THE ASYMPTOTIC SOLUTION OF BLASIUS PROBLEM FOR CONDUCTING FLUID WITH STRONG SUCTION

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Abstract

The asymptotic solution of Blasius problem for conducting fluid, when electrical conductivity of fluid is $\sigma = \sigma_0 \left(1 - \frac{u}{u_\infty}\right)^m$ and transverse magnetic field is perpendicular to the plate, has been considered.

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We consider a steady flow of conducting fluid past an half infinity porous flat plate, when all physical quantities are constant.

Let x and y be the coordinates measured along and perpendicular to the plate, u and v are the components of velocity in the respectively directions of a boundary layer. A transverse magnetic field $B_0(x)$ is perpendicular to the plate. We consider the case when σ electrical conductivity is given by:

$$\sigma = \sigma_0 \left(1 - \frac{u}{u_\infty}\right)^m, \ \ m > 1.$$

where $\sigma_0 = const$, $u_\infty = const$ is the velocity of the flow outside.

If we negligible Arqimed eliminate force and respect to $\frac{\partial p}{\partial x} = 0$, when the equations of dynamic and temperature, including viscous and Joule dissipation of a boundary layer are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho} u \left(1 - \frac{u}{u_\infty}\right)^m, \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$\rho c_\tau \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \lambda \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \sigma_0 B_0^2 u^2 \left(1 - \frac{u}{u_\infty}\right)^m; \tag{3}$$

the boundary conditions on $u(x,y)$ and $T(x,y)$ are:

$$u(x,0) = 0; \quad u(x,\infty) = u_\infty; \quad v(x,0) = -v_w(x); \tag{4}$$

$$T(x,0) = T_w; \quad T(x,\infty) = T_\infty, \tag{5}$$

where ρ is the density, ν and μ are the cinematic and dynamic viscosity, λ is the thermal conductivity, c_τ is the specific heat of the fluid, when volume is constant, $-v_w(x)$ is the suction velocity, $T_w = const$ is the temperature of plate, and $T_\infty = const$ is the temperature of the flow outside.

Using (2) and (4) boundary conditions in equations (1) and (3) we get:

$$u\,\frac{\partial u}{\partial x} - v_w\,\frac{\partial u}{\partial y} - \frac{\partial u}{\partial y}\int_0^y \frac{\partial u}{\partial x}\,dy = \nu\,\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho}\,u\biggl(1-\frac{u}{u_\infty}\biggr)^m;$$

$$u\,\frac{\partial T}{\partial x} - v_w\,\frac{\partial T}{\partial y} - \frac{\partial T}{\partial y}\int_0^y \frac{\partial u}{\partial x}\,dy =$$

$$= \frac{\lambda}{\rho c_\tau}\,\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_\tau}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma_0 B_0^2}{\rho c_\tau}\,u^2\biggl(1-\frac{u}{u_\infty}\biggr)^m.$$

These equations of boundary layer will have similar solution if we require, that $v_w(x) \sim x^{-\frac{1}{2}}$, $B_0(x) \sim x^{-\frac{1}{2}}$.

If we define the non-dimensional velocity and temperature by

$$u(x,y) = u_\infty f(\eta),$$
$$T(x,y) = (T_\infty - T_w)\varphi(\eta) + T_w,$$

where $\eta = \sqrt{\frac{u_\infty}{\nu x}}\,y$, then we obtain:

$$\begin{aligned} f'' + \beta f' + \frac{1}{2}\,f'\int_0^\eta f\,ds - Nf(1-f)^m = 0,\\ \frac{1}{Pr}\,\varphi'' + \beta\varphi' + \frac{1}{2}\,\varphi'\int_0^\eta f\,ds + Ecf'^2 + NEcf^2(1-f)^m = 0, \end{aligned} \qquad (6)$$

and the boundary conditions of f and φ becomes:

$$f(0) = 0, \quad f(\infty) = 1,$$
$$\varphi(0) = 0, \quad \varphi(\infty) = 1;$$

the following denotes are given:

$$\beta = v_w\sqrt{\frac{x}{\nu u_\infty}}\,, \qquad N = \frac{\sigma_0 B_0^2 x}{\rho u_\infty}\,,$$
$$Pr = \frac{\lambda}{c_\tau \rho \nu}\,, \qquad Ec = \frac{u_\infty^2}{c_\tau(T_w - T_\infty)}\,.$$

These systems are general equations of dynamic and heat boundary layer with external magnetic field and suction velocity.

As dynamic field does not depend on the temperature field we can solve the hydrodynamic problem and result empty to define temperature field. In order to find the asymptotic solution of equations (6), suppose that suction in the plate is strong, let us now introduce a new variable z and functions $F(z)$, $\phi(z)$ by

$$\begin{aligned} &z = \beta\eta, \\ &F(z) = f(\eta), \quad \phi(z) = f(\eta); \end{aligned}$$

then (6) system becomes:

$$\begin{aligned} &F'' + F' = -\varepsilon\left\{\frac{1}{2}F'\int_0^z F\,ds - N(1-F)^m F\right\} = A(z), \\ &\phi'' + Pr\phi' = -PrEcF'^2 - \\ &\quad -\varepsilon Pr\left\{\frac{1}{2}\phi'\int_0^z F\,ds + NEcF^2(1-F)^m\right\} = B(z), \end{aligned} \tag{7}$$

where $\varepsilon = \frac{1}{\beta^2}$.

The solutions of the system (7) can be written as

$$\begin{aligned} F &= F_0 + \int_0^\infty A(s)\,G(z,s)\,ds, \\ \phi &= \phi_0 + \int_0^\infty B(s)\,G_{\mathrm{Pr}}(z,s)\,ds, \end{aligned} \tag{8}$$

where G and G_{Pr} are Green's functions of following boundary problems respectively:

$$\begin{aligned} &G'' + G' = 0, \\ &G(0) = 0, \quad G(\infty) = 0; \\ &G''_{Pr} + P_r G'_{Pr} = 0, \\ &G_{Pr}(0) = 0, \quad G_{Pr}(\infty) = 0. \end{aligned}$$

We have:

$$G(z,s) = \begin{cases} e^{-z} - 1, & 0 < z < s; \\ e^{-z}(1 - e^s), & s < z < \infty; \end{cases}$$

$$G_{Pr}(z,s) = \begin{cases} \dfrac{1}{Pr}\,(e^{-Prz} - 1), & 0 < z < s; \\ \dfrac{e^{-Prz}}{Pr}\,(1 - e^{-Ps}), & s < z < \infty. \end{cases}$$

We search solution of the system (7) as a power series:

$$F(z) = \sum_{n=0}^{\infty} \varepsilon^n F_n(z), \quad \phi(z) = \sum_{n=0}^{\infty} \varepsilon^n \phi_n(z).$$

Then from (8) we find that

$$F_0(z) = 1 - e^{-z},$$

where $F_0(z)$ is the solution of following boundary problem

$$\begin{aligned} &F_0'' + F_0' = 0; \\ &F_0(0) = 0, \quad F_0(\infty) = 1; \end{aligned}$$

$$\begin{aligned} F_1(z) &= -\int_0^\infty \left\{ \frac{1}{2} F_0' \int_0^s F_0 \, d\alpha - N F_0 (1 - F_0)^m \right\} G(z,s) \, ds = \\ &= \left(\frac{z^2+1}{4} - \frac{2N}{m(m^2-1)} \right) e^{-z} - \frac{e^{-2z}}{4} + \frac{N e^{-mz}}{m(m-1)} - \frac{N e^{-(m+1)z}}{m(m+1)}, \end{aligned}$$

and $\phi_0(z)$ is the solution of following boundary problem:

$$\begin{aligned} &\phi_0'' + Pr\phi_0' = -PrEcF_0'^2; \\ &\phi_0(0) = 0, \quad \phi_0(\infty) = 1; \end{aligned}$$

The solution of this system is depended on Prandtl's number.

If $Pr \neq 2$, then $\phi_0 = 1 - \left(1 + \frac{PrEc}{2(Pr-2)}\right) e^{-Prz} + \frac{PrEc}{2(Pr-2)} \, e^{-2z}$.

If $Pr = 2$, then $\phi_0 = 1 + (Ecz - 1)e^{-2z}$.

If $Pr = 1$, then $\Phi_0 = 1 - \left(1 - \frac{Ec}{2}\right) e^{-z} - \frac{Ec}{2} \, e^{-2z}$.

Now we can calculate the skin friction, the displacement thickness δ^* and the momentum thickness δ^{**} in the first and two asymptotics

$$\begin{aligned} \tau &= \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \approx \frac{\mu}{\nu} \, u_\infty v_w \, \frac{\partial}{\partial z} \, (F_0 + \varepsilon F_1)|_{z=0} = \\ &= \frac{\mu}{\nu} \, u_\infty v_w \left[1 + \varepsilon \left(\frac{1}{4} - \frac{N}{m(m+1)} \right) \right]; \\ \delta^* &= \int_0^\infty \left(1 - \frac{u}{u_\infty} \right) dy \approx \frac{\nu}{v_w} \int_0^\infty [1 - (F_0 + \varepsilon F_1)] \, dz = \\ &= \frac{\nu}{v_w} \left[1 + \varepsilon \left(\frac{N(2m+1)}{m^2(m+1)^2} - \frac{5}{8} \right) \right]; \\ \delta^{**} &= \int_0^\infty \frac{u}{u_\infty} \left(1 - \frac{u}{u_\infty} \right) dy \approx \\ &\approx \frac{\nu}{v_w} \int_0^\infty (F_0 + \varepsilon F_1) \, (1 - F_0 - \varepsilon F_1) \, dz \approx \\ &\approx \frac{\nu}{v_w} \left[\frac{1}{2} + \varepsilon \left(\frac{N(2m+1)}{m^2(m+1)^2} + \frac{2N}{m(m+1)(m+2)} - \frac{5}{6} \right) \right]. \end{aligned}$$

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