeta)$ and $\Omega(\zeta)$ we obtain

$$\operatorname{Re}W(\xi_1) = 0, \ \ \xi_1 \in \ell_0 + \ell_2, \quad \operatorname{Im}W(\xi_1) = 0, \ \ \xi_1 \in \ell_1, \tag{32}$$

$$\operatorname{Im}\Omega(\xi_1) = 0, \ \ \xi_1 \in \ell_0 + \ell_1, \quad \operatorname{Re}\Omega(\xi_1) = 0, \ \ \xi_1 \in \ell_2. \tag{33}$$

Problems (32) and (33) are, in fact, the Keldysh–Sedov problems for a half-plane. The solutions of these problems satisfying the above-mentioned conditions are given by the following formulas (see [5]):

$$W(\zeta) = \frac{1}{2}\,(h + H)\chi_1(\zeta)P_{n+1}(\zeta), \quad \Omega(\zeta) = \frac{1}{2}\,(h - H)\chi_2(\zeta)Q_{m+1}(\zeta), \tag{34}$$

$$\chi_1(\zeta) = \left[(\zeta - d_0)(\zeta - c_0)(\zeta - b_0)\prod_{j=1}^{n}(\zeta - a_j)(\zeta - b_j)\right]^{-\frac{1}{2}}, \tag{35}$$

$$\chi_2(\zeta) = \left[(\zeta - d_0)(\zeta - a_0)(\zeta - b_0)\prod_{j=1}^{n}(\zeta - d_j)(\zeta - c_j)\right]^{-\frac{1}{2}}, \tag{36}$$

where

$$P_{n+1}(\zeta) = \sum_{k=0}^{n+1} \alpha_k \zeta^k, \quad Q_{m+1}(\zeta) = \sum_{q=0}^{m+1} \beta_q \zeta^q, \tag{37}$$

α_k, $k = \overline{0, n+1}$, β_q, $q = \overline{0, m+1}$, are real constants, $\alpha_{n+1} = \beta_{m+1} = 1$. Moreover, from (18), (23) and (28) we have

$$h = -\gamma H. \tag{38}$$

On the basis of (31), (33) and (38) we define $\omega'(\zeta)$ and $\Psi_0(\zeta)$ as follows:

$$\omega(\zeta) = \int_{\zeta_0}^{\zeta} \left[(1-\gamma)\chi_1(\xi_1)P_{n+1}(\xi_1) - (1+\gamma)\chi_2(\xi_1)Q_{m+1}(\xi_1)\right] d\xi_1 + \overset{(1)}{c}, \tag{39}$$

$$\begin{aligned}&\Psi(\omega(\zeta))\\ &= h\int_{\zeta_0}^{\zeta} \left[(1-\gamma)\chi_1(\xi_1)P_{n+1}(\xi_1) - (1+\gamma)\chi_2(\xi_1)Q_{m+1}(\xi_1)\right] d\xi_1 + \overset{(2)}{c}.\end{aligned} \tag{40}$$

Moreover, from (25) we get

$$\varphi(\omega(\zeta)) = M\omega(\zeta) + \overset{(3)}{c}, \tag{41}$$

where ζ_0 is an arbitrary point of the half-plane, and $\overset{(j)}{c} = (\overset{(j)}{c}_1, \overset{(j)}{c}_2)^\top$, $j = 1, 2, 3$, are the constant vectors.

Having the function we can define the equation of the unknown part of the boundary L, and by formulas (40) and (41) we can investigate the stressed state of the body.

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