THE SECOND BVP OF STATICS OF THE THEORY OF ELASTIC TRANSVERSALLY ISOTROPIC BINARY MIXTURES FOR AN INFINITE STRIP

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Abstract

In this present paper the basic two-dimensional second BVP of statics of elastic transversally isotropic binary mixtures is investigated for an infinite strip. The solution of the basic BVP for the anisotropic strip is given in [1]. The present paper is an attempt to use this method for BVP of elastic mixture theory for a transversally-isotropic elastic strip. Using the potential method and the Fourier method, we solve effectively (in quadratures) the second BVP that has not been solved before.

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We say that a body is subject to a plane deformation if the second components u'_2 and $u"_2$ of the partial displacements vectors $u'(u'_1, u'_2, u'_3)$ and $u"(u"_1, u"_2, u"_3)$ vanish and the other components are functions of the variables only x_1, x_3. Then the basic homogeneous equations of statics of the transversally isotropic elastic binary mixtures theory in the case of plane deformation can be written in the form [2]

$$C(\partial x)U = \begin{pmatrix} C^{(1)}(\partial x) & C^{(3)}(\partial x) \\ C^{(3)}(\partial x) & C^{(2)}(\partial x) \end{pmatrix} U = 0, \tag{1}$$

where the components of the matrix $C^{(j)}(\partial x) = \|C^{(j)}_{pq}(\partial x)\|_{2x2}$ are given in the form

$$C^{(j)}_{pq} = C^{(j)}_{qp}, j = 1,2,3; \quad p,q = 1,2, \quad C^{(j)}_{11}(\partial x) = c^{(j)}_{11}\frac{\partial^2}{\partial x_1^2} + +c^{(j)}_{44}\frac{\partial^2}{\partial x_3^2},$$
$$C^{(j)}_{12}(\partial x) = (c^{(j)}_{13} + c^{(j)}_{44})\frac{\partial^2}{\partial x_1 \partial x_3}, \quad C^{(j)}_{22}(\partial x) = c^{(j)}_{44}\frac{\partial^2}{\partial x_1^2} + c^{(j)}_{33}\frac{\partial^2}{\partial x_3^2},$$

$c^{(k)}_{pq}$ are constants, characterizing the physical properties of the mixture and satisfying certain inequalities caused by the positive definiteness of potential energy.$U = U^T(x) = (u', u'')$ is four-dimensional displacement vector-function, $u'(x) = (u'_1, u'_3)$ and $u''(x) = (u''_1, u''_3)$ are partial displacement vectors depending on the variables x_1, x_3. Throughout this paper superscript T denotes transposition.

The stress vector is defined as follows [2]

$$T(\partial x, n)U = \begin{pmatrix} T^{(1)}(\partial x, n) & T^{(3)}(\partial x, n) \\ T^{(3)}(\partial x, n) & T^{(2)}(\partial x, n) \end{pmatrix} U, \tag{2}$$

where

$$T^j(\partial x, n) = \begin{pmatrix} c_{11}^{(j)} n_1 \partial x_1 + c_{44}^{(j)} n_3 \partial x_3 & c_{13}^{(j)} n_1 \partial x_3 + c_{44}^{(j)} n_3 \partial x_1 \\ c_{44}^{(j)} n_1 \partial x_3 + c_{13}^{(j)} n_3 \partial x_1 & c_{44}^{(j)} n_1 \partial x_1 + c_{33}^{(j)} n_3 \partial x_3 \end{pmatrix}, \tag{3}$$

where n_1, n_3 are the components of normal vector, $\partial x_k = \dfrac{\partial}{\partial x_k}$.

Let D denote an infinite transversally isotropic binary mixtures strip $0 < x_3 < h, \infty < x_1 < \infty$. Introduce the definition of a regular vector function.

Definition 1. A vector-function $U(x)$ defined in the domain D, is called regular if it has integrable continuous second derivatives in D and $U(x)$ itself and its first derivatives are continuously extendable at every point of the boundary of D (i.e. $U(x) \in C^2(D) \cap C^1(\overline{D})$), and the following conditions at infinity are added:

$$U(x) = O(1), \quad \frac{\partial U}{\partial x_k} = O(|x|^{-1}), \quad |x|^2 = x_1^2 + x_3^2, k = 1, 3,$$

where $O(1)$ denotes a bounded function.

For the equation (1) the second BVP for an infinite strip is formulated as follows:

Problem 2. Find a regular function $U(x) = U(x_1, x_3)$, satisfying in D the equation (1), when on the boundary of the domain D the following conditions are given in the form

$$TU = f(x_1), \quad x_3 = 0, \quad TU = F(x_1), \quad x_3 = h,$$

where f and F are given vector functions satisfying certain smoothness conditions and also the conditions at infinity.

A solution of the second BVP is sought in the domain D in the form

$$\begin{aligned} U(x) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} \sum_{k=1}^{4} [R^{(k)T} L e^{ip\alpha_k x_3} X(p) + \\ \overline{R^{(k)T} L} e^{-ip\overline{\alpha_k}(h-x_3)} Y(p)] e^{ipx_1} \frac{dp}{ip}, \end{aligned} \tag{4}$$

where $X(p)$ and $Y(p)$ are unknown vector functions, $\alpha_k = i\sqrt{a_k}sgnp$, $\overline{\alpha_k} = -ip\sqrt{a_k}sgnp$, $a_k > 0$, $k = 1,..,4$ are the roots of the characteristic equation [see [3],[4]]. Let $a_1 < a_2 < a_3 < a_4$. Since $ip\alpha_k = -|p|\sqrt{a_k}, -ip\overline{\alpha_k} = -|p|\sqrt{a_k}$, the functions $\exp(ip\alpha_k x_3)$ and $exp(-ip\overline{\alpha_k}x_3)$ tend to zero as $p \to \infty$,

$$
\begin{aligned}
&R^{(k)} = \|R^{(k)}_{pq}\|_{4x4}, p, q = 1,2,3,4,\\
&R^{(k)}_{1j} = \alpha_k(c^{(1)}_{44}A^{(k)}_{1j} + c^{(3)}_{44}A^{(k)}_{j3}) + c^{(1)}_{44}A^{(k)}_{j2} + c^{(3)}_{44}A^{(k)}_{j4}, R^{(k)}_{2j} = -\alpha_k^{-1}R^{(k)}_{1j},\\
&R^{(k)}_{3j} = \alpha_k(c^{(3)}_{44}A^{(k)}_{1j} + c^{(2)}_{44}A^{(k)}_{j3}) + c^{(3)}_{44}A^{(k)}_{j2} + c^{(2)}_{44}A^{(k)}_{j4},\\
&R^{(k)}_{4j} = -\alpha_k^{-1}R^{(k)}_{3j}, j = 1,2,3,4,
\end{aligned} \tag{5}
$$

$A^{(k)}_{pq}$ are given in [3],[4] and $\overline{R^{(k)T}L}$ denotes the complex conjugate matrix of $R^{(k)T}L$,

$$
L = \frac{1}{\Delta_1\Delta_2}\begin{pmatrix} L_{33}\Delta_2 & 0 & -L_{13}\Delta_2 & 0\\ 0 & L_{44}\Delta_1 & 0 & -L_{24}\Delta_1\\ -L_{13}\Delta_2 & 0 & L_{11}\Delta_2 & 0\\ 0 & -L_{24}\Delta_1 & 0 & L_{22}\Delta_1 \end{pmatrix}, \tag{6}
$$

$$
\begin{aligned}
&L_{11} = -\Delta q_4[a_{44}B_1 + (b_{11} + 2a_{34})A_1 + a_{33}D_1],\\
&A_1 = -B_0 m_3, B_1 = B_o m_1, \ C_1 = -\frac{A_1 + B_1 m_2}{\alpha_1\alpha_2\alpha_3\alpha_4},\\
&L_{13} = \Delta q_4[a_{24}B_1 + (-b_{33} + a_{14} + a_{23})A_1 + a_{13}D_1],\\
&L_{22} = -\Delta q_4[a_{44}C_1 + (b_{11} + 2a_{34})B_1 + a_{33}A_1],\\
&D_1 = -A_1 m_2 - B_1\alpha_1\alpha_2\alpha_3\alpha_4,\\
&L_{24} = \Delta q_4[a_{24}C_1 + (-b_{33} + a_{14} + a_{23})B_1 + a_{13}A_1],\\
&L_{33} = -\Delta q_4[a_{22}B_1 + (b_{22} + 2a_{12})A_1 + a_{11}D_1],\\
&\Delta_1 = L_{11}L_{33} - L_{13}^2, \Delta_2 = L_{22}L_{44} - L_{24}^2,\\
&L_{44} = -\Delta q_4[a_{22}C_1 + (b_{22} + 2a_{12})B_1 + a_{11}A_1],\\
&\Delta_2 = [b_4(m_1 m_3 - 2\sqrt{a_1a_2a_3a_4}) + q_4\Delta m_0]q_4\Delta B_0 > 0,\\
&m_0 = (a_{11}a_{44} + a_{33}a_{22} - 2a_{13}a_{24}), \ \Delta_1 = \sqrt{a_1a_2a_3a_4}\Delta_2,\\
&m_1 = \sum_{k=1}^{4}\alpha_k, m_2 = \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 + \alpha_2\alpha_3 + \alpha_2\alpha_4 + \alpha_3\alpha_4,\\
&m_3 = \alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_4 + \alpha_1\alpha_3\alpha_4 + \alpha_2\alpha_3\alpha_4,\\
&B_0^{-1} = (\alpha_1 + \alpha_2)(\alpha_1 + \alpha_3)(\alpha_1 + \alpha_4)(\alpha_2 + \alpha_3)(\alpha_2 + \alpha_4)(\alpha_3 + \alpha_4).
\end{aligned}
$$

Between the coefficients a_{pq}, b_{pp} and $c_{pq}^{(j)}$ there are the relations

$$
\begin{aligned}
&a_{11}\Delta = c_{11}^{(2)}q_3 - c_{33}^{(1)}c_{13}^{(2)2} + 2c_{13}^{(2)}c_{13}^{(3)}c_{33}^{(3)} - c_{33}^{(2)}c_{13}^{(2)2} > 0,\\
&a_{12}\Delta = c_{13}^{(2)}(c_{13}^{(1)}c_{13}^{(2)} - c_{13}^{(3)2}) - c_{13}^{(1)}c_{11}^{(2)}c_{33}^{(2)} - c_{13}^{(2)}c_{11}^{(3)}c_{33}^{(3)}\\
&+c_{13}^{(3)}(c_{11}^{(2)}c_{33}^{(3)} + c_{11}^{(3)}c_{33}^{(2)}),\\
&a_{13}\Delta = -c_{11}^{(3)}q_3 + c_{33}^{(2)}c_{13}^{(1)}c_{13}^{(3)} + c_{33}^{(1)}c_{13}^{(2)}c_{13}^{(3)} - c_{33}^{(3)}(c_{13}^{(1)}c_{13}^{(2)} + c_{13}^{(3)2}),\\
&a_{14}\Delta = -c_{13}^{(3)}(c_{13}^{(1)}c_{13}^{(2)} - c_{13}^{(3)2}) + c_{13}^{(1)}c_{11}^{(2)}c_{33}^{(3)} + c_{13}^{(2)}c_{11}^{(3)}c_{33}^{(1)}\\
&-c_{13}^{(3)}(c_{11}^{(2)}c_{33}^{(1)} + c_{11}^{(3)}c_{33}^{(3)}),\\
&a_{23}\Delta = -c_{13}^{(3)}(c_{13}^{(1)}c_{13}^{(2)} - c_{13}^{(3)2}) + c_{13}^{(1)}c_{11}^{(3)}c_{33}^{(2)} + c_{13}^{(2)}c_{11}^{(1)}c_{33}^{(3)}\\
&-c_{13}^{(3)}(c_{11}^{(1)}c_{33}^{(2)} + c_{11}^{(3)}c_{33}^{(3)}),\\
&a_{22}\Delta = c_{33}^{(2)}q_1 - c_{11}^{(1)}c_{13}^{(2)2} + 2c_{13}^{(2)}c_{13}^{(3)}c_{11}^{(3)} - c_{11}^{(2)}c_{13}^{(2)2} > 0,\\
&a_{24}\Delta = -c_{33}^{(3)}q_1 + c_{11}^{(2)}c_{13}^{(1)}c_{13}^{(3)} + c_{13}^{(2)}c_{13}^{(3)}c_{11}^{(1)} - c_{11}^{(3)}(c_{13}^{(1)}c_{13}^{(2)} + c_{13}^{(3)2}),\\
&a_{33}\Delta = c_{11}^{(1)}q_3 - c_{33}^{(2)}c_{13}^{(1)2} + 2c_{13}^{(1)}c_{13}^{(3)}c_{33}^{(3)} - c_{33}^{(1)}c_{13}^{(3)2} > 0,\\
&a_{34}\Delta = c_{13}^{(1)}(c_{13}^{(1)}c_{13}^{(2)} - c_{13}^{(3)2}) - c_{13}^{(2)}c_{11}^{(1)}c_{33}^{(1)} - c_{13}^{(1)}c_{11}^{(3)}c_{33}^{(3)} + c_{13}^{(3)}(c_{11}^{(1)}c_{33}^{(3)} + c_{11}^{(3)}c_{33}^{(1)}),\\
&a_{44}\Delta = c_{33}^{(1)}q_1 - c_{11}^{(2)}c_{13}^{(1)2} + 2c_{13}^{(1)}c_{13}^{(3)}c_{11}^{(3)} - c_{11}^{(1)}c_{13}^{(3)2} > 0,\\
&\Delta = (c_{11}^{(1)}a_{11} + c_{11}^{(2)}a_{33} + 2c_{11}^{(3)}a_{13})\Delta - q_1q_3 + (c_{13}^{(1)}c_{13}^{(2)} - c_{13}^{(3)2}) > 0,\\
&b_{jj} = c_{44}^{(j)}q_4^{-1} > 0, j = 1,2,3.
\end{aligned}
$$

After elementary calculations, from (4) we obtain for the stress vector

$$
\begin{aligned}
TU = &\frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty}\sum_{k=1}^{4}[L^{(k)}L\exp(ip\alpha_k x_3)X(p)\\
&+\overline{L^{(k)}L}\exp(ip\overline{\alpha_k}(h-x_3)Y(p)]\exp(ipx_1)(n_1\alpha_k - n_3)dp,
\end{aligned}
\tag{7}
$$

where the matrix $L^{(k)}(\partial x) = \|L_{pq}^{(k)}(\partial x)\|_{4x4}$ is given in the form

$$
\begin{aligned}
&L^{(k)} = \begin{pmatrix}
\alpha_k^2 L_{22}^{(k)} & -\alpha_k L_{22}^{(k)} & \alpha_k^2 L_{24}^{(k)} & -\alpha_k L_{24}^{(k)}\\
-\alpha_k L_{22}^{(k)} & L_{22}^{(2)} & -\alpha_k L_{24}^{(k)} & L_{24}^{(k)}\\
\alpha_k^2 L_{24}^{(k)} & -\alpha_k L_{24}^{(k)} & \alpha_k^2 L_{44}^{(k)} & -\alpha_k L_{44}^{(k)}\\
-\alpha_k L_{24}^{(k)} & L_{24}^{(k)} & -\alpha_k L_{44}^{(k)} & L_{44}^{(k)}
\end{pmatrix},\\
&L_{22}^{(k)} = -\Delta q_4 d_k[a_{44} + \alpha_k^2(b_{11} + 2a_{34}) + a_{33}\alpha_k^4],\\
&L_{24}^{(k)} = \Delta q_4 d_k[a_{24} + \alpha_k^2(-b_{33} + a_{14} + a_{23}) + a_{13}\alpha_k^4],\\
&L_{44}^{(k)} = -\Delta q_4 d_k[a_{22} + \alpha_k^2(b_{22} + 2a_{12}) + a_{11}\alpha_k^4],
\end{aligned}
$$

In what follows, when the point x lies on the boundary D, i.e. $x_3 = 0$ or $x_3 = h$, in the expression for the stress vector we will always suppose that $n_1 = 0$ and $n_3 = -1$. Taking into account this remark and the boundary conditions,for determining the unknown vector functions $X(p)$ and $Y(p)$, from (7) we obtain the following equations

$$\begin{aligned} X(p)+\overline{Q}Y(p)&=\widehat{f}=\tfrac{1}{\sqrt{2\pi}}\int\limits_{\infty}^{\infty} f(\xi)e^{-ip\xi}d\xi,\\ QX(p)+Y(p)&=\widehat{F}=\tfrac{1}{\sqrt{2\pi}}\int\limits_{\infty}^{\infty} F(\xi)e^{-ip\xi}d\xi, \end{aligned}\tag{8}$$

Here we have introduced the notation

$$Q=\sum_{k=1}^{4}L^{(k)}Le^{-|p|\sqrt{a_k}h},\overline{Q}=\sum_{k=1}^{4}\overline{L^{(k)}L}e^{-|p|\sqrt{a_k}h}.$$

$\widehat{f},\widehat{F}$ are Fourier transforms of the functions f and F respectively. Hence

$$(E-\overline{Q}Q)X(p)=\widehat{f}-\overline{Q}\widehat{F},\ \ (E-Q\overline{Q})Y(p)=-Q\widehat{f}+\widehat{F},$$

where E is the unit matrix. Denote $D=det(E-\overline{Q}Q)$. After some cumbersome transformation we find that

$$\begin{aligned}
&D(p)=(1-\lambda_1^2)(1-\lambda_2^2)(1-\lambda_3^2)(1-\lambda_4^2)\\
&+\frac{16}{\Delta_1\Delta_2}\{\alpha_1\alpha_2h_{12}(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)+\alpha_1\alpha_3h_{13}(\lambda_3-\lambda_4)(\lambda_1-\lambda_2)\}^2\\
&+\frac{4\alpha_1\alpha_2\alpha_3}{\Delta_1\Delta_2}\{\alpha_1h_{12}h_{13}[(1-\lambda_2^2)(1-\lambda_3^2)(\lambda_1-\lambda_4)^2\frac{(\alpha_2-\alpha_3)^2}{\alpha_2\alpha_3}\\
&+(1-\lambda_4^2)(1-\lambda_1^2)(\lambda_2-\lambda_3)^2\frac{(\alpha_1-\alpha_4)^2}{\alpha_1\alpha_4}]\\
&+\alpha_2h_{12}h_{23}[(1-\lambda_1^2)(1-\lambda_3^2)(\lambda_2-\lambda_4)^2\frac{(\alpha_1-\alpha_3)^2}{\alpha_1\alpha_3}\\
&+(1-\lambda_2^2)(1-\lambda_4^2)(\lambda_1-\lambda_3)^2\frac{(\alpha_2-\alpha_4)^2}{\alpha_2\alpha_4}]\\
&+\alpha_1h_{13}h_{23}[(1-\lambda_1^2)(1-\lambda_2^2)(\lambda_3-\lambda_4)^2\frac{(\alpha_1-\alpha_2)^2}{\alpha_1\alpha_2}\\
&+(1-\lambda_3^2)(1-\lambda_4^2)(\lambda_1-\lambda_2)^2\frac{(\alpha_3-\alpha_4)^2}{\alpha_3\alpha_4}]\}=det(E-Q\overline{Q}),\\
&\lambda_k=e^{-|p|h\sqrt{a_k}},
\end{aligned}\tag{9}$$

where

$$\begin{aligned}
h_{12}&=\frac{\Delta q_4[-b_4(a_1a_2+a_3a_4)+m_0\Delta q_4]}{\sqrt{a_1a_2}(a_1-a_3)(a_1-a_4)(a_2-a_3)(a_2-a_4)},\\
h_{13}&=-\frac{\Delta q_4[-b_4(a_1a_3+a_2a_4)+m_0\Delta q_4]}{\sqrt{a_1a_3}(a_1-a_2)(a_1-a_4)(a_2-a_3)(a_3-a_4)},\\
h_{23}&=\frac{\Delta q_4[-b_4(a_2a_3+a_1a_4)+m_0\Delta q_4]}{\sqrt{a_3a_2}(a_1-a_2)(a_2-a_4)(a_1-a_3)(a_3-a_4)}.
\end{aligned}$$

From (9) we can show, that in the vicinity of the point $p = 0$ the function $D(p)$ has zero of sixth order and moreover $\lim\limits_{|p|\to\infty} D(p) = 1$. Then on the basis of the uniqueness theorem for the second BVP , we conclude that $D \neq 0,\ \ p \in (0, +\infty)$. Thus from the system (8) we uniquely define X and Y:

$$X = (E - \overline{Q}Q)^{-1}(\widehat{f} - \overline{Q}\widehat{F}), Y = (E - Q\overline{Q})^{-1}(\widehat{F} - Q\widehat{f}). \tag{10}$$

If we substitute this expressions into (7) the stress vector takes the form

$$\begin{aligned} TU = \frac{1}{\sqrt{2\pi}}\int\limits_{\infty}^{\infty}\sum_{k=1}^{4}[L^{(k)}Le^{ip\alpha_k x_3}(E - \overline{Q}Q)^{-1}(\widehat{f} - \overline{Q}\widehat{F}) \\ +\overline{L^{(k)}L}e^{ip\overline{\alpha_k}(h-x_3)}(E - Q\overline{Q})^{-1}(\widehat{F} - Q\widehat{f})]e^{ipx_1}dp. \end{aligned} \tag{11}$$

The conditions of vanishing of the principal vectors and of the principal moment of external forces

$$\begin{aligned} &\int\limits_{\infty}^{\infty} f(\xi)d\xi = \int\limits_{\infty}^{\infty} F(\xi)d\xi, \int\limits_{\infty}^{\infty} \xi(F_2(\xi) - f_2(\xi))d\xi = h\int\limits_{\infty}^{\infty} F_1(\xi)d\xi, \\ &\int\limits_{\infty}^{\infty} \xi(F_4(\xi) - f_4(\xi))d\xi = h\int\limits_{\infty}^{\infty} F_3(\xi)d\xi, \end{aligned}$$

will take the form

$$\widehat{f}(0) = \widehat{F}(0),\ \ \widehat{f'}_2(0) - \widehat{F'}_2(0) = ih\widehat{F'_1}(0),\ \ \widehat{f'}_4(0) - \widehat{F'}_4(0) = ihF'_3(0).$$

On the basis of this conditions we conclude, that the integrand in (11) remains bounded at the point $p = 0$.

Substituting $X(p)$ and Y into (4),we obtain an expression for the displacement vector

$$\begin{aligned} U(x) = \frac{1}{\sqrt{2\pi}}\int\limits_{\infty}^{\infty}\sum_{k=1}^{4}[R^{(k)T}Le^{ip\alpha_k x_3}(E - \overline{Q}Q)^{-1}(\widehat{f} - \overline{Q}\widehat{F}) \\ +\overline{R^{(k)T}L}e^{-ip\overline{\alpha_k}(h-x_3)}(E - Q\overline{Q})^{-1}(\widehat{F} - Q\widehat{f})]e^{ipx_1}\frac{dp}{ip}. \end{aligned} \tag{12}$$

But in this case the integrant possesses a singularity at the point $p = 0$. To get rid of the singularity, it is necessary to assume that

$$u(0) = 0, \left(\frac{\partial u'_3}{\partial x_1} - \frac{\partial u'_1}{\partial x_3}\right)_{x=0} = 0,\ \ \left(\frac{\partial u''_3}{\partial x_1} - \frac{\partial u''_1}{\partial x_3}\right)_{x=0} = 0. \tag{13}$$

Now the integrand in (12) is bounded at the point $p = 0$. Thus we have proved the following result.

For the solvability of second BVP problem it is necessary that the principal vector and the principal moment of external forces be equal to zero. Moreover it is also necessary that the boundary values be absolutely integrable vectors. To ensure the boundedness of the displacement vector, one has to require additionally the fulfilment of condition (13).

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