UNSTEADY MAGNITOHYDRODYNAMIC FLOW OF LOW-CONDUCTIVE LIQUID IN THE NEIGHBOURHOOD OF INFINITE ROTARY POROUS PLATE IN CASE OF VARIABLE SUCTION VELOCITY

L. Jikidze

Georgian Technical University
Tbilisi, Georgia,

(Received: 25.04.07; accepted: 18.08.07)

Abstract

In this paper a unsteady problem of the motion of conductive liquid, caused by rotation of infinite porous plate, when the suction velocity represents periodic function of a time has, been studied.

Key words and phrases: Suction velocity, Conductivity, Magnetic field, Flow, Porosity.

AMS subject classification: 76W05.

In paper [1] the problem was studied of unsteady flow of low-conductive liquid along an infinite porous plate for the case when liquid was evently infiltrated into the plate with constant velocity, where it has been under action of constant magnetic field with taking into account a heat transferring. The dynamic part of this problem, when the velocity changes with periodic low, was generalized in work [2]. The influence of change of infiltrating liquid's velocity with periodic law on liquid's temperature was investigated in work [3].

Paper [4] deals with the unsteady flow of liquid along infinite rotary porous plate when the coefficient of electro-conductivity is a periodic function of time.

In the present paper we consider the unsteady problem of low-conductive liquid flow caused by the rotation of infinite porous plate in constant magnetic field, when liquid's suction velocity is a periodic function of time.

For solution of the problem we use the equations of unsteady motion of low-conductive liquid in homogeneous magnetic field:

$$\begin{cases} \dfrac{\partial v_r}{\partial t} + v_r \dfrac{\partial v_r}{\partial r} + v_z \dfrac{\partial v_r}{\partial z} - \dfrac{v_\varphi^2}{r} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \nu\left(\Delta v_r - \dfrac{v_r}{r^2}\right) - \dfrac{\sigma B_0^2}{\rho} v_r, \\ \dfrac{\partial v_\varphi}{\partial t} + v_r \dfrac{\partial v_\varphi}{\partial r} + v_z \dfrac{\partial v_\varphi}{\partial z} + \dfrac{v_r v_\varphi}{r} = \nu\left(\Delta v_\varphi - \dfrac{v_\varphi}{r^2}\right) - \dfrac{\sigma B_0^2}{\rho} v_\varphi, \\ \dfrac{\partial v_z}{\partial t} + v_r \dfrac{\partial v_z}{\partial r} + v_z \dfrac{\partial v_z}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} + \nu \Delta v_z, \\ \dfrac{\partial v_r}{\partial r} + \dfrac{v_r}{r} + \dfrac{\partial v_z}{\partial z} = 0, \end{cases} \tag{1}$$

where $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$.

In general case the solution of system (1) must satisfy the following initial and boundary conditions:

$$\begin{cases} t = 0; \quad v_r = v_\varphi = v_z = 0; \\ z = 0, \quad v_r = 0, \quad v_\varphi = s\omega r, \quad v_z = v_w(t), \\ z = \infty, \quad v_r = v_\varphi = 0. \end{cases} \tag{2}$$

Taking into account certain geometric and mechanical considerations, we seek the solution of system (1) in the following form:

$$\begin{cases} v_r = \omega r f(\eta, t'), \quad v_\varphi = \omega r \varphi(\eta, t'), \quad v_z = \sqrt{v\omega}(\psi(\eta, t') + v_w(t')), \\ P = -\rho\nu\omega p(\eta), \quad \eta = \sqrt{\dfrac{\omega}{\nu}}\, z, \quad t' = \omega t. \end{cases} \tag{3}$$

According to [5] and [6] let us consider the case, when the influence of dissipative effects on liquid flow is small and intensive suction near a plate causes significant decrease of the radial component of liquid's velocity.

Due to this fact and that suction velocity changes by low

$$v_w(t) = v_0(1 + \varepsilon A \cos\omega t).$$

By putting (3) into system (1) gives the following system of equations:

$$\begin{cases} \dfrac{\partial^2 f}{\partial \eta^2} - v_0(1 + \varepsilon A\cos\omega t)\dfrac{\partial f}{\partial \eta} - m^2 f - \dfrac{\partial f}{\partial t} = -\varphi^2, \\ \dfrac{\partial^2 \varphi}{\partial \eta^2} - v_0(1 + \varepsilon A\cos\omega t)\dfrac{\partial \varphi}{\partial \eta} - m^2 \varphi - \dfrac{\partial \varphi}{\partial t} = 0, \\ \dfrac{\partial p}{\partial \eta} = -\dfrac{\partial^2 \psi}{\partial \eta^2} + \dfrac{\partial \varphi}{\partial \eta}(\psi + v_w) + \dfrac{\partial}{\partial t}(\psi + v_w), \\ \dfrac{\partial \psi}{\partial \eta} = -2f, \end{cases} \tag{4}$$

where $m^2 = \frac{\sigma B_0^2}{\omega\rho}$.

Since the flow is periodic, then system (4) has only boundary conditions

$$\begin{cases} \eta = 0, \quad f = 0, \quad \varphi = s, \quad \psi = 0, \\ \eta = \infty, \quad f = 0, \quad \varphi = 0. \end{cases} \tag{5}$$

We seek the solution of problem (4)–(5) in the following form:

$$\begin{cases} f(\eta, t) = f_0(\eta) + \varepsilon[f_1(\eta)\cos\omega t + f_2(\eta)\sin\omega t], \\ \varphi(\eta, t) = \varphi_0(\eta) + \varepsilon[\varphi_1(\eta)\cos\omega t + \varphi_2(\eta)\sin\omega t]. \end{cases} \tag{6}$$

If we put (6) into systems (4) and (5), neglecting summands which contain ε^2 as a factor and equating the coefficients of functions $\cos\omega t$ and $\sin\omega t$, we receive the following equations and corresponding boundary conditions:

$$\begin{cases} f_0'' - v_0 f_0' - m^2 f_0 = -\varphi_0^2, \quad f_0(0) = f_0(\infty) = 0, \\ \varphi_0'' - v_0\varphi_0' - m^2\varphi_0 = 0, \quad \varphi_0(0) = s, \quad \varphi_0(\infty) = 0, \end{cases} \tag{7}$$

$$\begin{cases} f_1'' - v_0 f_1' - m^2 f_1 - \omega f_2 = A v_0 f_0' - 2\varphi_0\varphi_1, \quad f_1(0) = f_1(\infty) = 0, \\ f_2'' - v_0 f_2' - m^2 f_2 + \omega f_1 = -2\varphi_0\varphi_2, \quad f_2(0) = f_2(\infty) = 0, \end{cases} \tag{8}$$

$$\begin{cases} \varphi_1'' - v_0\varphi_1' - m^2\varphi_1 - \omega\varphi_2 = A v_0\varphi_0', \quad \varphi_1(0) = \varphi_1(\infty) = 0, \\ \varphi_2'' - v_0\varphi_2' - m^2\varphi_2 + \omega\varphi_1 = 0, \quad \varphi_2(0) = \varphi_2(\infty) = 0. \end{cases} \tag{9}$$

The solution of system (7) has the form:

$$\begin{cases} f_0 = \dfrac{s^2}{3m^2 - 2kv_0}\,(e^{-k\eta} - e^{-2k\eta}), \\ \varphi_0 = s e^{-k\eta}, \end{cases} \tag{10}$$

where $k = \frac{1}{2}\big(\sqrt{v_0^2 + 4m^2} - v_0\big)$.

From systems (8) and (9), by some transformation, we receive new system of equations with corresponding initial and boundary conditions:

$$\begin{cases} F'' - v_0 F' - (m^2 - i\omega)F = A v_0 f_0' - 2\varphi_0\Phi, \quad F(0) = F(\infty) = 0, \\ G'' - v_0 G' - (m^2 + i\omega)G = A v_0 f_0' - 2\varphi_0 Q, \quad G(0) = G(\infty) = 0, \end{cases} \tag{11}$$

$$\begin{cases} \Phi'' - v_0\Phi' - (m^2 - i\omega)\Phi = A v_0\varphi_0', \quad \Phi(0) = \Phi(\infty) = 0, \\ Q'' - v_0 Q' - (m^2 + i\omega)Q = A v_0\varphi_0', \quad Q(0) = Q(\infty) = 0. \end{cases} \tag{12}$$

Here we introduce new functions: $F = f_1 + if_2$, $G = f_1 - if_2$, $\Phi = \varphi_1 + i\varphi_2$, $Q = \varphi_1 - i\varphi_2$.

The solution of problem (12) has the form

$$\begin{cases} \Phi(\eta) = -\dfrac{Av_0ks}{i\omega}\,[e^{-k\eta} - e^{-(a-ib)\eta}], \\ Q(\eta) = \dfrac{Av_0ks}{i\omega}\,[e^{-k\eta} - e^{-(a+ib)\eta}], \end{cases} \tag{13}$$

and the solution of problem (11) - the following form:

$$\begin{cases} F(\eta) = -\dfrac{Av_0ks^2}{(3m^2 - 2v_0k)\omega i}\,[e^{-k\eta} - 2e^{-2k\eta} + e^{-(a-ib)\eta}] - \\ \qquad - \dfrac{2Av_0ks^2}{\omega(\alpha + i\beta)}\,(e^{-k\eta} - 1)e^{-(a-ib)\eta}, \\ G(\eta) = \dfrac{Av_0ks^2}{(3m^2 - 2v_0k)\omega i}\,[e^{-k\eta} - 2e^{-2k\eta} + e^{-(a+ib)\eta}] - \\ \qquad - \dfrac{2Av_0ks^2}{\omega(\alpha - i\beta)}\,(e^{-k\eta} - 1)e^{-(a+ib)\eta}, \end{cases} \tag{14}$$

where we use the following notations:

$$a = \frac{1}{2}\left[-v_0 + \sqrt{\frac{1}{2}\Big(v_0 + 4m^2 + \sqrt{(v_0^2 + 4m^2)^2 + 16\omega^2}\Big)}\right],$$
$$b = \frac{\omega}{2a + v_0}, \quad \alpha = 2b(k + a) + v_0b - \omega,$$
$$\beta = (k + a)^2 + v_0(k + a) - b^2 - m^2.$$

For unknown functions f_1, f_2, φ_1, φ_2 we have

$$\varphi_1(\eta) = -\frac{Av_0ks}{\omega}\,e^{-a\eta}\sin b\eta,$$
$$\varphi_2(\eta) = \frac{Av_0ks}{\omega}\,(e^{-k\eta} - e^{-a\eta}\cos b\eta),$$
$$f_1(\eta) = \frac{Av_0ks^2}{(3m^2 - 2v_0k)\omega}\,e^{-a\eta}\sin b\eta - \frac{2Av_0ks^2}{\omega(\alpha^2 + \beta^2)}\,e^{-a\eta}(e^{-k\eta} - 1)\times$$
$$\times(\alpha\cos b\eta - \beta\sin b\eta),$$
$$f_2(\eta) = \frac{Av_0ks^2}{(3m^2 - 2v_0k)\omega}\,(e^{-k\eta} - 2e^{-2k\eta} + e^{-a\eta}\cos b\eta) +$$
$$+ \frac{2Av_0ks^2}{\omega(\alpha^2 + \beta^2)}\,e^{-a\eta}(e^{-k\eta} - 1)(\alpha\sin b\eta + \beta\cos b\eta).$$

The received solutions are valid for an infinite plate, but for quite large values of radius R it is possible to neglect the influence of the end of a plate and to determine the moment of resistance force against rotation:

$$M = -2\pi\rho v \int_0^R r^2 \left(\frac{\partial v_\varphi}{\partial z}\right)_{z=0} dr = \frac{-\pi\rho\omega\sqrt{\nu\omega}\, R^4}{2} \Big\{ - ks + \\ + \varepsilon \, \frac{A v_0 ks}{\omega} \left[(a-k)\sin\omega t - b\cos\omega t\right] \Big\}.$$

The coefficient of resistance moment

$$C_M = -\frac{2\pi}{\sqrt{\mathrm{Re}}} \Big\{ - ks + \varepsilon \, \frac{A v_0 ks}{\omega} \left[(a-k)\sin\omega t - b\cos\omega t\right] \Big\},$$

where $\mathrm{Re} = \frac{R^2\omega}{\nu}$ is Reinold's number.

From formulas received above the influence of suction velocity, angular velocity of rotation and magnetic field on physical characteristics of liquid flow are clearly shown.

We can control the liquid flow and surface friction by changing the amplitude and frequency of oscillation motion, as wall as suction velocity v_0 and parameter s of rotation.

References

1. U. Suryaprakasarao, The response of laminar skin friction, temperature and heat transfer to fluctuations in the stream velocity in the presence of a transverse magnetic field. I. Z. Angew. Math. Mech., 1962, 42, 133–141.

2. I. Pop, Buletinul Inst. Politechnic. Din IASI, Ser. Noua, 13(17), 3-4, 1967, p. 173-178.

3. G. Abesadze, Proceedings of Georgian Polytechnical Institute, 1975, 3 (176), 223–227 (in Russian).

4. L. Jikidze, Science and technology, 2004, No. 1-3, 49-53 (in Georgian).

5. L. Jikidze, Proceedings of Tbilisi State University, 2005, 354, 261–267 (in Russian).

6. J. Sharikadze, L. Jikidze, Transactions of Int. Symposium of the Problems of Continuum Mechanics, Tbilisi, 1997, 221–223 (in Russian).