HEAT TRANSFER WITH THE FLOW OF CONDUCTING FLUID IN CIRCULAR PIPES WITH FINITE CONDUCTIVITY UNDER UNIFORM TRANSVERSE MAGNETIC FIELDS

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Abstract

The flow of conducting, viscous fluids in circular pipes under transverse magnetic field is studied theoretically.The correlation of Hartman's figure, Poisale's figure, Reynold's figure and conductivity of walls are considered.

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Exploration of flows of electrically conducting fluid with two approaches are considered either in noninductive and inductive ways. Magnetohydrodynamics main equations in noninductive ($R_m \ll 1$) approach will be done as follows [4–7]:

$$\begin{cases} \dfrac{\partial \vec{V}}{\partial t} + (\vec{V}\nabla)\vec{V} = -\dfrac{1}{\rho}\,\mathrm{grad}\,p + \nu\Delta\vec{V} - \dfrac{\sigma}{\rho}\left(\vec{H}\times\left(\vec{V}\times\vec{H}\right)\right), \\ \mathrm{div}\,\vec{V} = 0, \quad \mathrm{div}\,\vec{H} = 0, \quad \mathrm{rot}\,\vec{H} = 0, \\ \rho C_V\left(\dfrac{\partial T}{\partial t} + \left(\vec{V}\nabla\right)T\right) = K\Delta T + \Phi + \sigma\left(\vec{V}+\vec{H}\right)^2, \end{cases} \tag{1}$$

where $\sigma(\vec{V}\times\vec{H})^2$ is a **Jole** heat, and Φ is a dissipation function as a result of friction and will be gauged as follows:

$$\Phi = 2\eta\left\{\frac{1}{2}\left[\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)^2 + \left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z}\right)^2\right] + \right.$$
$$\left. + \left(\frac{\partial V_x}{\partial x}\right)^2 + \left(\frac{\partial V_y}{\partial y}\right)^2 + \left(\frac{\partial V_z}{\partial z}\right)^2\right\}. \tag{2}$$

Let us consider flow of viscous incompressible weakly conducting fluid taking into account heat transfers under effects of external homogenous magnetic field (H_0) in square pipe. Here we may suppose that the conditions are created when the tension of electric field is equal to zero ($E = 0$). Induced magnetic field inside fluid is less in contrast with external magnetic field and it is ignored.

It is well known that the fluid speed has only one constituent: $\vec{V} = V_z(x,t)$ directed along axis OZ, and temperature T is considered to be the function of axis x and t ($T = T(x,t)$).

Taking into consideration of above mentioned the system of magnetohydrodynamic equations in non-dimensional values is as follows [1, 2, 8]:

$$\begin{cases} \dfrac{\partial U}{\partial \tau} - \dfrac{\partial^2 U}{\partial R^2} - \dfrac{1}{R}\dfrac{\partial U}{\partial R} + M^2 U = f(\tau), \\ P_r \dfrac{\partial \theta}{\partial \tau} - \dfrac{\partial^2 \theta}{\partial R^2} - \dfrac{1}{R}\dfrac{\partial \theta}{\partial R} = \left(\dfrac{\partial U}{\partial R}\right)^2 + M^2 U^2, \end{cases} \tag{3}$$

where $U = \frac{V}{V_0}$, $R = \frac{r}{a}$, $t = \frac{a^2 \tau}{\nu}$, $\theta = \frac{k}{\eta V_0^2}$, $f(\tau) = -\frac{a^2}{\nu V_0 \rho}\frac{\partial P}{\partial z}$ are non-dimensional values, and V_0 and a-typical speed and length, correspondingly, $M = H_0 a\sqrt{\frac{\sigma}{\eta}}$ is **Hartmann's** number, $R_m = \frac{V_0 a}{\nu_m}$ is Raynold's magnetic number, $P_r = \frac{\eta C_V}{k}$ is **Prandtl's number**, $\alpha = \frac{\omega a^2}{\nu}$ is simulation criterion, established by pulsating flow. ρ, ω, ν, η, C_V, k, σ, ν_m are density, frequency, kinematic viscosity, dynamic viscosity, heat capacity, heat conduction, electrical conduction and fluid magnetic viscosity coefficient, correspondingly.

Extreme conditions generally are as follows:

$$\begin{cases} U(R,0) = 0, \quad U(1,\tau) = \varphi_1(\tau), \quad \theta_{1,2}(R,0) = q_{1,2}(R), \\ \theta(1,\tau) = \theta_1(1,\tau) + \theta_2(1,\tau) = q_1^{(1)}(\tau) + q_2^{(2)}(\tau) = q(\tau), \end{cases} \tag{4}$$

where $\theta_1(R,\tau)$ is temperature while in equation of heat-transfer is taken into account only viscous heat, and $\theta_2(R,\tau)$ is temperature while in equation of heat-transfer is taken into account only **Jole** heat.

It is well known that pulsating flow of fluid is caused only by pulsating drop of pressure ($f(\tau) = Ae^{i\alpha\tau}$), the pipe is not wheeled and change of temperature on the surface of the pipe is not equal to zero ($\varphi_1(\tau) = 0$, $\theta_{1,2}(R,0) = 0$, $\theta_{1,2}(1,\tau) = B_{1,2}e^{2i\alpha\tau}$). In equation of heat-transfer is taken into account either viscous heat - $\left(\frac{\partial U}{\partial R}\right)^2$, or **Jole** heat $(MU)^2$.

Let us search solution of for the task (3)–(4) in following view [3]:

$$\begin{cases} U(R,\tau) = \varphi(R)e^{i\alpha\tau}, \\ \theta_1(R,\tau) = \psi_1(R)e^{2i\alpha\tau}, \\ \theta_2(R,\tau) = \psi_2(R)e^{2i\alpha\tau}. \end{cases} \tag{5}$$

Finally, for speed and heat transfer we will get:

$$U(R,\tau)=\frac{A}{M^2+i\alpha}\left(1-\frac{I_0(\sqrt{M^2+i\alpha}\,R)}{I_0(\sqrt{M^2+i\alpha}\,)}\,e^{i\alpha\tau}\right),$$

$$\theta_1(R,\tau)=\frac{B_1 I_0(\sqrt{2i\alpha P_r}\,R)}{I_0(\sqrt{2i\alpha P_r}\,)}\,e^{2i\alpha\tau}+$$

$$+\frac{A^2e^{2i\alpha\tau}}{\sqrt{2i\alpha\tau P_r}\,(M^2+i\alpha)I_0^2(\sqrt{M^2+i\alpha}\,)}\Bigg\{\Bigg[K_0(\sqrt{2i\alpha P_r}\,R)\times$$

$$\times\int\frac{I_1^2(\sqrt{M^2+i\alpha}\,R)I_0(\sqrt{2i\alpha P_r}\,R)\,dR}{I_0(\sqrt{2i\alpha P_r}\,R)K_1(\sqrt{2i\alpha P_r}\,R)+I_1(\sqrt{2i\alpha P_r}\,R)K_0(\sqrt{2i\alpha P_r}\,R)}\Bigg]-$$

$$-\Bigg[I_0(\sqrt{2i\alpha P_r}\,R)\times$$

$$\times\int\frac{I_1^2(\sqrt{M^2+i\alpha}\,R)K_0(\sqrt{2i\alpha P_r}\,R)\,dR}{I_0(\sqrt{2i\alpha P_r}\,R)K_1(\sqrt{2i\alpha P_r}\,R)+I_1(\sqrt{2i\alpha P_r}\,R)K_0(\sqrt{2i\alpha P_r}\,R)}\Bigg]\Bigg\},$$

$$\theta_2(R,\tau)=\frac{B_2 I_0(\sqrt{2i\alpha P_r}\,R)}{I_0(\sqrt{2i\alpha P_r}\,)}\,e^{2i\alpha\tau}+$$

$$+\frac{A^2M^2e^{2i\alpha\tau}}{\sqrt{2i\alpha\tau P_r}\,(M^2+i\alpha)I_0^2(\sqrt{M^2+i\alpha}\,)}\Bigg\{\Bigg[K_0(\sqrt{2i\alpha P_r}\,R)\times$$

$$\times\int\frac{[I_0(\sqrt{M^2+i\alpha})-I_0(\sqrt{M^2+i\alpha}\,R)]^2I_0(\sqrt{2i\alpha P_r}\,R)\,dR}{I_0(\sqrt{2i\alpha P_r}\,R)K_1(\sqrt{2i\alpha P_r}\,R)+I_1(\sqrt{2i\alpha P_r}\,R)K_0(\sqrt{2i\alpha P_r}\,R)}\Bigg]-$$

$$-\Bigg[I_0(\sqrt{2i\alpha P_r}\,R)\times$$

$$\times\int\frac{[I_0(\sqrt{M^2+i\alpha}\,)-I_0(\sqrt{M^2+i\alpha}\,R)]^2K_0(\sqrt{2i\alpha P_r}\,R)\,dR}{I_0(\sqrt{2i\alpha P_r}\,R)K_1(\sqrt{2i\alpha P_r}\,R)+I_1(\sqrt{2i\alpha P_r}\,R)K_0(\sqrt{2i\alpha P_r}\,R)}\Bigg]\Bigg\},$$

$$\theta(R,\tau)=\theta_1(R,\tau)+\theta_2(R,\tau),$$

where I_0, K_0 and I_1, K_1 are correspondingly the functions of zero and first order of **Bessel** and **MacDonald's** ($I_0'=I_1'$, $K_0'=-K_1$).

Viscosity strength on the wall and fluid consumption through pipe profile are calculated as follows:

$$F=-\frac{A\eta}{\sqrt{M^2+i\alpha}}\cdot\frac{I_1(\sqrt{M^2+i\alpha}\,R)e^{i\alpha\tau}}{I_0(\sqrt{M^2+i\alpha}\,)},$$

$$Q=\frac{\pi A}{M^2+i\alpha}\cdot\left(1-\frac{\sum\limits_{k=0}^{\infty}\frac{(\frac{1}{2}\sqrt{M^2+i\alpha}\,)^{2k}}{(K+1)!\Pi(K)}}{\sum\limits_{k=0}^{\infty}\frac{(\frac{1}{2}\sqrt{M^2+i\alpha}\,)}{K!\Pi(K)}}\right)e^{i\alpha\tau},$$

where $\Pi(K)$ Gauss function ($\Pi(K) = \int\limits_0^\infty e^{-x} x^k \, dx$).

Calculations show that pulsating flow of weakly conducting viscous incompressible fluid in square pipe in presence of external homogenous magnetic field is hampered and maximum speed transfers from axis of pipe towards walls. The most intensive effect of retard is observed while the walls of channel are ideally conducting. At minor **Hartman's** figures viscous dissipation plays more important role than **Jole** heat. The fluid temperature in square pipe under pulsation drop of pressure is reduced with the length of **Hartman's** number and reduction of **Prandtl** number. This result corresponds with statement on retarded effect of magnetic field on fluid flow.

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