GENERAL THEORY OF ELASTIC MICROPOLAR THIN SHELLS

S.H. Sargsyan

Chair for Mathematical Analysis and Differential Equations
Gyumri State Pedagogical Institute, Armenia
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Abstract

In the paper boundary value problem of three-dimensional asymmetrical micropolar, momental elasticity theory with free rotation is investigated for thin shell. It is established that the general stress-deformed state is formed by the interior stress-deformed state and boundary layers. For the approximate definition of interior stress-deformed state and boundary layers the asymptotic method of integration of three-dimensional boundary value problem of asymmetrical elasticity theory with free rotation is applied. For the set problem of three-dimensional elasticity theory with free rotation depending on the values of dimensionless physical constants of shell material there are built three different asymptotics. The initial approximation, correspondingly, for the first asymptotics leads to the theory of micropolar shell with free rotation, for the second asymptotics leads to the theory of micropolar shells with constraint rotation and for the third asypmtotics leads to the so-called theory of micropolar shells with "small shift rigidity". Corresponding micropolar boundary layers are built and studied. The fields of application of each of the constructed theories of micropolar shell are listed.

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1 Introduction

The construction of elasticity theory of thin plates and shells was realized with the help of three main methods: a) method of hypothesis, b) method of expansion along the thickness of coordinates of plates and shells c) the asymptotic method. Alongside with the classic theory of thin plates and shells built according to Kirchhoff- Love hypothesis, Ambartsumyan-Reissner-Timoshenko refined theory of plates and shells is generally acknowledged [1]. In [2] for the construction of the theory of micropolar plates and shells symbiosis of main principles of asymmetrical elasticity theory and refined theory of plates and shells is used. The power series expansion method with respect to the thickness of plates and shells, which was first stated in the works of Cauchy and Poisson, was further developed in several other works (see e.g. [3]). In the works of I.N. Vekua, we see rather a perspective method of expansion on the thickness coordinates and also the method for constructing the theory of plates and shells without Kirchhoff-Love hypothesis [4]. Great perspectives for the construction of the general theory of thin plates and shells were opened by the development of the methods of immediate asymptotic integration of equations of three-dimensional elasticity theory (Green A. [5], Vorovich I. [6], Goldenveizer A. [7], Agalovyan L. [8], Sargsyan S. [9], etc.) In [10], on the basis of the asymptotic method of integration of equations of three-dimensional elasticity theory depending on the values of dimensionless constants of plate's material, there are built the theories of generalized flat stress state and the theories of bending of micropolar plates with both free rotations, with constraint rotation and with small shift rigidity. Here the corresponding boundary layer theories are also built and investigated. In the present paper the results obtained in [11], [12] are generalized and systemized. On the basis of the asymptotic of integration of equations of asymmetrical elasticity theory depending on the values of dimensionless elastic constants of shell's material, three types of general applied two-dimensional theories of micropolar shells are constructed: the general applied two-dimensional theories of micropolar shells with free rotation, the general applied two-dimensional theories of micropolar shells with constraint rotation and the general applied two-dimensional theories of micropolar shells with small shift rigidity. Micropolar boundary layers around the shell's lateral surface are also built and investigated. Here the problems of matching of interior and boundary value problems are investigated and there are also obtained boundary conditions for applied two-dimensional theory of micropolar shells and boundary conditions for boundary layer problems.

2 Statement of the problem

Let us consider a shell with constant thickness 2h as a three-dimensional elastic body. Tensor equations of static problem of asymmetrical elasticity theory with independent fields of transition and rotation (ATE with IFTR) look as follows [13]:

Balance equations:

$$\nabla_j \sigma^{ji} = 0, \qquad \nabla_j \mu^{ji} + e^{ijk} \sigma_{jk} = 0. \tag{1.1}$$

Elasticity correlations:

$$\begin{cases} \sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + \lambda\gamma_{kk}\delta_{ij}, \\ \mu_{ji} &= (\gamma+\varepsilon)\chi_{ji} + (\gamma-\varepsilon)\chi_{ij} + \beta\chi_{kk}\delta_{ij}; \end{cases} \tag{1.2}$$

Geometrical correlations:

$$\gamma_{ji} = \nabla_j u_i - e_{kji}\omega^k, \quad \chi_{ji} = \nabla_j \omega_i. \tag{1.3}$$

Here σ^{ij}, μ^{ij} are the corresponding components of force and momental stress tensor; γ_{ji},χ_{ji}– are the corresponding components of deformation and bending-torsion tensors; $\vec{u}$–is the transition vector; $\vec{\omega}$ is the vector of free turning; $\lambda, \mu, \alpha, \beta, \gamma, \varepsilon$ are the elastic constants of the shell's micropolar material; $i, j, k = 1, 2, 3$.

Further, we shall use three-dimensional orthogonal coordinate system $\alpha_i (i = 1, 2, 3)$ which is accepted in the theory of shells[7]. Afterwards, we shall attach the corresponding boundary conditions to the defining equations of ATE with IFTR (1.1)-(1.3). For the boundary conditions on the shell's facial surface we shall accept boundary conditions of the first boundary value problem of ATE with IFTR, which can be presented as follows:

$$\sigma_{3i} = \mp q_i^{\pm}, \quad \mu_{3i} = \mp m_i^{\pm} \quad \text{when} \quad \alpha_3 = \pm h \ (i = 1, 2, 3). \tag{1.4}$$

The boundary conditions on the shell's lateral surface $\Sigma = \Sigma_1 \cup \Sigma_2$ (which represents itself a closed end $\alpha_1 = \alpha_{10}$) will be considered as given in general case of boundary conditions with mixed boundary value problems of ATE with IFTR.

$$\sigma_{ji} {\cdot} n_j = p_i^*, \quad \mu_{ji} {\cdot} n_j = m_i^* \quad \text{in} \ \Sigma_1 (i, j, k = 1, 2, 3), \ \vec{u} = \vec{u}^*, \ \vec{\omega} = \vec{\omega}^* \ \text{in} \ \Sigma_2, \tag{1.5}$$

where p_i^*, m_i^* $(i = 1, 2, 3)$ are the components of the given exterior force and moments on Σ_1 ; $\vec{u}^*$, $\vec{\omega}^*$ are the given vectors of transition and free turning on Σ_2 .

It is supposed that the shell's thickness is small in comparison with the characteristic curvature radiuses of shell's midplane.

We shall proceed from the following general conception: in the static case thin three-dimensional body's general stress-deformable state (SDS) which forms the shell, consists of interior SDS embracing the whole shell and the boundary layer located near the shell's lateral surface. For the approximate definition of both interior and lateral SDS we will apply the asymptotic method [10].

For the construction of the interior iteration process we shall introduce new free variables:

$$\alpha_i = R\lambda^{-p}\xi_i, \quad \alpha_3 = R\lambda^{-l}\zeta \quad (i = 1, 2), \tag{1.6}$$

where R is the characteristic curvature radius of midplane, p, l are whole numbers satisfying to inequalities $l > p \geq 0$; λ is a large constant dimensionless geometrical parameter, which is defined by formula $h = R\lambda^{-l}$.

While defining the shell's interior and lateral SDS, the values of physical constants of the shell's micropolar material play an important role. For this purpose we shall introduce the following dimensionless parameters:

$$\frac{\alpha}{\mu}, \quad \frac{\beta}{R^2\mu}, \quad \frac{\gamma}{R^2\mu}, \quad \frac{\varepsilon}{R^2\mu}. \tag{1.7}$$

It is convenient to introduce the asymmetrical tensor τ_{ij} of force stress [7] and the analogues tensor v_{ij} for momental stress [11].

$$\tau_{ii} = \left(1 + \frac{\alpha_3}{R_j}\right)\sigma_{ii}, \tau_{ij} = \left(1 + \frac{\alpha_3}{R_j}\right)\sigma_{ij}, \tau_{i3} = \left(1 + \frac{\alpha_3}{R_j}\right)\sigma_{i3}(i \leftrightarrow 3),$$

$$\tau_{33} = \left(1 + \frac{\alpha_3}{R_1}\right)\left(1 + \frac{\alpha_3}{R_2}\right)\sigma_{33}, \quad v_{ii} = \left(1 + \frac{\alpha_3}{R_j}\right)\mu_{ii},$$

$$\begin{aligned} v_{ij} &= \left(1 + \frac{\alpha_3}{R_j}\right)\mu_{ij}, v_{i3} = \left(1 + \frac{\alpha_3}{R_j}\right)\mu_{i3}(i \leftrightarrow 3), \\ v_{33} &= \left(1 + \frac{\alpha_3}{R_1}\right)\left(1 + \frac{\alpha_3}{R_2}\right)\mu_{33}, \end{aligned}$$

2. The applied two-dimensional theory of micropolar elastic thin shells with free fields of transition and rotation.

We suppose that the dimensionless physical constants (1.7) have the following values:

$$\frac{\alpha}{\mu} \sim 1, \quad \frac{\beta}{R^2\mu} \sim 1, \frac{\varepsilon}{R^2\mu} \sim 1, \quad \frac{\gamma}{R^2\mu} \sim 1. \tag{2.1}$$

Following the asymptotic method while constructing the interior problem, our aim is to approximately bring the three-dimensional equations (with free variables ξ_1, ξ_2, ζ) (1.1)-(1.6) to the two-dimensional equations (with free variables ξ_1 and ξ_2). So, at first it is important to get rid of differentiation on ζ in (1.1)-(1.6) and afterwards, in the obtained equations, their dependence on ζ and the asymptotic order on the large parameter λ must be underlined.

For the interior problem in case of asymptotic approximation $O(\lambda^{p-l})$ in (2.1) for the searched values in the three-dimensional thin shell's plane we shall receive the following representations:

$$\tau_{ii} = \mu\lambda^l(\tau_{ii}^0 + \lambda^{-l+c}\zeta\tau_{ii}^1), \qquad v_{ii} = R\mu\lambda^{l-c}(v_{ii}^0 + \lambda^{-l+c}\zeta v_{ii}^1),$$

$$\tau_{ij} = \mu\lambda^l(\tau_{ij}^0 + \lambda^{-l+c}\zeta\tau_{ij}^1), \qquad v_{ij} = R\mu\lambda^{l-c}(v_{ij}^0 + \lambda^{-l+c}\zeta v_{ij}^1),$$

$$\begin{aligned} \tau_{i3} &= \mu\lambda^{l-p+c}(\tau_{i3}^0 + \lambda^{-l+2p-c}\zeta\tau_{i3}^1) \ \ (i \leftrightarrow 3), \\ v_{i3} &= R\mu\lambda^{l-p}(v_{i3}^0 + \lambda^{-l+2p-c}\zeta v_{i3}^1) \ \ (i \leftrightarrow 3), \end{aligned} \tag{2.2}$$

$$\tau_{33} = \mu\lambda^{c}(\tau_{33}^0 + \zeta\tau_{33}^1 + \lambda^{-l+2p-c}\zeta^2\tau_{33}^2), \quad v_{33} = R\mu\lambda^{0}(v_{33}^0 + \zeta v_{33}^1 + \lambda^{-l+2p-c}\zeta^2 v_{33}^2),$$

$$V_i = R\lambda^{l-p}(V_i^0 + \lambda^{-l+c}\zeta V_i^l), \qquad \omega_i = \lambda^{l-p-c}(\omega_i^0 + \lambda^{-l+c}\zeta\omega_i^1),$$

$$V_3 = R\lambda^{l-2p+c}(V_3^0 + \lambda^{-l+2p-c}\zeta V_3^1), \qquad \omega_3 = \lambda^{l-2p}(\omega_3^0 + \lambda^{-l+2p-c}\zeta\omega_3^1).$$

While describing ATE of the interior problem, there only remains to find out the role of variables $(\xi\ \eta)$, which show the place of points on the shell's midplane surface.

From this point of view, instead of force and momental stresses it is expedient to introduce their equivalent forces, moments and hypermoments[10,11].

$$T_{ii} = \int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{ii}d\alpha_3, \qquad L_{ii} = \int_{-h}^{h}(1+\alpha_3/R_j)\mu_{ii}d\alpha_3,$$

$$S_{ij} = \int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{ij}d\alpha_3, \qquad L_{ij} = \int_{-h}^{h}(1+\alpha_3/R_j)\mu_{ij}d\alpha_3,$$

$$G_{ii} = -\int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{ii}\alpha_3 d\alpha_3, \qquad \Lambda_{ii} = -\int_{-h}^{h}(1+\alpha_3/R_j)\mu_{ii}\alpha_3 d\alpha_3,$$

$$H_{ij} = \int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{ij}\alpha_3 d\alpha_3, \qquad \Lambda_{ij} = \int_{-h}^{h}(1+\alpha_3/R_j)\mu_{ij}\alpha_3 d\alpha_3,$$

$$N_{3i} = -\int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{3i}d\alpha_3, \qquad L_{3i} = -\int_{-h}^{h}(1+\alpha_3/R_j)\mu_{3i}d\alpha_3,$$

$$N_{i3} = -\int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{i3}d\alpha_3, \qquad L_{i3} = -\int_{-h}^{h}(1+\alpha_3/R_j)\mu_{i3}d\alpha_3, \tag{2.3}$$

$$M_{3i} = -\int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{3i}\alpha_3 d\alpha_3, \qquad \Lambda_{3i} = -\int_{-h}^{h}(1+\alpha_3/R_j)\mu_{3i}\alpha_3 d\alpha_3,$$

$$M_{i3} = -\int_{-h}^{h}(1+\alpha_3/R_j)\sigma_{i3}\alpha_3 d\alpha_3, \qquad \Lambda_{i3} = -\int_{-h}^{h}(1+\alpha_3/R_j)\mu_{i3}\alpha_3 d\alpha_3,$$

$$\begin{aligned} N_{33} &= \int_{-h}^{h}(1+\alpha_3/R_1)(1+\alpha_3/R_2)\sigma_{33}d\alpha_3, \\ L_{33} &= \int_{-h}^{h}(1+\alpha_3/R_1)(1+\alpha_3/R_2)\mu_{33}d\alpha_3, \end{aligned}$$

$$M_{33} = \int_{-h}^{h} (1+\alpha_3/R_1)(1+\alpha_3/R_2)\sigma_{33}\alpha_3 d\alpha_3,$$
$$\Lambda_{33} = \int_{-h}^{h} (1+\alpha_3/R_1)(1+\alpha_3/R_2)\mu_{33}\alpha_3 d\alpha_3.$$

We shall as well use the concepts of transition and turnings of points on the shell's midplane surface.

$$u_i = V_i|_{\zeta=0}, \quad w = -V_3|_{\zeta=0}, \quad \Omega_i = \omega_i|_{\zeta=0}, \quad \Omega_3 = -\omega_3|_{\zeta=0}.$$

Here, as the main result, we shall introduce the defining system of two-dimensional equations on the level of asymptotic accuracy $O(\lambda^{p-l})$, which is a mathematical model of micropolar shell with independent fields of transition and rotation:

Balance equations:

$$\frac{1}{A_i}\frac{\partial T_{ii}}{\partial \alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_j}{\partial \alpha_i}(T_{ii}-T_{jj}) + \frac{1}{A_j}\frac{\partial S_{ji}}{\partial \alpha_j} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}(S_{ji}+S_{ij}) - (q_i^+ - q_i^-) = 0,$$

$$\frac{1}{A_i}\frac{\partial L_{ii}}{\partial \alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_j}{\partial \alpha_i}(L_{ii}-L_{jj}) + \frac{1}{A_j}\frac{\partial L_{ji}}{\partial \alpha_j} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}(L_{ji}+L_{ij}) +$$
$$+(-1)^j(N_{3j}-N_{j3}) - (m_i^+ - m_i^-) = 0, \tag{2.4}$$
$$\frac{T_{11}}{R_1} + \frac{T_{22}}{R_2} + \frac{1}{A_1A_2}\left[\frac{\partial(A_2N_{13})}{\partial \alpha_1} + \frac{\partial(A_1N_{23})}{\partial \alpha_2}\right] + (q_3^+ + q_3^-) = 0,$$
$$\frac{L_{11}}{R_1} + \frac{L_{22}}{R_2} + \frac{1}{A_1A_2}\left[\frac{\partial(A_2L_{13})}{\partial \alpha_1} + \frac{\partial(A_1L_{23})}{\partial \alpha_2}\right] - (S_{12}-S_{21}) + (m_3^+ + m_3^-) = 0.$$

Elasticity correlations:

$$T_{ii} = \frac{2Eh}{1-\nu^2}\left[\Gamma_{ii} + \nu\Gamma_{jj}\right], \qquad S_{ij} = 2h[(\mu+\alpha)\Gamma_{ij} + (\mu-\alpha)\Gamma_{ji}],$$

$$\begin{aligned} L_{ii} &= 2h\left[\tfrac{4\gamma(\beta+\gamma)}{\beta+2\gamma}\chi_{ii} + \tfrac{2\gamma\beta}{\beta+2\gamma}\chi_{jj}\right] - h\tfrac{\beta}{\beta+2\gamma}m, \\ L_{ij} &= 2h[(\gamma+\varepsilon)\chi_{ij} + (\gamma-\varepsilon)\chi_{ji}], \end{aligned} \tag{2.5}$$

$$N_{i3} = -2h\frac{4\alpha\mu}{\alpha+\mu}\Gamma_{i3} - \frac{\alpha-\mu}{\alpha+\mu}N_{3i}, \qquad L_{i3} = -2h\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\chi_{i3} + \frac{\gamma-\varepsilon}{\gamma+\varepsilon}L_{3i}.$$

Geometrical correlations:

$$\gamma_i = -\frac{1}{A_i}\frac{\partial w}{\partial \alpha_i} - \frac{u_i}{R_i}, \qquad \Gamma_{i3} = \gamma_i + (-1)^j\Omega_j, \qquad \chi_{i3} = -\frac{1}{A_i}\frac{\partial \Omega_3}{\partial \alpha_i} - \frac{\Omega_i}{R_i},$$

$$\begin{aligned} \Gamma_{ii} &= \tfrac{1}{A_i}\tfrac{\partial u_i}{\partial \alpha_i} + \tfrac{1}{A_iA_j}\tfrac{\partial A_i}{\partial \alpha_j}u_j - \tfrac{w}{R_i}, \\ \Gamma_{ij} &= \tfrac{1}{A_i}\tfrac{\partial u_j}{\partial \alpha_i} - \tfrac{1}{A_iA_j}\tfrac{\partial A_i}{\partial \alpha_j}u_i + (-1)^j\Omega_3, \end{aligned} \tag{2.6}$$

$$\chi_{ii}=\frac{1}{A_i}\frac{\partial\Omega_i}{\partial\alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\Omega_j-\frac{\Omega_3}{R_i},\qquad \chi_{ij}=\frac{1}{A_i}\frac{\partial\Omega_j}{\partial\alpha_i}-\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\Omega_i,$$

where

$$N_{3i}=h(q_i^+-q_i^-),\qquad L_{3i}=h(m_i^+-m_i^-).$$

Here, $T_{ii}(i=1,2)$, $S_{ij}(i,j=1,2;i\neq j)$, $N_{i3}(i=1,2)$ are the averaged forces, $L_{ii}(i=1,2)$, $L_{ij}(i,j=1,2;i\neq j)$, $L_{i3}(i=1,2)$ averaged moments, $\Gamma_{ii}(i=1,2)$, $\Gamma_{ij}(i,j=1,2;i\neq j)$, $\Gamma_{i3}(i=1,2)$ are the components of deformation tensor on the shell's midplane surface, $\chi_{ii}(i=1,2)$, $\chi_{ij}(i,j=1,2;i\neq j)$, $\chi_{i3}(i=1,2)$ are the components of bending-torsion tensor on the midplane surface of shell.

If we found the solution of equations (2.4)-(2.7) of applied two-dimensional theory of micropolar shell with independent fields of transition and rotation then the rest of the averaged values and the wanted values of the three-dimensional theory on the three-dimensional shell's plane will be defined by the corresponding formulae.

While studying the boundary micropolar elastic phenomena, we shall proceed from the equations of three-dimensional theory of ATE with IFTR (1.1)- (1.3). We shall consider that the shell's boundary, around which we shall have to investigate the stress state, is given by the equation $\alpha_1=\alpha_{10}$ and we shall change the free variables by the formulae:

$\alpha_1-\alpha_{10}=R\lambda^{-l}\xi_1$, $\alpha_2=R\lambda^{-p}\xi_2$, $\alpha_3=R\lambda^{-l}\zeta$, where R,λ,l,p have the same meaning as in (1.6).

We shall consider that in the boundary stress-deformed state the wanted values do not change their asymptotic behavior while differentiating on ξ_1,ξ_2,ζ.

The four boundary layer problems (plane and anti-plane, force and momental) are defined by separate differential equations. Studying the boundary layers on ATE with IFTR and their corresponding properties, we shall obtain several equations which satisfy to the attenuating character of boundary solutions.

General SDS in the shell is defined by the following structural formula:

$$(\mathrm{SDS})_{complete}=(\mathrm{SDS})_{interior}+\lambda^{r}\,(\mathrm{SDS})^{a}_{boundary}+\lambda^{\Theta}(\mathrm{SDS})^{n}_{boundary}$$

numbers r,Θ are the indices of micropolar boundary layer's intensity. Boundary layer equations are homogeneous. Numbers r and Θ are to be chosen in such a way that they could satisfy to the boundary conditions (1.5) on the shell's lateral surface. Considering the problem of matching the interior iteration process and micropolar boundary layers, we shall obtain the following boundary conditions on the boundary contour of the shell's midplane on the level of asymptotic approximation $O(\lambda^{p-l})$ for the system of equations of applied two-dimensional theory of micropolar shells (in case of the first variant of boundary conditions (1.5)):

$$T_{11}|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} p_1^* d\alpha_3, \qquad S_{12}|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} p_2^* d\alpha_3,$$

$$L_{11}|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} m_1^* d\alpha_3, \qquad L_{12}|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} m_2^* d\alpha_3, \qquad (2.8)$$

$$N_{13}|_{\alpha_1=\alpha_{10}} = -\int_{-h}^{h} p_3^* d\alpha_3, \qquad L_{13}|_{\alpha_1=\alpha_{10}} = -\int_{-h}^{h} m_3^* d\alpha_3.$$

Equations (2.4)-(2.7) and boundary condition (2.8) are mathematical models of micropolar shell with free fields of transition and rotation. This is the equation of the twelfth order with six boundary conditions on each boundary cut. Plane and anti-plane force and momental boundary layers will be defined by separate differential equations with corresponding boundary conditions.

3. Applied two-dimensional theory of micropolar elastic thin shells with constraint rotation.

Let's suppose that the dimensionless physical constants of the shell's material (1.7) are now expressed in the following way:

$$\frac{\alpha}{\mu} \sim 1, \quad \frac{\beta}{R^2\mu} \sim \lambda^{-2l}\beta^*, \quad \frac{\gamma}{R^2\mu} \sim \lambda^{-2l}\gamma^*, \quad \frac{\varepsilon}{R^2\mu} \sim \lambda^{-2l}\varepsilon^* \quad (\beta^*, \gamma^*, \varepsilon^*, \sim 1). \qquad (3.1)$$

For the interior problem, in this case, on the order of asymptotic approximation $O(\lambda^{p-l})$ for the wanted sizes in the three-dimensional thin shell plane we shall obtain the following asymptotic representations:

$$\tau_{ii} = \mu\lambda^l(\tau_{ii}^0 + \lambda^{-l+2p-c}\zeta\tau_{ii}^1), \quad \tau_{ij} = \mu\lambda^l(\tau_{ij}^0 + \lambda^{-l+2p-c}\zeta\tau_{ij}^1 + \lambda^{-l+2p-c}\widetilde{\tau}_{ij}),$$

$$\tau_{3i} = \mu\lambda^p(\tau_{3i}^0 + \zeta\tau_{3i}^1 + \lambda^{-l+2p-c}\zeta^2\tau_{3i}^2 + \lambda^{-l+2p-c}\widetilde{\tau}_{3i}),$$

$$\tau_{i3} = \mu\lambda^p(\tau_{i3}^0 + \zeta\tau_{i3}^1 + \lambda^{-l+2p-c}\zeta^2\tau_{i3}^2 + \lambda^{-l+2p-c}(\widetilde{\tau}_{i3} + \widetilde{\widetilde{\tau}}_{i3})),$$

$$\begin{aligned}\tau_{33} = {} & \mu\lambda^c(\tau_{33}^0 + \zeta\tau_{33}^1 + \lambda^{-l+2p-c}\zeta^2\tau_{33}^2 + \lambda^{-2l+4p-2c}\zeta^3\tau_{33}^3 \\ & + \lambda^{-2l+4p-2c}(\widetilde{\tau}_{33} + \widetilde{\widetilde{\tau}}_{33})),\end{aligned} \qquad (3.2)$$

$$\begin{aligned} v_{ii} &= R\mu\lambda^{-l+2p-c}(v_{ii}^0 + \widetilde{v}_{ii}), \quad v_{ij} = R\mu\lambda^{-l+2p-c}v_{ij}^0, \\ v_{33} &= R\mu\lambda^{-l+2p-c}(v_{33}^0 + \widetilde{v}_{33}), \end{aligned}$$

$$\begin{aligned} v_{i3} &= R\mu\lambda^{-l+p}(v_{i3}^0 + \lambda^{-l+2p-c}\zeta v_{i3}^1 + \lambda^{-l+2p-c}(\widetilde{v}_{i3} + \widetilde{\widetilde{v}}_{i3})) \quad (i \leftrightarrow 3), \\ \omega_i &= \lambda^{l+p-c}\omega_i^0, \end{aligned}$$

$$V_i = R\lambda^{l-p}(V_i^0 + \lambda^{-l+2p-c}\zeta V_i^1), \quad V_3 = R\lambda^{l-c}V_3^0,$$
$$\omega_3 = \lambda^l(\omega_3^0 + \lambda^{-l+2p-c}\zeta\omega_3^1 + \lambda^{-l+2p-c}\widetilde{\omega}_3).$$

Asymptotics (3.1), (3.2) have the following peculiarities:

1) components of vector rotation in the points of the shell's midplane surface are expressed by the components of transition vector at these points as in the elasticity theory;

2) part of the values are connected and form a two-dimensional theory, and the definition of the other values μ_{31}, μ_{32}, μ_{33} are brought to separate boundary value problems for differential equations of second order (as ordinary differential equations) with free variable ζ, where variables ξ_1 and ξ_2 are entered as parameters. The main result is the defining system of two-dimensional equations, which is represented as a defining system of applied two-dimensional theory of micropolar shells with constraint rotation:

Balance equations:

$$\frac{1}{A_i}\frac{\partial T_{ii}}{\partial\alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_j}{\partial\alpha_i}(T_{ii}-T_{jj})+\frac{1}{A_j}\frac{\partial S_{ji}}{\partial\alpha_j}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}(S_{ji}+S_{ij})-(q_i^+-q_i^-)=0,$$

$$\frac{1}{A_i}\frac{\partial(G_{ii}-(-1)^jL_{ij})}{\partial\alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_j}{\partial\alpha_i}\left[(G_{ii}-(-1)^jL_{ij})-(G_{jj}+(-1)^jL_{jj})\right]-$$
$$-\frac{1}{A_j}\frac{\partial}{\partial\alpha_j}(H_{ji}+(-1)^jL_{ij})-\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\left[(H_{ji}-(-1)^jL_{jj})+(H_{ij}-(-1)^jL_{ii})\right]-$$
$$-N_{i3}+h(q_i^+-q_i^-)+(-1)^j(m_j^+-m_j^-)=0, \tag{3.3}$$
$$\frac{T_{11}}{R_1}+\frac{T_{22}}{R_2}+\frac{1}{A_1A_2}\left[\frac{\partial(A_2N_{13})}{\partial\alpha_1}+\frac{\partial(A_1N_{23})}{\partial\alpha_2}\right]+(q_3^++q_3^-)=0.$$

Elasticity correlations

$$G_{ii} = -\frac{2Eh^3}{3(1-v^2)}\left[K_{ii}+vK_{jj}\right],$$
$$H_{ij} = \frac{Eh^3}{3(1+v)}\left[K_{12}+K_{21}\right]+(-1)^j\frac{1}{2}\left(1-\frac{hk_1}{th(hk_1)}\right)\widetilde{L}_{33},$$

$$T_{ii}=\tfrac{2Eh}{1-v^2}\left[\Gamma_{ii}+v\Gamma_{jj}\right],$$
$$S_{ij}=\tfrac{Eh}{1+v}\left[\Gamma_{12}+\Gamma_{21}\right]+(-1)^j\tfrac{1}{2}\left(m_3^++m_3^-\right), \tag{3.4}$$

$$L_{ii}=4h\gamma\chi_{ii}+\frac{\beta}{\beta+2\gamma}\widetilde{L}_{33}, \qquad L_{ij}=2h\left[(\gamma+\varepsilon)\chi_{ij}+(\gamma-\varepsilon)\chi_{ji}\right],$$

$$\widetilde{L}_{33}=\frac{th(hk_1)}{k_1}\left[4\gamma(\chi_{11}+\chi_{22})-(m_3^+-m_3^-)\right], \quad k_1=\sqrt{\frac{4\alpha}{\beta+2\gamma}}.$$

Geometrical correlations

$$\Gamma_{ii} = \frac{1}{A_i}\frac{\partial u_i}{\partial \alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}u_j - \frac{w}{R_i}, \quad \Gamma_{ij} = \frac{1}{A_i}\frac{\partial u_j}{\partial \alpha_i} - \frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}u_i,$$
$$\beta_i = \frac{1}{A_i}\frac{\partial w}{\partial \alpha_i} + \frac{u_i}{R_i},$$

$$K_{ii} = \tfrac{1}{A_i}\tfrac{\partial \beta_i}{\partial \alpha_i} + \tfrac{1}{A_iA_j}\tfrac{\partial A_i}{\partial \alpha_j}\beta_j, \quad K_{ij} = \tfrac{1}{A_i}\tfrac{\partial \beta_j}{\partial \alpha_i} - \tfrac{1}{A_iA_j}\tfrac{\partial A_i}{\partial \alpha_j}\beta_i,$$
$$\Omega_i = (-1)^i\beta_i, \tag{3.5}$$

$$\Omega_3 = \frac{1}{2}(\Gamma_{21} - \Gamma_{12}), \quad \chi_{ii} = \frac{1}{A_i}\frac{\partial \Omega_i}{\partial \alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}\Omega_j - \frac{\Omega_3}{R_i},$$
$$\chi_{ij} = \frac{1}{A_i}\frac{\partial \Omega_j}{\partial \alpha_i} - \frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}\Omega_i.$$

If the solution to the two-dimensional equations (3.3)-(3.5) of the applied two-dimensional theory of micropolar shells with constraint rotation is known then all the calculated values of the three-dimensional theory in the shell's three-dimensional plane are defined by corresponding formula.

Studying the micropolar boundary layers (which are two plane and anti-plane mixed force-momental boundary layers), their corresponding properties and the matching of interior problem and the boundary layer we shall obtain boundary conditions of two-dimensional theory (in case of the first variant of boundary conditions (1.5)):

$$T_{11}|_{\alpha_1=\alpha_{12}} = \int_{-h}^{h} p_1^* d\alpha_3, \quad S_{12}|_{\alpha_1=\alpha_{12}} = \int_{-h}^{h} p_2^* d\alpha_3,$$

$$[L_{12} - G_{11}]\,|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} (m_2^* + \alpha_3 P_1^*) d\alpha_3,$$

$$\left[-N_{13} + \tfrac{1}{A_2}\tfrac{\partial}{\partial \alpha_2}(H_{12} - L_{11})\right]\Big|_{\alpha_1=\alpha_{10}}$$
$$= \int_{-h}^{h} \left[P_3^* + \tfrac{1}{A_2}\tfrac{\partial}{\partial \alpha_2}(\alpha_3 P_2^* - m_1^*)\right] d\alpha_3. \tag{3.6}$$

Equations (3.3)-(3.5) and boundary conditions (3.6) are mathematical models of micropolar shell with constraint rotation. This system of equations is of eighth order with four boundary conditions on each boundary cut.

The mentioned boundary layer problems are defined as separate differential equations and by corresponding boundary conditions.

4. Applied two-dimensional theory of micropolar shell with small shift rigidity.

Now let's consider that the physical constants of shell material (1.7) are presented as follows:

$$\frac{\alpha}{\mu} = \lambda^{-2l+2p}\alpha_*, \qquad \frac{\beta}{R^2\mu} = \beta_*, \qquad \frac{\varepsilon}{R^2\mu} = \varepsilon_*, \qquad \frac{\gamma}{R^2\mu} = \gamma_*, \tag{4.1}$$

where $\alpha_*,\ \beta_*,\ \varepsilon_*,\ \gamma_* \sim 1$.

For the values of interior problem on the level of asymptotic accuracy $O(\lambda^{p-l})$ in the three-dimensional thin shell plane we shall obtain the following asymptotic presentation:

$$\tau_{ii} = \mu\lambda^l(\tau_{ii}^0 + \lambda^{-l+2p-c}\zeta\tau_{ii}^1), \quad \tau_{ij} = \mu\lambda^l(\tau_{ij}^0 + \lambda^{-l+2p-c}\zeta\tau_{ij}^1), \quad v_{ii} = R\mu\lambda^{l+2p-c}v_{ii}^0,$$

$$\tau_{i3} = \mu\lambda^p(\tau_{i3}^0 + \zeta\tau_{i3}^1 + \lambda^{-l+2p-c}\zeta^2\tau_{i3}^2)\ (i \leftrightarrow 3), \quad v_{ij} = R\mu\lambda^{l+2p-c}v_{ij}^0,$$

$$\tau_{33} = \mu\lambda^c(\tau_{33}^0 + \lambda^{-c}\zeta\tau_{33}^1 + \lambda^{-l+2p-c}\zeta^2\tau_{33}^2 + \lambda^{-2l+4p-2c}\zeta^3\tau_{33}^3), \tag{4.2}$$

$$\begin{aligned} v_{i3} &= R\mu\lambda^{l+p}(v_{i3}^0 + \lambda^{-l+2p-c}\zeta v_{i3}^1)\ \ (i \leftrightarrow 3), \\ v_{33} &= R\mu\lambda^{2p}(v_{33}^0 + \zeta v_{33}^1 + \lambda^{-l+2p-c}\zeta^2 v_{33}^2), \end{aligned}$$

$$\begin{aligned} V_i &= R\lambda^{l-p}(V_i^0 + \lambda^{-l+2p-c}\zeta V_i^1), \ \ V_3 = R\lambda^{l-c}V_3^0, \\ \omega_i &= \lambda^{l+p-c}\omega_i^0, \ \ \omega_3 = \lambda^l(\omega_3^0 + \lambda^{-l+2p-c}\zeta\omega_3^1). \end{aligned}$$

Let's mention that in case of asymptotics (4.1), (4.2) in the two-dimensional equations of micropolar shell the values of "pure momental" character get separated and form separate system of equations. For the "stress" part we shall obtain a shift theory of shells, where the corners of turning are conditioned by "pure momental" part of the problem.

Let's formulate the following separate groups of equations.

Equations of "pure momental" part of micropolar shell problem:

Balance equations

$$\frac{1}{A_i}\frac{\partial L_{ii}}{\partial \alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_j}{\partial \alpha_i}(L_{ii} - L_{jj}) + \frac{1}{A_j}\frac{\partial L_{ji}}{\partial \alpha_j} +$$

$$+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial \alpha_j}(L_{ji} + L_{ij}) - (m_i^+ - m_i^-) = 0,$$

$$\frac{L_{11}}{R_1} + \frac{L_{22}}{R_2} + \frac{1}{A_1A_2}\left[\frac{\partial(A_2L_{13})}{\partial\alpha_1} + \frac{\partial(A_1L_{23})}{\partial\alpha_2}\right] + (m_3^+ + m_3^-) = 0. \tag{4.3}$$

Elasticity correlations

$$L_{ii} = 2h\left[\frac{4\gamma(\beta+\gamma)}{\beta+2\gamma}\chi_{ii} + \frac{2\gamma\beta}{\beta+2\gamma}\chi_{jj}\right],$$

$$L_{ij} = 2h\left[(\gamma+\varepsilon)\chi_{ij} + (\gamma-\varepsilon)\chi_{ji}\right], \quad L_{i3} = -2h\frac{4\gamma\varepsilon}{\gamma+\varepsilon}\chi_{I3} + \frac{\gamma-\varepsilon}{\gamma+\varepsilon}h(m_i^+ - m_i^-). \tag{4.4}$$

Geometrical correlations

$$\chi_{i3} = -\frac{1}{A_i}\frac{\partial\Omega_3}{\partial\alpha_i} - \frac{\Omega_i}{R_i}, \quad \chi_{ii} = \frac{1}{A_i}\frac{\partial\Omega_i}{\partial\alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\Omega_j - \frac{\Omega_3}{R_i},$$

$$\chi_{ij} = \frac{1}{A_i}\frac{\partial\Omega_j}{\partial\alpha_i} - \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\Omega_i. \tag{4.5}$$

Equations of "pure force" part of micropolar shell problem:
Balance equations

$$\frac{1}{A_i}\frac{\partial T_{ii}}{\partial\alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_j}{\partial\alpha_i}(T_{ii} - T_{jj}) + \frac{1}{A_j}\frac{\partial S_{ji}}{\partial\alpha_j} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}(S_{ji} + S_{ij}) - (q_i^+ - q_i^-) = 0,$$

$$\frac{T_{11}}{R_1} + \frac{T_{22}}{R_2} + \frac{1}{A_1A_2}\left[\frac{\partial(A_2N_{13})}{\partial\alpha_1} + \frac{\partial(A_1N_{23})}{\partial\alpha_2}\right] + (q_3^+ + q_3^-) = 0, \tag{4.6}$$

$$-N_{3i} + \frac{1}{A_i}\frac{\partial G_{ii}}{\partial\alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_j}{\partial\alpha_i}(G_{ii} - G_{jj}) -$$

$$-\frac{1}{A_j}\frac{\partial H_{ji}}{\partial\alpha_j} - \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}(H_{ji} + H_{ij}) + h(q_i^+ - q_i^-) = 0$$

Elasticity correlations

$$T_{ii} = \frac{2Eh}{1-v^2}(\Gamma_{ii} + v\Gamma_{jj}), \quad S_{ij} = S_{ji} = 2h\mu(\Gamma_{ij} + \Gamma_{ji}), \quad N_{i3} = N_{3i} - 8h\alpha\Gamma_{i3},$$

$$G_{ii} = -\frac{2Eh^3}{3(1-v^2)}(K_{ii} + vK_{jj}), \quad H_{ij} = H_{ji} = \frac{2\mu h^3}{3}(K_{ij} + K_{ji}). \tag{4.7}$$

Geometrical correlations

$$\beta_i = \frac{1}{A_i}\frac{\partial w}{\partial\alpha_i} + \frac{u_i}{R_i}, \quad \Gamma_{i3} = -\beta_i + (-1)^j\Omega_j, \quad \Gamma_{ii} = \frac{1}{A_i}\frac{\partial u_i}{\partial\alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}u_j - \frac{w}{R_i},$$

$$\begin{aligned} \Gamma_{ij} &= \frac{1}{A_i}\frac{\partial u_j}{\partial\alpha_i} - \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}u_i, \quad K_{ii} = \frac{1}{A_i}\frac{\partial\beta_i}{\partial\alpha_i} + \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\beta_j, \\ K_{ij} &= \frac{1}{A_i}\frac{\partial\beta_j}{\partial\alpha_i} - \frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}\beta_i. \end{aligned} \tag{4.8}$$

If we treat the coefficient $8h\alpha$ with shift-momental rigidity (the physical constant α is the shift module like the classical module μ) then the presented theory (4.6)-(4.8) with the account of (4.1) we can treat as shell theory "with small shift rigidity".

On the basis of construction of corresponding micropolar boundary layers and the study of interaction problem of interior problem with boundary layer problem we shall obtain separate boundary conditions for the system of equations (4.3)-(4.5) and (4.6)-(4.8):

$$\begin{aligned} L_{11}|_{\alpha_1=\alpha_{10}} &= \int_{-h}^{h} m_1^* d\alpha_3, \quad L_{12}|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} m_2^* d\alpha_3, \\ L_{13}|_{\alpha_1=\alpha_{10}} &= \int_{-h}^{h} m_3^* d\alpha_3, \end{aligned}$$

$$\left(-N_{13} + \frac{1}{A_2}\frac{\partial H_{12}}{\partial \alpha_2}\right)\Bigg|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} p_3^* d\alpha_3 + \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}\int_{-h}^{h} p_2^* \alpha_3 d\alpha_3, \quad (4.9)$$

$$\begin{aligned} T_{11}|_{\alpha_1=\alpha_{10}} &= \int_{-h}^{h} p_1^* d\alpha_3, \quad S_{12}|_{\alpha_1=\alpha_{10}} = \int_{-h}^{h} p_2^* d\alpha_3, \\ G_{11}|_{\alpha_1=\alpha_{10}} &= -\int_{-h}^{h} p_1^* \alpha_3 d\alpha_3. \end{aligned}$$

System of equations (4.3)-(4.5) and (4.6)-(4.8) with corresponding boundary conditions of (4.9) form a mathematical model of micropolar shell with "small shift rigidity".

3 Conclusion

Here the asymptotic approach of constructing the mathematical models of thin shells on the basis of asymmetrical elasticity with free fields of transition and rotation is presented. The essential point consists in the fact that certain mathematical model of micropolar shell depends on values of physical dimensionalless parameters of shell material, where scale factor is also included. This means that depending on the mentioned scale factor and the shell's thin-walledness, the constructed mathematical models of micropolar shell can be used in the investigation of problems of structural mechanics and particularly for problems of micro- and nano-mechanics.

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