NUMERICAL AND ANALYTICAL MODELLING OF SPILLED OIL PENETRATION IN SOILS FOR RISK ASSESSMENT

T. Davitashvili¹, A. Ebel² ¹I.Vekua Institute of Applied Mathematics of Tbilisi State University, 2 University St., 0186, Tbilisi, Georgia. E-mail: tedavitashvili@gmail.com ²Rhenish Institute for Environmental Research, University of Cologne Aachener Str. 209 50931 Cologne, Germany. E-mail: eb@eurad.uni-koeln.de

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Abstract

In this paper some environmental problems resulting from oil spillage along oil pipeline routes are discussed. Oil penetration into soil with flat surface containing pits is studied by numerical modelling. Some analytical and numerical solutions of the diffusion and filtration equations are given and analyzed.

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1 Introduction

The Druzhba pipeline is the world's longest oil pipeline (about 4,000 km). It transports oil from central Russia to the energy-hungry western regions of Russia, Belarus, Ukraine, Slovakia, the Czech Republic, Hungary. The northern branch crosses the remainder of Belarus to reach Poland and Germany. There have recently been proposals to extend this branch to the German North Sea port of Wilhelmshaven, which would reduce oil tanker traffic in the Baltic Sea and make it easier to transport Russian oil to the United States. Today, the Druzhba pipeline is one of the principal arteries for the transportation of hydrocarbons across Europe to western Europe.

The next longest is the Baku-Tbilisi-Ceyhan oil pipeline (BTC) transporting crude oil (1,760 km) from the Caspian to the Mediterranean Sea. It passes through: Baku, (Azerbaijan); Tbilisi, (Georgia); and Ceyhan (Turkey).

According to the experience of European transit countries the transit of oil and gas causes great losses regarding the ecological situation thus

counteracting the intended political and economical benefits. In addition to ordinary pollution of the environment it is possible that non-ordinary situations like pipeline and railway accidents arise. As foreign experience with pipelines shows, the main reasons of crashes and spillages (and fires as a consequence) are the destruction of pipes as a result of corrosion, defects of welding and natural phenomena. Also terrorist attacks and sabotage may occur. The probability of crashes for oil pipeline transport rises with the age of the pipelines in service, and with the extent of their network. For example, 250 ruptures, which were accompanied by spillages of the transferred products, occurred every year from 1973 to 1983 in the US pipeline network with a total length of about 250,000 km. In West Europe it has been found that 10 - 15 leakages happen every year in a pipeline network of around 16,000 km length, resulting in a loss of 0.001. It is obvious that safety of the neighborhood of oil and gas pipelines is an urgent problem for Russia, many other east and west European countries and for the Caucasus. The application of models to the investigation of the impact of pollution on the environment and on the population caused by pipelines and rail road transport offers a chance to design sensible economical, social and political abatement strategies. Models can efficiently support the development of prognostic and management methods for environmental protection. With this goal in mind we have applied mathematical models describing oil penetration into soils in case of pipeline damage in the general case and for special conditions taking into account the specific properties of several types of soils found along pipeline routs.

2 To the Non-Linear Theory of Oil Filtration in the Soil

By pipes or some other transportation means, in emergency cases, oil can spread on the ground and cause a pollution of soil. Character of the pollution and ecological damage will depend on intensivity of the oil source and on composition of the soil structure.

Interaction of the oil and soil represents a difficult non-linear problem of liquid filtration into porous media [1,2]. Soil is not a homogeneous solid medium [1]. It consists of the hard particles (of skeleton), grains of quartz (sand) and other rocks, particles of clay, and organic substances. Mineral and organic components form a hard (though not quite rigid) "matrix" or "skeleton" with irregular-shaped pores filled partly by air and partly - by water. Contact between the hard particles is provided by water, therefore a diffusion coefficient of the soil depends mostly on its humidity f (expressed in percentage on weight). The diffusion coefficient of the soil can roughly be presented in the form [1]:

$$D = D_1 + f \frac{D_2}{100},\tag{1}$$

where D_1 - is diffusion coefficient of skeleton, D_2 - diffusion coefficient of water.

Diffusion coefficient of soil at spreading of oil into the soil clearly will also be a function of W, where W is a non-dimensional magnitude representing a ratio of the oil mass in unit of volume to all weight of a ground in the same volume. Dependence of D on W for non-linear processes of filtration can be represented as follows [3]:

$$D^{(W)} = d_0 \left(\frac{W}{\sigma}\right)^n,\tag{2}$$

where d_0 is a diffusion coefficient of soil at $n = 0, \sigma$ -is a porosity coefficient of soil.

At the penetration of oil deeply into the soil the equation of indissolubility of oil with account of diffusive and external sources can be written in the form [1-3]:

$$\frac{\partial W}{\partial z} + V_z \frac{\partial W}{\partial z} = \frac{\partial}{\partial z} \left[D(W) \frac{\partial W}{\partial z} \right] + Q, \tag{3}$$

Here V_z - is a velocity of penetration of oil in the depth of soil, Q- is an external source of oil on the ground.

Since convective term $V_z \frac{\partial W}{\partial z}$ expresses hydrodynamic process of oil filtration into the depth of soil, without loss of generality it can be represented as a non-linear negative source of filtration:

$$V_z \frac{\partial W}{\partial z} \approx S_0 \left(\frac{W}{\sigma}\right)^{n+1}.$$
(4)

In this expression the higher degree of non-linearity is conditioned by the fact, that hydrodynamic process of liquid filtration in the ground is faster than diffusive one. The positive source of oil on the surface of ground Q will obviously be only linear function of the amount of oil spreading from pipeline, so the function Q(w) takes the form:

$$Q(W) = q_0 \frac{W}{\sigma}.$$
(5)

Coefficients S_0 and q_0 with the dimension c^{-1} show ratio of the capacity of oil to the work, made during the oil filtration and the oil spreading on ground.

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Substituting (2), (4) and (5) in equation (3) and introducing denotations: $D_0 = \frac{d_0}{\sigma^n}$, $k_0 = \frac{S_0}{\sigma^{n+1}}$, $Q_0 = \frac{q_0}{\sigma}$, finally we obtain the non-linear equation of the oil filtration in soil:

$$\frac{\partial W}{\partial t} = \frac{D_0}{n+1} \frac{\partial^2}{\partial z^2} W^{n+1} - k_0 W^{n+1} + Q_0 W.$$
(6)

Reception of the general analytical solution of nonlinear equation (6) is practically impossible. Further we shall be limited to some exact analytical solutions of equation (6), which can describe a real picture of filtration of oil in soils of various structures.

3 Analytical Solution of Equation (6) at Presence of Nonlinear Diffusive and Filtration Processes

So we research a case of simultaneous action of nonlinear mechanisms of diffusions of ground in absence of a secondary source of oil on a surface (Q(W) = 0). This condition will be fulfilled if a point-wise source of oil has such an intensity of spreading, when the soil completely absorbs oil by diffusive and filtration mechanisms and the additive (secondary) source on the surface of ground can not generate.

In the considered case filtration equation (6) will take the form:

$$\frac{\partial W}{\partial t} = \frac{D_0}{n+1} \frac{\partial^2 W^{n+1}}{\partial z^2} - k_0 W^{n+1}.$$
(7)

This nonlinear equation fortunately also has a solution for any n. We have found the solution. It takes the form [6]:

$$W = W_0 \frac{ch^{2/n} \left(\frac{\pi}{2} \frac{z}{z_0}\right)}{\left(1 + \frac{t}{t_0}\right)^{1/n}}.$$
(8)

Formula (8) shows that a process of oil spreading in soil in this case is also bounded in space, but the magnitude W decreases in time. This fact verifies again that the filtration mechanism delays the spreading of oil in time and contest with the diffusive mechanism.

Substituting (8) in equation (7), we define uniquely parameters z_0 and t_0 :

$$z_0 = \frac{\pi}{n} \sqrt{\frac{(2+n)D_0}{k_0}}; \quad t_0 = \frac{2}{n} \frac{1}{k_0 W_0^n}.$$
(9)

From (9) it follows that z_0 is completely defined by the properties of ground D_0 and k_0 , and t_0 -by the property of ground k_0 and the initial value of the intensity of the boundary regime of oil spreading W_0^n . Equation

(7) has a stationary $\left(\frac{\partial}{\partial t}=0\right)$ solution: $U = U_0 e^{\frac{z}{\sigma}}$; where $U = W^{n+1}$; $\sigma = \sqrt{\frac{(n+1)D_0}{k_0}}$ is a characteristic depth of the oil penetration.

Equation (8) again shows a slowdowning action of filtration on diffusive processes of oil propagation in the soil. In Fig. 1 is represented the results of calculations according to (8) for the value of parameters: $W_0 = 0.7$; $z_0 = 2.7$; n = 2. Analysis of the Fig. 1 shows a slowdowning action of filtration on diffusive processes of oil propagation in the soil and a process of oil spreading in soil is bounded in space.

Thus, in spite of the fact that obtaining of the general analytical solution of nonlinear equation (6) is practically impossible, the above-considered exact nonlinear solutions can behave as theoretical models of oil filtration in soil in different cases, which can take place in the real emergency situations of oil transportation. The found solutions can also be used as tests at numerical resolution of euqation (6).

We should note that the found solutions (8) have properties of organized structure and universal character [7]. Nonlinear solution (8) shows an origin of the structure in physical medium, its development in time and finally collapse of the structure. Solution (8) shows development in time of the already formed structure and its passing to the stable stationary structure, which does not change more in time.

4 Numerical Investigation of Spreading of Spilled Oil

As it is known, filtration of oil and oil products into soil in general form can be described by means of the following nonlinear parabolic equation [2-5]:

$$\frac{\partial S}{\partial t} + K_{11} \frac{\partial S^n}{\partial z} = D_{11} \frac{\partial^2 S^{n+m+1}}{\partial x^2} + D_{11} \frac{\partial^2 S^{n+m+1}}{\partial z^2}, \tag{10}$$

where $S = W - W_0 W$ is the ratio of the unit oil mass existing in the soil to unit soil mass: W_0 is the ratio of the unit soil mass connected with water to the unit soil mass; K_{11} and D_{11} are filtration and diffusion coefficients of oil filtration into soil, respectively and they can be written as follows [3,4]:

$$K_{11} = \frac{-K_1}{(\sigma - \omega_0)^n} \frac{Y_\omega}{Y_{oil}}, \quad D_{11} = \frac{-K_1 a_1}{(n + m + 1)(\sigma - \omega_0)^m},$$



Figure 1:

where K_1 is a water filtration coefficient at its full saturation; σ a is a porosity of soil: Y_{ω} and Y_{oil} are cinematic coefficients of water and oil viscosity, respectively: n and m are parameters, which describe the degree of non-linearity of filtration and diffusion processes: a_1 is a parameter describing the pressure in pores and can be written as follows:

$$a_1 = \frac{P_0 Y_\omega}{(\sigma - \omega_0) Y_{oil}}$$

where P_0 is a pressure of the liquid in case of its full saturation.

Now we solve the following practical problem. Suppose a big amount of oil is spilled on the Earth surface which has a cylindrical form with a basis Ω_1 and height H_0 . And takes the area $\Omega_1 = \Omega_1 * H_0$. We are interested in the problem of oil filtration and diffusion in the soil, so we seek for solution of the problem (10) in the rectangular parallelepiped $G = \{0 \le x \le l_1, 0 \le y \le l_2, 0 \le z \le l_3\},\$

We suppose, that axis Ox and Oy is oriented in the way, that the upper boundary coincides with the Earth surface, and axis Oz is directed vertically down.

The problem is solved with the following initial conditions

$$S(O, x, y, z) = -W_0, \quad \text{at} \quad x, y, z \notin \Omega,$$

$$S(O, x, y, z) = \sigma - W_0, \quad \text{at} \quad x, y, z \in \Omega,$$
(11)

and boundary conditions

$$S(x, y, O, t) = \sigma - W_0 \quad \text{at} \quad x, y \in \Omega, \tag{12}$$

$$S(x, y, O, t) = -W_0 \quad \text{at} \quad x, y \notin \Omega, \tag{13}$$

$$\frac{\partial S}{\partial z} = 0, \quad \text{at} \quad z = l_3,$$
 (14)

$$\frac{\partial S}{\partial x} = 0, \quad \text{at} \quad x = l; \ x = 0,$$
 (15)

$$\frac{\partial S}{\partial y} = 0, \quad \text{at} \quad y = 0, \ y = l_2,$$
 (16)

In general, problem (10), with initial (11) and boundary conditions (12)-(16) is solved numerically[5].

We have carried out numerical experiments for four type of soils. Numerical integration was carried out during t = 240 days for the meadowalluvial, sandy and sub-sandy soils, t = 265 days for the meadow-marshy soils, t = 424 days for the brown- black, peat soils and t = 270 days for the gray-brown, salt soils. For an example some of the received results are given in Fig. 2 and 3.

The numerical calculations showed, that the process of oil infiltration in all considered soils proceeds qualitatively equally i.e. in all considered soils it is possible to distinguished a stage of absorption of the oil in the soil and a stage of distribution of the oil to depth and width of soils [4,6]. In our case the stage of the oil absorption proceeds about 5 days in the all types of soils (with the exception of the grey-brown, and solt soils when the stage of the oil absorption proceeds about 6.3 days). For this time, an oil spillage with thickness 5cm is fully absorbed by a surface layer of soil. During this stage the value of W (oil concentration) (for the meadow-alluvial, sandy and sub-sandy type of soils) is maximal on the soil surface, and it quickly falls with depth i.e. on the depth of 62cm the concentration of oil- W is minimal and extended with width. The front of pollution at t = 0.5, 2, 5and 10 days reaches the depths z = 9, 27, 48, 75 cm, respectively. During the second stage of infiltration, concentration of oil-W on a surface of soil gradually decreases (see Fig.2).

The process of infiltration in the brown- black, peat soils is the most intensively and the maximal values of concentration at t = 5, 10 days reaches



Figure 2: The distribution of the concentrations W(z) of the oil at t = 0.5, 2, 5, 7, 10, and 240 days for the meadow-alluvial, sandy and sub-sandy soils.



Figure 3: The distribution of the concentrations W(z) of the oil at t = 0.5 2, 5, 6, 15, and 265 days for the meadow-marshy soils.

on the depths Z = 26, 50cm respectively. The process of infiltration in the grey-brown and solt soils is the least intensively. The maximal values of concentration at t = 5, 10 days reaches on the depths Z = 12, 21cm respectively The front of oil pollution at t = 5, 10 days reaches the depths Z = 25, 37 cm. The process of infiltration in the meadow-alluvial, sandy and sub-sandy soils proceeded qualitatively equally of the brown- black, peat soils but this process is less intensive than in brown- black, peat soils. The front of oil pollution at t = 5, 10 days reaches the depths Z = 153, 203cm respectively and the maximal values of concentration at t = 5, 10days reaches on the depths Z = 24, 45cm respectively. The process of infiltration in the meadow-marshy soils proceeded qualitatively equally of the meadow-alluvial, sandy and sub-sandy soils but this process is less intensive than in meadow-alluvial, sandy and sub-sandy soils. The front of oil pollution at t = 1, 2 and 5 days reaches the depths Z = 15, 30 and 48cm respectively and the maximal values of concentration at t = 15 and 265 days reaches on the depths Z = 63, 120cm respectively (see Fig.3).

5 Soil Pollution by Spilled Into Pit Oil

It is very important to study the soil pollution by oil products on the urban territory, as it is directly connected with the possible pollution of water supply network and underground waters. In the territory of Germany and Georgia, there are many small and medium enterprises of consumption and service of oil and gas production.. Enterprises of such type include: auto, railway and air transport services, petrol stations, oil and gas cisterns and tanks. Each of them is potential pollutant of the environment, therefore prediction of pollution of soil and underground waters for cities and regional centers represents a very actual problem. In the urban conditions, there are many cases when spilled oil cover as the flat surface, as the pit of rectangular type (see Fig.5). In this case we conventionally divide the problem of oil and oil-products filtration in the soil into two stages. Namely, on the first stage oil filtration in the soil takes place (with account of oil evaporation). before the flat surface will be cleared from the oil. On the second stage, oil infiltration into the soil from the pit proceeds intensively and then it is continued by the oil infiltration process spilled on the surrounding places of the pit, and interaction of these two process strengthen the saturation of the soil porosity by oil and following process of oil infiltration. Hence, from the mathematical point of view, we have the problem of oil filtration into soil with non-stationary boundary conditions, which are varying both in space and in time. With purpose to simplify the problem and represent

it in a better way, we restrict our self to the two-dimensional model of oil filtration into soil, which show quite well all main properties of threedimensional model.

Let us consider Cartesian coordinate system oriented on Oxz in such a way that Ox axis is directed along the ground surface, and Oz axis is directed vertically down. Suppose that oil with thickness ho was spilled on the surface and filled a pit with depth hi. Our aim is to study the problem of filtration of spilled oil into soil (both from the surface and from the pit) with account of oil evaporation process and boundary conditions varying in time and space.

Let us search solution of the equation (10) in the area $\Omega_2 = \Omega/\Omega_1$, where $\Omega = \{0 \le x \le x_5, 0 \le z \le z_2\}$ and $\Omega_1 = \{x_2 < x < x_3, 0 \le z \le z_1\}$. Let us denote border of Ω_2 by S_2 , border of Ω_1 by S_1 and $\{x_1 \le x \le x_2Ux_3 \le x \le x_4, z = 0\}$ by S_1^* . Let us search solution of problem (1) in Ω_2 with the following initial and boundary conditions:

$$S(O, x, z) = \begin{cases} \sigma - \omega_0 & \text{if} \quad x, z \in S_1 \cup S_1^*, & \text{when} \quad t = 0, \\ -\omega_0 & \text{if} \quad x, z \in \Omega_2, & \text{and} \quad x, z \notin S_1 \cup S_1^*, \end{cases}$$
(17)

$$\frac{\partial S}{\partial x} = 0$$
 when $x = 0$ and $x = x_5$, (18)

$$\frac{\partial S}{\partial t} = 0 \quad \text{when} \quad z = z_2 \tag{19}$$

$$\frac{\partial S}{\partial z} = 0 \quad \text{when} \quad z = 0 \quad \text{and} \quad x \in]0, x_1] \cup]x_4, x_5], \tag{20}$$

$$\frac{\partial S}{\partial z} = \begin{cases} \sigma - \omega_0 & \text{if } h(t) = h_0 - \iiint_{t=S_2} SdS - h^*(t) > 0 \\ 0 & \text{if } h(t) = h_0 - \iiint_{t=S_2} SdS - h^*(t) = 0 \end{cases}$$
(21)

and if $x \in S_1^*$ and z = 0,

$$\frac{\partial S}{\partial z} = \begin{cases} \sigma - \omega_0 & \text{if } h(t) = h_0 - \iiint_{S_1} SdS - h^{**}(t) > 0, \\ 0 & \text{if } h(t) = h_0 - \iiint_{t=S_1} SdS - h^{**}(t) = 0 \\ h_2 = 0 & \text{and } x \in [x_2, x_3] & \text{and } z = z_1, \end{cases}$$
(22)

$$\frac{\partial S}{\partial z} = \begin{cases} \sigma - \omega_0 & \text{if } h(t) = h_0 - \iiint_t S dS - h^{**}(t) \ge z_1 - z, \\ 0 & \text{if } h(t) = h_0 - \iiint_t S dS - h^{**}(t) \le z_1 - z \end{cases}$$
(23)

$$h_0 = 0 \text{ and } z \in [O_2, z_1] \text{ and } x = x_2 \text{ or } x = x_3$$
 (24)

where function h(t) describes the change of the height of the spilled oil in time, caused by oil infiltration and evaporation in the equation (20) and function h(t) describes the same processes, but in the pit with boundary conditions (21) and (22); functions h * (t) and h * *(t) describe change of the oil height on the surface and in the pit due to oil evaporation.

The sizes of the spilled oil on the surface of ground were 2m.*0.03m. along the axes Ox and Oz, respectively. The pit was located in the central part of the spilled oil and had the following sizes: 0.6m *0.5m along the axes Ox and Oz, respectively.

We approximate (10) with respect to time according to difference scheme of Adams-Beshfort,

$$S^{(l+1)*} = S^{(l)} + \frac{\Delta t}{2} f^{(l)}, \quad S^{(l+1)} = S^{(l)} + \Delta t \left[\frac{3}{2} f^{(l+1)*} - \frac{1}{2} f^{(l)} \right]$$

This is a scheme of the second order accuracy with respect to time. With respect to space is approximated according to Schumann scheme , which also is of the second order accuracy.

The calculations were made for the meadow-alluvial, sandy and subsandy soil. The values of the parameter we have used were the following: $Q_0 = 0.00001; v_{wat} = 0.0106 \text{ cm/s}; v_{0ll} = 0.28 \text{ cm/s}; K_{\omega} = 0.2510 \text{ cm/s}; D_{\omega} = 2.42 \text{ cm/h}.$

Numerical calculations were carried out during t-180 days. Numerical calculation have shown that the process of oil filtration into soils was various for different period of the numerical integration of the (10). Particularly it was possible to distinguish three stages during the numerical calculations. The first stage continued until inequality 0 < -z < h was true. For the second and third stages we have analogous inequalities 0 < z, < H; and z. > h, respectively.

The first stage contained intensive processes of oil evaporation and oil filtration in the soil. This process continued about 1.8 days until ground's surface was not cleared from the spilled oil (of course with the exception of the pit area). During the second stage there existed both processes: oil filtration and evaporation in soil, and oil infiltration into the soil from neighboring areas of the pit , until the pit was not cleared from the spilled oil. Numerical calculations have shown that this stage continued about 31

days. The third stage of oil infiltration in soil was less intensive than the first and second stages. We have kept the process of oil infiltration in soil under our observation until a velocity of front of oil distribution to soil was not infinitesimal i.e. until the process became almost stable. Numerical calculations have shown that third stage continued about 147 days.

As we have mentioned above (numerical calculations have shown that) the process of oil filtration in soil during the first stage was much more intensive than during other two stages. These phenomena that were presented for the first stage was stipulated (conditioned) by the existence of the mutual process of oil filtration in soil from the ground surface and pit boundaries simultaneously. For the first stage distribution of oil in soil was almost similar both in the vertical and in the horizontal directions. This picture of oil distribution in soil was especially evident in neighboring areas of the lateral boundaries of the pit. Although we can denote that oil penetration in soil in the vertical direction was more intensive than in the horizontal direction in neighboring areas of the lower border of the pit. That is why a line of front of the oil penetration in soil has sine character at the end of the first stage process.

The second stage mainly was characterized by oil propagation in soil from the pit borders but we note that during the second stage we have used non-stationary boundary conditions at the lateral borders of the pit, taking into account oil filtration and evaporation processes simultaneously, until the pit clearing process has not accomplished. For the second stage, we have studied oil propagation in soil from the pit borders, on the one hand with account of evaporation from the pit and on the other hand without it. Numerical calculations have shown that the results of calculations were rather different. For the first case the pit clearing process accomplished in 31 days and maximum depth of oil penetration in soil reached 3.1m at the end of the second process, whereas without of evaporation the pit clearing process accomplished in 47 days and maximum depth of oil penetration in soil reached 4.05m at the end of the second stage.

On the third stage of oil infiltration to the soil there oil penetration in soil took place as in vertical as well in the horizontal directions. We note that this process in the vertical direction was more intensive than in the horizontal direction and in 180 days the line of the front of oil distribution in soil was not very well expressed, but had sine character. The maximum depth of the front of oil distribution in soil was observed on the depth 5. 7m almost along the symmetric axes of the pit, and the maximum distance of oil penetration in the horizontal direction was observed about 4. 1m from the pit symmetric axes.

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