ON THE COMPRESSION OF LINEAR ALGEBRAIC EQUATION SYSTEM

N. Nanobashvili
I. Vekua Institute of Applied Mathematics,
Iv. Javakhishvili Tbilisi State University
0143 University Street 2, Tbilisi, Georgia

(Received: 24.07.07; accepted: 12.09.07)

Abstract

A conception of the uniqueness of some formal system is closely connected with the conception of redundancity which is the one of the fundamental ideas in the theory of information. This paper deals with the problem of reducing a system of linear equations to a single compressed equation. Formal search for the correction of the coefficients of the equation is shown, on the basis of which an equation represented in redundancy classes uniquely reflects the system of linear equations.

Key words and phrases: Data Compression, linear algebraic system.
AMS subject classification: 68P30.

Let us consider the system of linear equations

$$\sum_{i=1}^{n} a_{ij}x_j = c_j,\ j = 1, \cdots, m; \tag{1}$$

and let

$$\sum_{i=1}^{n} a_{ij}y_j = c_j^{'},\ j = 1, \cdots, n; \tag{2}$$

is the representation of the system (1) in the redundancy class $V_{j,n}$, where
$y_j = v_{jy}; j = 1, \cdots, n.$

According to the results of the work [2] we can transform the system (1) to the following compressed equation

$$\sum_{i=1}^{n} a_i z_i = c(\mathrm{mod}\ 2^n), \tag{3}$$

where

$$a_j = \sum_{i=1}^{n} 2^{i-1} v_{ij}(\mathrm{mod}\ 2^n), \tag{4}$$

$$c = \sum_{i=1}^{n} 2^{i-1} c_i (\text{mod } x2^n). \tag{5}$$

The equation (3) is undefined on the real number field, indeed, in the redundancy class this equation has the uniquely determined solution, but for the coincidence of solutions of systems (2) and (3) one needs a complimentary procedure.For this purpose we need to define the coefficients $a_i; i = 1, \cdots, n$ accordingly.

Without loss of generality we propose the following:

1. For the system (1) the ranks of the system matrix and expanded matrix are equal.

2. The coefficients of the system (1) are positive integers.

3. $m = n$.

Using the results of the work [1] the following theorem is proved

Theorem 1. The compressed equation (3) can be transformed to the following equation

$$\sum_{i=1}^{n} A_i y_i = c(\text{mod } 2^n), \tag{6}$$

where solutions of the equation (6) coincide with the solutions of the system (2), if coefficients of the equation (3) will be transformed by the following correction scheme

$$A_i = a_i + \frac{a_i}{\text{mod } m}; \; i = 1, \cdots, n; \tag{7}$$

Example: Let us consider the system

$$\begin{cases} x_1 + x_2 + x_3 = 6, \\ x_1 + 2x_2 + x_3 = 8, \\ x_1 + x_2 + 3x_3 = 12. \end{cases} \tag{8}$$

The solutions of this system are $x_1 = 1; x_2 = 2; x_3 = 3$.

In the redundancy class this system is representable by

$$\begin{cases} y_1 + y_2 + y_3 = 00063 \text{ (mod 8)}, \\ y_1 + 2y_2 + y_3 = 00005 \text{ (mod 8)}, \\ y_1 + y_2 + 3y_3 = 00063 \text{ (mod 8)}, \end{cases} \tag{9}$$

where according to [1]

$$y_1 = v_1 = 1 \cdot 00001 = 00001 \text{ (mod 8)},$$

$$y_2 = 2v_2 = 2 \cdot 00011 = 00022 \text{ (mod 8)},$$

$$y_3 = 4v_3 = 4 \cdot 00012 = 00040 \text{ (mod 8)},$$

$$c_0 = 000063 + 2 \cdot 00005 + 4 \cdot 00063 = 00061 \text{ (mod 8)}.$$

Now, using the transformation (3) we obtain the original form of the compressed equation

$$7z_1 + 27z_2 + 43z_3 = 00061 \text{ (mod 8)}. \tag{10}$$

In the decimal system the equation (10) has the form

$$7z_1 + 23z_2 + 35z_3 = 49 \text{ (mod 10)}. \tag{11}$$

The solutions of the system (9) and (10) are not coincide and hence is not adequate mathematical scheme of the system (8). Thus it needs some corrections.

Let us use the theorem 1. According to (7) we have

$$\begin{aligned}
&A_1 = 7 + \left[\frac{7}{\text{mod } 8}\right] = 7 + \left[\frac{7}{8}\right] = 7, \\
&A_2 = 23 + \left[\frac{23}{8}\right] = 23 + 2 = 25, \\
&A_3 = 35, \\
&c_0 = 49.
\end{aligned}$$

The equation obtained after the correction is of the form

$$7y_1 + 25y_2 + 35y_3 = 49 \text{ (mod 10)} \tag{12}$$

This equation is the compressed adequate scheme of the system (8). Below we will prove it.

Substituting (12) over mod8 we obtain [3]

$$7y_1 + y_2 + 3y_3 = 00061 \text{ (mod 10)}. \tag{13}$$

The equation (13) is true ,when

$$y_1 = 0001;\ y_2 = 00022;\ y_3 = 00040;$$

$$x_1 = 1;\ x_2 = 2;\ x_3 = 3.$$

Let us replace first and third equations in (8)

$$\begin{cases} x_1 + 2x_2 + x_3 = 8, \\ x_1 + x_2 + x_3 = 6, \\ x_1 + x_2 + 3x_3 = 12. \end{cases} \tag{14}$$

After correction the compressed equation will be of the form

$$7y_1 + 16y_2 + 39y_3 = 39 \text{ (mod 10)} \tag{15}$$

or

$$7y_1 + 0.y_2 + y_3 = 00047 \text{ (mod 8)}, \tag{16}$$

to this equation corresponds the following

$$y_1 = 00001;\ y_2 = 00022;\ y_3 = 00040;$$

$$x_1 = 1;\ x_2 = 2;\ x_3 = 3.$$

After replacing first, second and third equations in (14) we obtain

$$\begin{cases} x_1 + x_2 + 3x_3 = 12, \\ x_1 + x_2 + x_3 = 6, \\ x_1 + 2x_2 + x_3 = 8. \end{cases} \tag{17}$$

After correction we have

$$7y_1 + 43y_2 + 9y_3 = 27 \text{ (mod 10)} \tag{18}$$

to which corresponds

$$7y_1 + 3y_2 + y_3 = 00025 \text{ (mod 8)}, \tag{19}$$

and we obtain

$$y_1 = 00001;\ y_2 = 00022;\ y_3 = 00040;$$

$$x_1 = 1;\ x_2 = 2;\ x_3 = 3.$$

Replacing first and third equations in (17) we obtain

$$\begin{cases} x_1 + x_2 + 3x_3 = 12, \\ x_1 + 2x_2 + x_3 = 8, \\ x_1 + x_2 + x_3 = 6. \end{cases} \tag{20}$$

After correction we have

$$7y_1 + 25y_2 + 9y_3 = 49 \text{ (mod 10)}. \tag{21}$$

To the last equation corresponds the following

$$7y_1 + y_2 + y_3 = 00061 \text{ (mod 10)}.$$

From this equation we obtain

$$y_1 = 00001;\ y_2 = 00022;\ y_3 = 00040;$$

$$x_1 = 1;\ x_2 = 2;\ x_3 = 3.$$

Note 1. The system (10) is isomorphical to the system (9), but their solutions are not coincide.

Note 2. The example shows that replacement of equations in the system does not change the solution,but coefficients and right hand side will be changed. This fact is important for applications.

References

1. Nanobashvili N. D., On one improved scheme for the compress of information (Russian).Proc.of TSU, vol.224, 1981.

2. Nanobashvili N. D., The informaticion approach for the solution of linear equations in redundancy classes (Russian). Proc.of TSU, vol.224,1981.

3. Bukhshtab A.A., Number theory. Moskow, Nauka., 1966 (Russian).