THE MIXED PROBLEM OF THE THEORY OF ELASTICITY MIXTURE FOR A RECTANGLE WEAKENED BY UNKNOWN EQUALLY STRONG HOLES

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Abstract

In the present paper we investigate the mixed problem of statics in the linear theory of elasticity mixtures for a rectangle weakened by equally strong holes. Using the methods of the theory of elastic functions a stressed state of the plate, a form and mutual location of the hole contours are defined.

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1^0. The homogeneous equation of statics of the theory of elastic mixture in the complex form is written as [3]

$$\frac{\partial^2 U}{\partial z \partial \overline{z}} + \mathcal{K} \frac{\partial^2 \overline{U}}{\partial \overline{z}^2} = 0, \tag{1}$$

where $z = x_1 + ix_2$, $\overline{z} = x_1 - ix_2$, $\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)$, $\frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)$, $U = (u_1 + iu_2, u_3 + iu_4)^T$, $u^1 = (u_1, u_2)^T$, $u'' = (u_3, u_4)^T$ are particle displacements,

$$\mathcal{K} = -\frac{1}{2} e m^{-1}, \quad e = \begin{bmatrix} e_4 & e_5 \\ e_5 & e_6 \end{bmatrix}, \quad m^{-1} = \frac{1}{\Delta_0} \begin{bmatrix} m_3 & -m_2 \\ -m_2 & m_1 \end{bmatrix};$$

$$\Delta_0 = m_1 m_3 - m_2^2, \quad m_k = e_k + \frac{1}{2} e_{3+k}, \quad k = 1, 2, 3, \quad e_1 = a_2/d_2,$$

$$e_2 = -c/d_2, \quad e_3 = a_1/d_2, \quad a_1 = \mu_1 - \lambda_5, \quad a_2 = \mu_2 - \lambda_5, \quad c = \mu_3 + \lambda_5,$$

$$d_2 = a_1 a_2 - c^2, \quad e_1 + e_4 = b/d_1, \quad e_2 + e_5 = -C_0/d_1, \quad e_3 + e_4 = 1/d_1$$

$$a = a_1 + b_1, \quad b = a_2 + b_2, \quad c_0 = c + d, \quad b_1 = \mu_1 + \lambda_1 + \lambda_5 - \alpha_2 \rho_2/\rho,$$

$$b_2 = \mu_2 + \lambda_5 + \alpha_2 \rho_1/\rho, \quad d = \mu_2 + \lambda_3 - \lambda_5 - \alpha_2 \rho_1/\rho \equiv \mu_3 + \lambda_4 - \lambda_5 + \alpha_2 \rho_2/\rho,$$

$$\alpha_2 = \lambda_3 - \lambda_4, \quad \rho = \rho_1 + \rho_2, \quad d = ab - c_0^2.$$

Here ρ_1 and ρ_2 partial densities, and μ_1, μ_2, μ_3, λ_p, $p = \overline{1,5}$, are constants characterizing physical properties [5].

In [2,3] M. Basheleishvili obtained the representations

$$U=(u_1+iu_2,u_3+iu_4)^T=m\varphi(z)+\frac{1}{2}\,ez\overline{\varphi'(z)}+\overline{\psi(z)}, \tag{2}$$

$$TU=\begin{pmatrix}(TU)_2-i(TU)_1\\(TU)_4-i(TU)_3\end{pmatrix}=\frac{\partial}{\partial s(x)}\Big[(A-2E)\varphi(z)+Bz\overline{\varphi'(z)}+2\mu\overline{\psi(z)}\Big], \tag{3}$$

where $\varphi(z)=(\varphi_1,\varphi_2)^T$ and $\psi(z)=(\psi_1,\psi_2)^T$ are arbitrary analytic vector-functions,

$$A=2\mu m,\quad \mu=\begin{bmatrix}=\mu_1 & \mu_3\\ \mu_3 & \mu_2\end{bmatrix},\quad m=\begin{bmatrix}m_1 & m_2\\ m_2 & m_3\end{bmatrix},\quad B=\mu e,\quad E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$\dfrac{\partial}{\partial s(x)}=-n_2\dfrac{\partial}{\partial x_1}+n_1\dfrac{\partial}{\partial x_2}$, n_1 and n_2 are the projections of the unit vector of the normal onto the axes ox_1 and ox_2; $(Tu)_p$, $p=\overline{1,4}$, are components of the stresses vector [2]

$$(TU)_1=r'_{11}n_1+r'_{21}n_2,\quad r'_{11}=a\theta'+c_0\theta''-2\frac{\partial}{\partial x_2}(\mu_1u_2+\mu_3u_4),$$

$$r'_{21}=-a_1\omega'-c\omega'+2\frac{\partial}{\partial x_1}(\mu_1u_2+\mu_3u_4),$$

$$(TU)_2=r'_{12}n_1+r'_{22}n_2,\quad r'_{12}=a_1\omega'+c\omega''+2\frac{\partial}{\partial x_2}(\mu_1u_1+\mu_3u_3),$$

$$r'_{22}=a\theta'+c_0\theta''-2\frac{\partial}{\partial x_1}(\mu_1u_1+\mu_3u_3),$$

$$(TU)_3=r'_{11}n_1+r''_{21}n_2,\quad r''_{11}=c_0\theta'+b\theta''-2\frac{\partial}{\partial x_2}(\mu_3u_2+\mu_2u_4),$$

$$r''_{21}=c\omega'a_2-\omega''+2\frac{\partial}{\partial x_1}(\mu_3u_2+\mu_2u_4),$$

$$(TU)_4=r'_{12}n_1+r''_{22}n_2,\quad r''_{12}=c\omega'+a_2\omega''+2\frac{\partial}{\partial x_2}(\mu_3u_1+\mu_2u_3),$$

$$r''_{22}=c_0\theta'+b\theta''-2\frac{\partial}{\partial x_1}(\mu_3u_1+\mu_2u_3);$$

$$\theta'=\operatorname{div}u'=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2},\quad \theta''=\operatorname{div}u''=\frac{\partial u_3}{\partial x_1}+\frac{\partial u_4}{\partial x_2},$$

$$\omega'=\operatorname{rot}u'=\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2},\quad \omega''=\operatorname{rot}u''=\frac{\partial u_4}{\partial x_1}-\frac{\partial u_3}{\partial x_2}.$$

Let us now consider the vectors:

$$\overset{(1)}{=}\tau(r'_{11},r''_{11})^T,\quad \overset{(2)}{=}\tau(r'_{22},r''_{22})^T,\quad \tau=\overset{(1)}{\tau}+\overset{(2)}{\tau}, \tag{4}$$

$$\overset{(1)}{\eta}=(r'_{21},r''_{21})^T,\quad \eta^{(2)}=(r'_{12},r''_{12})^T,\quad \eta=\overset{(1)}{\eta}+\overset{(2)}{\eta},\quad \varepsilon^*=\overset{(1)}{\eta}-\overset{(2)}{\eta}. \tag{5}$$

After lengty but elementary calculations we obtain

$$\tau = \overset{(1)}{\tau} + \overset{(2)}{\tau} = 2(2E - A - B)\,\mathrm{Re}\,\Phi(z), \tag{6}$$

$$\varepsilon^* = \overset{(1)}{\eta} - \overset{(2)}{\eta} = 2(A - B - 2E)\,\mathrm{Im}\,\Phi(z), \tag{7}$$

$$\overset{(1)}{\tau} - \overset{(2)}{\tau} - i\eta = 2(B\overline{z}\phi'(z) + 2\mu\Psi(z)), \tag{8}$$

here $\Phi(z) = \varphi'(z)$, $\Psi(z) = \psi'(z)$; $\det(2E - A - B) > 0$, [4].

Now we consider the right orthogonal coordinate system (nS). By n we denote the outer normal vector to L at point $t = t_1 + it_2$, and by S the tangent vector. Suppose that $n = (n_1, n_2)^T = (\cos\alpha, \sin\alpha)^T$ and $S = (-n_2, n_1)^T = (-\sin\alpha, \cos\alpha)^T$, where $\alpha = \alpha(t)$ is size of the angle made by the outer normal n and the ox_1 axis.

Next we construct the vectors:

$$\sigma_n = \begin{pmatrix} (Tu)_1 n_1 + (Tu)_2 n_2 \\ (Tu)_3 n_1 + (Tu)_4 n_2 \end{pmatrix} = \overset{(1)}{\tau}\cos^2\alpha + \overset{(2)}{\tau}\sin^2\alpha + \eta\sin\alpha\cos\alpha, \tag{9}$$

$$\sigma_s = \begin{pmatrix} (Tu)_2 n_1 - (Tu)_1 n_2 \\ (Tu)_4 n_1 - (Tu)_3 n_2 \end{pmatrix} = \frac{1}{2}(\overset{(2)}{\tau} - \overset{(1)}{\tau})\sin 2\alpha + \frac{1}{2}\eta\cos 2\alpha - \frac{1}{2}\varepsilon^*, \tag{10}$$

$$\sigma_s^* = \sigma_s + \frac{1}{2}\varepsilon^*, \tag{11}$$

$$\begin{aligned} \sigma_t = \begin{pmatrix} [r'_{21}n_1 - r'_{11}n_2, \quad r'_{22}n_1 - r'_{12}n_2]^T S \\ [r''_{21}n_1 - r''_{11}n_2, \quad r''_{22}n_1 - r''_{12}n_2]^T S \end{pmatrix} = \\ = \overset{(1)}{\tau}\sin^2\alpha + \overset{(2)}{\tau}\cos^2\alpha - \eta\sin\alpha\cos\alpha \end{aligned} \tag{12}$$

From (8)–(12) and (6)–(8) we obtain on L

$$\sigma_n + \sigma_t = \overset{(1)}{\tau} + \overset{(2)}{\tau} = 2(2E - A - B)\,\mathrm{Re}\,\Phi(t), \tag{13}$$

$$\sigma_n - i\sigma_s = (2E - A)\overline{\Phi(t)} - B\Phi(t) + (B\bar{t}\Phi'(t) + 2\mu\Psi(t))e^{2i\alpha(t)}, \tag{14}$$

and with elementary calculation we get on L

$$\sigma_n + 2\mu\Big(\frac{\partial U_s}{\partial s} + \frac{U_n}{\rho_0}\Big) + i\Big[\sigma_s - 2\mu\Big(\frac{\partial U_n}{\partial s} - \frac{U_s}{\rho_0}\Big)\Big] = 2\Phi(t), \tag{15}$$

where $1/\rho_0$ is the curvature of the curve L at the point $t = t_1 - it_2$,

$$U_n = \begin{pmatrix} u_1 n_1 + u_2 n_2 \\ u_3 n_1 + u_4 n_2 \end{pmatrix}, \quad U_s = \begin{pmatrix} u_2 n_1 - u_1 n_2 \\ u_4 n_1 - u_3 n_2 \end{pmatrix}. \tag{16}$$

(2), (3), (6), (8) and (13)-(15) represent analogues to Kolosov-Muskhelishvili formulas in the theory of elastic mixtures.

2^0. In the present work we consider the similar problem which have been investigated in the plane theory of elasticity by R. Bantsuri and Sh. Mzhavanadze in [1].

Let a middle surface of an elastic isotropic of the mixture occupy on the plane $z = x_1 + ix_2$ the $(n+1)$ connected domain whose outer boundary is formed by straight lines $x_1 = \pm x_1^0$ and $x_2 = \pm x_2^0$, and inner boundary is a union of simple closed smooth contours (a rectangle with holes).

Suppose that on the outer boundary of the domain D the value of the vector (10) is equal to zero, i.e. $\sigma_s = 0$, the vector U_n (see (16_1)) on the rectangle sides take constant values, and the inner boundary the value of the vector (9) i.e. $\sigma_n = p = \text{const}$.

Consider the problem: Find a stressed state of the plate, a form and mutual location of the hole contours so as the vector (12), i.e. σ_t, on them to take constant value.

For solving of the problem we use method given in [1].

In the case under consideration we assume that the domain D is symmetric with respect to the ox_1-axis. This allows us to consider a part of the domain D lying in the upper half-plane. We denote it by D_1.

Owing to the symmetry, for the domain D_1 we have the same boundary conditions as for D. The mathematical ground of this fact can be found in [4] and [7].

Thus for the domain D_1 we obtain the following boundary conditions:

$$\begin{gathered} \sigma_s = 0,; \quad U_n = U_0(t) \quad \text{on} \quad L_1 + L_2, \\ \sigma_s = 0, \quad \sigma_n = P, \quad \sigma_t = K^0, \quad \text{on} \quad L_0, \end{gathered} \tag{17}$$

where $U_0(t)$ is the given piecewise constant vector-function, where $P = (P_1, P_2)^T$ is the given and $K^0 = (K_1^0, K_2^0)^T$ is an unknown constant. The value K^0 will be defined later in solving the problem. L_0 is a union of unknown arcs, L_1 and L_2 are the unions of segments which are parallel with respect to the ox_1 and ox_2-axes.

$$\begin{gathered} L_1 = \bigcup_{k=0}^{n+2} A_{2k}A_{2k+1}, \quad L_2 = A_1A_2 \cup A_{2n+3}A_{2n+4}, \\ L_0 = \bigcup_{k=1}^{n} A_{2k+1}A_{2k+2}, \quad A_0 = A_{2n+5} = ix_2^0, \quad A_1 = x_1^0 + ix_2^0, \\ A_2 = -x_1^0, \quad A_{2n+3} = x_1^0, \quad A_{2n+4} = x_1^0 + ix_2^0. \end{gathered}$$

The points and their affixes are denoted by one and the same symbols.

By virtue of (13)-(15) and the conditions (17) the stressed state of the body is described by two analytic vector-functions $\phi(z) = (\phi_1, \phi_2)^T$ and $\Psi(\zeta) = (\psi_1, \psi_2)^T$ which on the boundary of the domain satisfy the conditions

$$\operatorname{Re}\Phi(t) = \frac{1}{2}(2E - A - B)^{-1}(K^0 + P), \quad t \in L_0, \tag{18}$$

$$\operatorname{Im}\Phi(t) = 0, \quad t \in L_1 + L_2, \tag{19}$$

$$(B\overline{t\Phi'(t)} + 2\mu\Psi(t))e^{2i\alpha(t)} = P + B\Phi(t) + (A - 2E)\overline{\Phi}(t), \quad t \in L_0, \tag{20}$$

where $\alpha(t)$ is the angle between the outer normal to the boundary of the domain D_1 at the point t and the ox_1-axis. In our case $\alpha(t) = 0$ or $\alpha(t) = \pi$, if $t \in L_2$, $\alpha(t) = \frac{\pi}{2}$ or $\alpha(t) = -\frac{\pi}{2}$, if $t \in L_1$ and $\alpha(t)$ is unknown if $t \in L_0$.

The functions $\Phi_j(z)$ and $\Psi_j(z)$, $j = 1, 2$, satisfy in neighborhoods of the points A_k, $k = \overline{1, 2n+4}$ the following conditions:

$$|\Phi_j(z)| < \text{const}\,|z - c|^{-\delta}, \quad |\Psi_j(z)| < \text{const}\,|z - c|^{-\delta}, \quad j = 1, 2, \quad 0 \le \delta \le 1. \tag{21}$$

Let the functions $z = \omega(\zeta)$ map conformally the upper half-plane of the complex variable $\zeta = \xi + i\xi_2$ onto the domain D_1. Here we use the following notation: $\omega^{-1}(A_k) = a_k$, $k = \overline{0, 2n+5}$, $\omega^{-1}(L_k) = l_k$, $k = 0, 1, 2$. We may assume that $a_0 = -\infty$, $a_{2n+5} = +\infty$, $a_2 = -1$ and $a_{2n+3} = 1$.

Since the contours of the hole are smooth, the function $\omega(z)$ in the neighborhoods of the points ($k = \overline{1, 2n+4}$) is representable in the form [1]

$$\omega(\zeta) = (\zeta - c)^{1/2}\omega_0(\zeta) + \omega(c), \tag{22}$$

where $\omega_0(\zeta)$ is the function, holomorphic in the neighborhood of the point, and $\omega(c) \neq 0$ (c denotes any of the points $k = \overline{1, 2n+4}$). In the neighborhood of the point $\zeta = \infty$ we have

$$\omega(\zeta) = c_0 + \frac{c_1}{\zeta} + \frac{c_2}{\zeta^2} + \cdots, \tag{23}$$

where c_k ($k = 0, 1, 2, \dots$) are the constants.

The change of the variable $\zeta = \omega(\zeta)$ results in

$$e^{2i\alpha(t)} = -\omega'(\xi_1)/\overline{\omega'(\xi_1)}. \tag{24}$$

Taking into account the fact that $z = \omega(\zeta)$ and also the condition (17), from (18) and (19) we easily obtain

$$\operatorname{Im}\Phi_0(\xi_1) = 0, \quad \text{on} \quad l_1 + l_2; \quad \operatorname{Re}\Phi_0(\xi_1) = H, \quad \text{on} \quad l_0; \tag{25}$$

where $\Phi(z) = \Phi(\omega(z))$, $H = \frac{1}{2}(2E - A - B)^{-1}(K^0 + p)$, $\Phi_0 = (\Phi_{01}, \Phi_{02})$.

Taking into consideration (21)-(23), we establish that $\phi_0(\zeta)$ is bounded as $\zeta \to \infty$ and in the neighborhood of the points a_k, $k = \overline{1, 2n+4}$, satisfies the conditions

$$|\Phi_{oj}(\zeta)| < \text{const}\,|\zeta - c|^{-\delta}, \quad 0 \le \delta < \frac{1}{2}, \quad i = 1, 2.$$

Consider the vector-function $F(\zeta)=\Phi_0(\zeta)-H$. To define $F(\zeta)$, in view of (25), we get

$$\operatorname{Im}F(\xi_1)=0 \quad \text{on} \quad l_1+l_2, \quad \operatorname{Re}F(\xi_1)=0 \quad \text{on} \quad l_0. \tag{26}$$

$F(\zeta)$ has the same estimates as $\Phi_0(\zeta)$.

(26) is, in fact, the homogeneous Keldysh-Sedov's problem for the half-plane and under the above conditions this problem has only a trivial solution $F(z)=0$. Therefore

$$\Phi_0(\zeta)=H. \tag{27}$$

Tasking into account (27) and (24), the boundary condition (20) takes the form

$$h\omega'(\xi_1)+\overline{\omega'(\xi_1)\Psi_0(\xi_1)}=0 \quad \text{on} \quad l_0, \tag{28}$$

and by virtue of (17) we have (see (14))

$$\operatorname{Im}\Psi_0(\xi)=0 \quad \text{on} \quad l_1+l_2; \tag{29}$$

where $\Psi_0(\zeta)=\Psi(\omega(\zeta))$, $h=\dfrac{\mu^{-!}}{4}(P-K^0)$, $\Psi_0=(\Psi_{01},\Psi_{02})^T$.

Obviously,

$$\operatorname{Re}\omega(\xi_1)=A(\xi_1) \quad \text{on} \quad l_2; \quad \operatorname{Im}\omega(\xi_1)=B(\xi_1) \quad \text{on} \quad l_1, \tag{30}$$

where $A(\xi_1)$ and $B(\xi_1)$ are the piecewise constant vector-functions, in particular, $A(\xi_1)=x_1^0$, for $\xi_1\in(a_{2n+3},a_{2n+4})$, $A(\xi_1)=-x_1^0$, for $\xi\in(a_1,a_2)$, $B(\xi_1)=x_2^0$, for, $\xi_1\in(-\infty,a_1)\cup(a_{2n+4},\infty)$, $B(\xi_1)=0$, for $\xi_1\in\bigcup\limits_{k=1}^{n+1}(a_{2k},a_{2k+1})$.

Introduce the notation

$$W(\zeta)=\frac{1}{2}\,\omega'(\zeta)(\Psi_0(\zeta)+h), \quad \Omega(\zeta)=\frac{1}{2}\,\omega'(\zeta)(\Psi_0(\zeta)-h). \tag{31}$$

It is not difficult to state that $|\zeta|\to\infty$, as

$$W(\zeta)=O(\zeta^{-2}), \quad \Omega(\zeta)=O(\zeta^{-2}) \tag{32}$$

and in the neighborhood of the points a_k, $k=\overline{1,2n+4}$, satisfy the conditions

$$|W_j(\zeta)|<\text{const}\,|\zeta-c|^{-\delta}, \quad |\Omega_j(\zeta)|\leq\text{const}\,|\zeta-c|^{-\delta}, \quad 0\leq\delta\leq\frac{1}{2}, \quad j=1,2. \tag{33}$$

Taking into account (28)-(33), to find $W(\zeta)$ and $\Omega(\zeta)$, we obtain the boundary problem: Find the $W(\zeta)$ and $\Omega(\zeta)$, analytic in the upper half-plane ($\operatorname{Im}\zeta>0$), which on the axis $\xi_2=0$ satisfy the boundary conditions

$$\operatorname{Re}W(\xi_1)=0 \quad \text{on} \quad l_0+l_2, \quad \operatorname{Im}W(\xi_1)=0 \quad \text{on} \quad l_1, \tag{34}$$

$$\operatorname{Im}\Omega(\xi_1)=0 \quad \text{on} \quad l_0+l_1, \quad \operatorname{Re}\Omega(\xi_1)=0 \quad \text{on} \quad l_2, \tag{35}$$

and also the conditions (32) and (33) in the neighborhoods of the point $\zeta=\infty$ and a_k $(k=\overline{1,2n+4})$.

The problems (32)-(35) are, in fact, the homogeneous Keldysh-Sedov's problem for a half-plane. The solution of the problems is given by the formulas ([5])

$$W(\zeta)=h\chi_1(\zeta)P_n(z), \quad \Omega(\zeta)=h\chi_2(\zeta)D_0, \tag{36}$$

where

$$\begin{gathered}\chi_1(\zeta)=\left[\prod_{k=1}^{2n+4}(\zeta-a_k)\right]^{-\frac{1}{2}},\\ \chi_2(\zeta)=\Big[(\zeta-a_1)(\zeta-a_2)(\zeta-a_{2n+3})(\zeta-a_{2n+4})\Big]^{-\frac{1}{2}};\end{gathered} \tag{37}$$

$P_n(\zeta)=\sum\limits_{k=0}^{n}C_k\zeta^k$, $h=\dfrac{1}{4}\mu^{-1}(P-K^0)$; D_0, $C_0,C_1,\dots,C_n$ are the real constants, under $\chi_1(\zeta)$ and $\chi_2(\zeta)$ are understood one-valued branches satisfying the conditions $\chi_k(\zeta)<0$, $k=1,2$, when $\xi_2=0$ and $\xi_1>a_{2n+4}$.

From (31) and (36) we find

$$\omega(z)=\int_{\zeta_0}^{\zeta}(\chi_1(\xi_1)P_n(\xi_1)-D_0\chi_2(\xi_1))d\xi_1+\omega(z_0). \tag{38}$$

Taking into account the fact that $\Psi_0(\zeta)=\Psi(\omega(\zeta))=\Psi'(\omega(\zeta))$ and $\Phi_0(\zeta)=\Phi(\omega(\zeta))=\varphi'(\omega(\zeta))$ from (31), (36) and (27) we obtain

$$\psi(\omega(\zeta))=h\int_{\zeta_0}^{z}(\chi_1(\xi_1)P_n(\xi)+D_0\chi_2(\xi_1))d\xi_1+\psi(\zeta_0)), \tag{39}$$

$$\varphi(\omega(\zeta))=H\omega(\zeta)+C, \tag{40}$$

where ζ_0 is an arbitrary point of the domain $\operatorname{Im} z\geq 0$, C is a constant to be defined and $H=\frac{1}{2}(2E-A-B)^{-1}(P+K^0)$.

The above-formulated problem we can solve by the formulas (38)-(40).

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