SOLUTION OF THE BOUNDARY VALUE PROBLEMS OF STATICS OF THERMOELASTICITY THEORY OF BINARY MIXTURE FOR CIRCLE AND CIRCLE EXTERNAL DOMAIN

I. Tsagareli
I. Vekua Institute of Applied Mathematics,
Iv. Javakhishvili Tbilisi State University
0143 University Street 2, Tbilisi, Georgia

(Received: 27.04.07; accepted: 09.09.07)

Abstract

Solutions to the first and the second boundary problems of the statics of the thermoelasticity theory of binary mixture for circle and circle external domain are given by series

Key words and phrases: Binary mixtures for thermoelastic solids, Boundary-value problems, The theorems of uniqueness.

AMS subject classification: 74E30, 74H25.

A particular case of diffusions model will be considered when the coefficient of diffusion is zero. Besides, the source of heat doesn't exists and the volume forces equal zero. In these conditions the equations of thermoelasticity theory statics of two isotropic elastic mixtures will be written thus [1,2]:

$$\begin{gathered}
a_1\Delta u^{(1)} + b_1 \operatorname{graddiv} u^{(1)} + c\Delta u^{(2)} + d \operatorname{graddiv} u^{(2)} = \gamma_1 \operatorname{grad} u_3,\\
c\Delta u^{(1)} + d \operatorname{graddiv} u^{(1)} + a_2\Delta u^{(2)} + b_2 \operatorname{graddiv} u^{(2)} = \gamma_2 \operatorname{grad} u_3,\\
\Delta u_3 = 0.
\end{gathered} \tag{1}$$

Let us consider, that a domain filled with mixture of two isotropic material is a circle D^+ with a radius-R with a center in the beginning or an infinite plane D^- with a circular hole with a radius-R.

Let us state I and II basic problems for the system (1): in domain D^+ or D^- should be found such regular solution of system $U(x) = (u^{(1)}, u^{(2)}, u_3)$ which satisfies the boundary conditions:

$$\mathrm{I}^{\pm}. \quad \{u^{(i)}\}^{\pm}_{r=R} = f^{(i)}(z), \quad \{u_3\}^{\pm}_{r=R} = f_3(z); \tag{2}$$

$$\mathrm{II}^{\pm}. \quad \{RU^{(i)}\}^{\pm}_{r=R} = f^{(i)}(z), \quad \{u_3\}^{\pm}_{r=R} = f_3(z); \tag{3}$$

where U-is a five-dimensional vector, $u^{(1)}$, $u^{(2)}$-are particular displacements of mixture, u_3 is a change of the temperature by deformation, $f^{(i)} = (f_1^{(i)}, f_2^{(i)})$, f_3-are the functions given on the boundary of the circle, $x =$

$(x_1, x_2) = (r, \psi) \in D^{\pm}$, $z = (R, \psi) \in S$, $r^2 = x_1^2 + x_2^2$, a_1, b_1, c, d, γ_1, γ_2-are the known constants defining the mixture's elastic and thermal properties [1-3]; $RU = [(RU)^{(1)}, (RU)^{(2)}]$-is a thermalstress vector; $RU = Pu - \gamma n u_3$, where Pu is a stress vector of the elastic mixture [3],

$$(RU)_j^{(1)} = \sum_{q=1}^{2} [(\lambda_1\theta_1 + \lambda_3\theta_2)\delta_{jq} + 2\mu_1\varepsilon_{qj}^{(1)} + 2\mu_3\varepsilon_{qj}^{(2)} - 2\lambda_5 h_{qj}]n_q -$$
$$-\rho^{-1}\alpha_2(\rho_2\theta_1 + \rho_1\theta_2)n_j - \gamma_1 u_3 n_j, \tag{4}$$
$$(RU)_j^{(2)} = \sum_{q=1}^{2} [(\lambda_4\theta_1 + \lambda_2\theta_2)\delta_{jq} + 2\mu_3\varepsilon_{qj}^{(1)} + 2\mu_2\varepsilon_{qj}^{(2)} + 2\lambda_5 h_{qj}]n_q +$$
$$+\rho^{-1}\alpha_2(\rho_2\theta_2 + \rho_1\theta_2)n_j - \gamma_2 u_3 n_j \quad j = 1, 2,$$
$$\varepsilon_{qj}^{(p)} = \frac{1}{2}(\partial_q u_j^{(p)} + \partial_j u_q^{(p)}), \quad h_{qj} = \frac{1}{2}[\partial_q(u_j^{(1)} - u_j^{(2)}) - \partial_j(u_q^{(1)} - u_q^{(2)})],$$
$$\theta_p = \operatorname{div} u^{(p)}, \quad \partial_p = \frac{\partial}{\partial x_p}, \quad p, q, j = 1, 2,$$

$n = (n_1, n_2)$-is the external normal of the circle.

To determine the uniqueness of solution in the statics case, the problem is divided into two problems: Dirichlet problem for Laplace equation is solved separately (toward u_3) and the boundary problem $I^{\pm}$ or $II^{\pm}$ of elastic mixture statics (toward $u_1^{(i)}$, $u_2^{(i)}$) is solved separately.

The uniqueness of solution of each of these problems are studied (ex. [4,5]), so it is possible to the uniqueness of the whole problem. The theorems of uniqueness of boundary problems I and II of the thermoelastic mixture may be established thus:

Theorem 1. I$^{\pm}$ *problems have unique regular solution;*

Theorem 2. II^{+} *problem's two arbitrary regular solutions may differ from each other only by a rigid displacement vector:*

$$u^{(j)} = q^{(j)} + p^{(1)} \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}, \quad j = 1, 2,$$

where $q^{(j)} = (q_1^{(j)}, q_2^{(j)})$, $q_1^{(j)}$, $q_2^{(j)}$, $p^{(1)}$*-are arbitrary constants.*

Theorem 3. *Two arbitrary regular solutions of problem* II^{-} *may differ only by a constant vector which have components:* $u^{(j)} = q^{(j)}$, $u_3 = 0$, $j = 1, 2$.

Let us solve the problems.

Problem I^{-}. To find u_3 we solve Dirichlet problem in the domain D^{-} external to circle for Laplace $(1)_3$ equation and (2) condition. It will be shown like a series this way [4]:

$$u_3(x) = \sum_{m=0}^{\infty} f_{3m}(x), \tag{5}$$

$$f_{3m}(x) = \frac{1}{\pi}\Big(\frac{R}{r}\Big)^m \int_0^{2\pi} \cos m(\theta-\psi) f_3(\theta)d\theta, \tag{6}$$

$$x_1 = r\cos\psi, \quad x_2 = r\sin\psi, \quad y_1 = R\cos\theta, \quad y_2 = R\sin\theta, \quad y = (y_1, y_2) \in S.$$

f_{3m} is a homogeneous harmonic function.

We form the particular $u^{(j)} = (u_1^{(j)}, u_2^{(j)})$ displacements are as a sum of two vectors:

$$u^{(j)} = W_1^{(j)} + W_2^{(j)}, \quad j = 1, 2, \tag{7}$$

in which $W_1^{(j)}(x)$ is suitable to (1) homogeneous solution of system $(1)_0$ by the condition (2), and $W_2^{(j)}(x)$ is the solution of system (1) suitable by homogeneous condition $(2)_0$.

Let us find $W_1^{(j)}$. Let us use the representation of general solution of system $(1)_0$ in D^- by four harmonic functions [6]:

$$W_1^{(1)}(x) = \operatorname{grad}\Phi_1 + r^2 \operatorname{grad}\Big\{\Big[(\xi_1+\frac{1}{2})r\frac{\partial}{\partial r} + 2\xi_1\Big]\Phi_2 +$$
$$+\beta_1\Big(r\frac{\partial}{\partial r}+2\Big)\Phi_3\Big\} - xr\frac{\partial}{\partial r}[(2\xi_1-1)\Phi_2 + 2\beta_1\Phi_3] + \Psi'(x), \tag{8}$$

$$W_1^{(2)}(x) = \operatorname{grad}\Phi_4 + r^2 \operatorname{grad}\Big\{(r\frac{\partial}{\partial r}+2)\xi_2\Phi_2 +$$
$$+\Big[(\beta_2+\frac{1}{2})r\frac{\partial}{\partial r} + 2\beta_2\Big]\Phi_3\Big\} - xr\frac{\partial}{\partial r}[(2\xi_2\Phi_2 + (2\beta_1-1)\Phi_3] + \Psi''(x),$$

$$\Psi'(x) = \begin{cases} A_0x + B_0\widetilde{x}, & x \in D^+, \\ \frac{1}{r^2}(A_0x + B_0\widetilde{x}), & x \in D^- \end{cases},$$

$$\Psi''(x) = \begin{cases} C_0x + D_0\widetilde{x}, & x \in D^+, \\ \frac{1}{r^2}(C_0x + D_0\widetilde{x}), & x \in D^- \end{cases};$$

$$\xi_1 = \frac{1}{2\Delta_1}(cd - b_1a_2 - \Delta_1), \ \beta_1 = \frac{1}{2\Delta_1}(cb_2 - a_2d), \ \Delta_1 = a_1a_2 - c^2,$$

$$\xi_2 = \frac{1}{2\Delta_1}(cb_1 - a_1d), \ \beta_2 = \frac{1}{2\Delta_1}(cd - a_1b_2 - \Delta_1), \ x = (x_1, x_2), \ \widetilde{x} = (-x_2, x_1).$$

$\Phi_k(x)$-are arbitrary harmonic functions. We seek Φ_k by series:

$$\Phi_k(x) = \sum_{m=0}^{\infty}\Big(\frac{R}{r}\Big)^m (X_{mk}\cdot\nu_m(\psi)), \quad k = 1, 2, 3, 4, \tag{9}$$

where X_{mk}-is the required two-component vector; $\nu_m(\psi)=(\cos m\psi, \sin m\psi)$. Let us put the (9) in the (8). Let us expand the functions $f^{(i)}$ in Fourier series and pass to the boundary where $r\to R$. To find unknown X_{mk} values for each m will be obtained a system determinant of which is

$$D_m=\frac{1}{R^4}(A_1m+A_2),\ A_1=2\xi_2\beta_1,\ A_2=(2\xi_1-1)(1-2\beta_1)+4\xi_2\beta_2.$$

On the basis of the uniqueness of the problem solution we may conclude that $D_m\neq 0$. We will put system solution X_{mk} in the (9), and the found expressions in the (8), We will obtain:

$$\begin{aligned}W_1^{(i)}=\int_0^{2\pi}\Bigg\{\sum_{m=1}^{\infty}\frac{1}{mD_m\pi R^3}\Bigg\{&\operatorname{grad}\{[(t_im-p_i\}\cos\theta-(t_im-\eta_i)\sin\theta]f_1^{(i)}+\\&+[(t_im-\eta_i)\cos\theta+(t_im-p_i)\sin\theta]f_2^{(i)}+\\&+2\sum_{k=1}^{2}\Big\{(1-\delta_{ik})(m-1)[\varepsilon_2(\theta)f_1^{(k)}+\varepsilon_1(\theta)f_2^{(k)}]\Big\}+\\+\frac{r^2}{R^2}\operatorname{grad}\Big\{\Big[(e_i+\frac{1}{2})r\frac{\partial}{\partial r}+2e_i\Big]E_i(\theta)&+\delta_i\Big(r\frac{\partial}{\partial r}+2\Big)\sum_{k=1}^{2}(1-\delta_{ik})E_k(\theta)\Big\}-\\-\frac{x}{R^2}r\frac{\partial}{\partial r}\Big[(2e_i-1)E_i(\theta)+2\delta_i\sum_{k=1}^{2}&(1-\delta_{ik})E_k(\theta)\Big]\Big\}\Big(\frac{R}{r}\Big)^m\cos m(\theta-\Psi)+\\+\frac{R^4}{2r^2}[(x\cos\theta-\widetilde{x}\sin\theta)f_1^{(i)}&+(\widetilde{x}\cos\theta+x\sin\theta)f_2^{(i)}]\Bigg\}d\theta,\quad i=1,2,\end{aligned}\tag{10}$$

where $\eta_1=1-2\beta_2$, $\eta_2=1-2\xi_1$, $t_1=\xi_1\eta_1+\frac{1}{2}\eta_1+2\beta_1\xi_1$, $t_2=\beta_2\eta_2+\frac{1}{2}\eta_2+2\beta_2\xi_1$, $p_1=2(\xi_1\eta_1+2\xi_2\beta_2)$, $p_2=2(\beta_2\eta_2+\xi_2\beta_1)$, $\varepsilon_1(\theta)=\cos\theta+\sin\theta$, $\varepsilon_2(\theta)=\cos\theta-\sin\theta$,

$$\begin{aligned}E_1(\theta)&=\eta_1\varepsilon_2(\theta)f_1^{(1)}+\eta_1\varepsilon_1(\theta)f_2^{(1)}+2\delta_1\varepsilon_2(\theta)f_1^{(2)}+2\delta_1\varepsilon_1(\theta)f_2^{(2)},\\E_2(\theta)&=2\delta_2\varepsilon_2(\theta)f_1^{(1)}+2\delta_2\varepsilon_1(\theta)f_2^{(1)}+\eta_2\varepsilon_2(\theta)f_1^{(2)}+\eta_2\varepsilon_1(\theta)f_2^{(2)}.\end{aligned}$$

We will seek for the values of vector $W_2^{(j)}(x)$ by the following way:

$$W_2^{(i)}=(r^2-R^2)\operatorname{grad}\sum_{m=1}^{\infty}\alpha_m^{(i)}f_{3m}(x),\quad i=1,2,\tag{11}$$

where $\alpha_m^{(1)}$ and $\alpha_m^{(2)}$ are the unknown constants; f_{3m} is defined by (6).

Let us put the series (11) and (5) in the (1). Finally we will obtain:

$$W_2^{(i)}=(r^2-R^2)\frac{k_i}{b}\operatorname{grad}\sum_{m=1}^{\infty}\frac{1}{m}f_{3m}(x),\tag{12}$$

$$b = 2[(2c+d)^2 - (2a_1+b_1)(2a_2+b_2)], \quad k_2 = \gamma_2(2a_1+b_1) - \gamma_1(2c+d),$$
$$k_1 = -\frac{\gamma_1 b + 2k_2(2c+d)}{2(2a_1+b_1)}.$$

Thus, I^- problem solution $U = (u^{(1)}, u^{(2)}, u_3)$ will be represented by formulae (5) and (7), where $W_1^{(i)}$ is defined by series (10), and $W_2^{(i)}$-by series (12).

Problem II^+**.** We consider, that the functions given on the circle: $f^{(i)} \in C^3(S)$, $f_3 \in C^1(S)$. Besides, $\int_S f^{(i)}(y)d_yS = 0$, $\int_S [y \times f^{(i)}(y)]d_yS = 0$, $i = 1, 2$.

In D^+ the solution of equation $(1)_3$ by condition (3) is represented by series (5), in which

$$f_{3m}(x) = \frac{1}{\pi}\left(\frac{r}{R}\right)^m \int_0^{2\pi} \cos m(\theta - \psi) f_3(\theta)d\theta. \tag{13}$$

According to (3) we may write:

$$\{(RU)^{(i)}\}^+_{r=R} = \{(Pu)^{(i)}\}^+_{r=R} - \gamma_i n u_3^+ = f^{(i)},$$
$$\{(Pu)^{(i)}\}^+_{r=R} = f^{(i)}(z) + \gamma_i n(z) u_3^+(z), \quad i = 1, 2, \tag{14}$$

where Pu is defined by (4), if we will take $\gamma_1 = \gamma_2 = 0$ in the (4).

We seek for [(1), (14)] problem solution by sum (7), in which $W_1^{(i)}(x)$ is the solution of the homogeneous system $(1)_0$ which satisfies the condition:

$$\{(Pu)^{(i)}\}^+_{r=R} = f^{(i)}(z), \tag{15}$$

and $W_2^{(i)}(x)$ is the solution of system (1) (except the last equation) with the following boundary condition:

$$\{(Pu)^{(i)}\}^+_{r=R} = \gamma_i n(z) u_3^+(z). \tag{16}$$

To find $W_1^{(i)}$ we use the expression (8) in which Φ_k homogeneous functions will be presented by form of:

$$\Phi_k(x) = \sum_{m=0}^{\infty} \left(\frac{r}{R}\right)^m (X_{mk} \cdot \nu_m(\Psi)), \quad k = 1, 2, 3, 4. \tag{17}$$

We want the functions $f^{(i)}$ be expanded in Fourier series. Let us put the (17) and the (8) in the (4). To find unknown A_0, B_0, C_0 and D_0 constants and the required X_{mk} from (15) we obtain systems of linear equations:

$$\varepsilon_1 A_0 + \varepsilon_2 C_0 = \gamma_0^{(1)}, \quad \varepsilon_6 B_0 + \varepsilon_7 D_0 = \eta_0^{(1)},$$

$$\varepsilon_2 A_0 + \varepsilon_3 C_0 = \gamma_0^{(2)}, \quad \varepsilon_7 B_0 + \varepsilon_8 D_0 = \eta_0^{(2)}, \tag{18}$$
$$\sum_{p=1}^{4} a_{kp} X_{mp} = \ni_{mk}, \quad k = 1,2,3,4, \quad m = 1,2,\ldots,$$

where

$$\ni_{m1} = \frac{\gamma_m^{(1)}}{m}, \quad \ni_{m2} = \frac{\eta_m^{(1)}}{m} \quad \ni_{m3} = \frac{\gamma_m^{(2)}}{m}, \quad \ni_{m4} = \frac{\eta_m^{(2)}}{m}$$
$$a_{11} = \frac{1}{R^2}(2\mu_1 m - \varepsilon_1); \quad a_{12} = 2\Big[\mu_1\Big(\xi_1 + \frac{1}{2}\Big) + \xi_2\mu_3\Big]m^2 +$$
$$+\Big(\xi_2\varepsilon_2 - 2\xi_1\varepsilon_4 + \varepsilon_1\Big(\xi_1 + \frac{3}{2}\Big) - 2\xi_2\varepsilon_5\Big)m + \varepsilon_1;$$
$$a_{13} = 2\Big[\mu_1\beta_1 + \Big(\beta_2 + \frac{1}{2}\Big)\mu_3\Big]m^2 + \Big[\varepsilon_2\Big(\beta_2 + \frac{3}{2}\Big) +$$
$$+\varepsilon_1\beta_1 - 2\beta_2\varepsilon_5 - 2\beta_1\varepsilon_4\Big]m + \varepsilon_2; a_{14} = \frac{1}{R^2}(2\mu_3 m - \varepsilon_2);$$
$$a_{21} = \frac{1}{R^2}(2\lambda_5 m - \varepsilon_6); \quad a_{22} = 2\Big[\lambda_5\Big(\xi_1 + \frac{1}{2}\Big) - \xi_2\lambda_5\Big]m^2 +$$
$$+\Big(3\xi_1\varepsilon_6 + 3\xi_2\varepsilon_7 + \frac{1}{2}\varepsilon_6 - a_1\Big)m + 2\varepsilon_6\xi_1 + 2\xi_2\varepsilon_7;$$
$$a_{23} = 2\Big[\lambda_5\beta_1 - \lambda_5\Big(\beta_2 + \frac{1}{2}\Big)\Big]m^2 + \Big(3\varepsilon_6\beta_1 + 3\varepsilon_7\beta_2 + \frac{1}{2}\varepsilon_7 - c\Big)m +$$
$$+2\varepsilon_6\beta_1 + 2\varepsilon_7\beta_2; \quad a_{24} = -\frac{1}{R^2}(2\lambda_5 m + \varepsilon_7);$$
$$a_{31} = \frac{1}{R^2}(2\mu_3 m - \varepsilon_2); \quad a_{32} = 2\Big[\mu_3\Big(\xi_1 + \frac{1}{2}\Big) + \xi_2\mu_2\Big]m^2 +$$
$$+\Big[\varepsilon_2\Big(\xi_1 + \frac{3}{2}\Big) + \varepsilon_3\xi_2 - 2\varepsilon_5\xi_1 - 2\varepsilon_9\xi_2\Big]m + \varepsilon_2;$$
$$a_{33} = 2\Big[\mu_3\beta_1 + \mu_2\Big(\beta_2 + \frac{1}{2}\Big)\Big]m^2 + \Big[\varepsilon_3\Big(\beta_2 + \frac{3}{2}\Big) + \varepsilon_2\beta_1 - 2\varepsilon_5\beta_1 - 2\varepsilon_9\beta_2\Big]m + \varepsilon_3;$$
$$a_{34} = \frac{1}{R^2}(2\mu_2 m - \varepsilon_3); \quad a_{41} = -\frac{1}{R^2}(2\lambda_5 m + \varepsilon_7);$$
$$a_{42} = -2\Big[\lambda_5\Big(\xi_1 + \frac{1}{2}\Big) - \lambda_5\xi_2\Big]m^2 + \Big(3\varepsilon_7\xi_1 + 3\varepsilon_8\xi_2 + \frac{1}{2}\varepsilon_7 - c\Big)m + 2\varepsilon_7\xi_1 + 2\varepsilon_8\xi_2;$$
$$a_{43} = -2\Big[\lambda_5\beta_1 - \lambda_5\Big(\beta_2 + \frac{1}{2}\Big)\Big]m^2 + \Big(3\varepsilon_7\beta_1 + 3\varepsilon_8\beta_2 + \frac{1}{2}\varepsilon_8 - a_2\Big)m + 2\varepsilon_7\beta_1 + 2\varepsilon_8\beta_2;$$
$$a_{44} = \frac{1}{R^2}(2\lambda_5 m - \varepsilon_8), \quad \varepsilon_1 = a_1 + b_1, \quad \varepsilon_2 = c + d, \quad \varepsilon_3 = a_2 + b_2,$$
$$\varepsilon_4 = \lambda_1 - \frac{\alpha_2\rho_2}{\rho}, \quad \varepsilon_5 = \lambda_3 - \frac{\alpha_2\rho_1}{\rho} = \lambda_4 + \frac{\alpha_2\rho_2}{\rho}, \quad \varepsilon_6 = \mu_1 + \lambda_5,$$
$$\varepsilon_7 = \mu_3 - \lambda_5, \quad \varepsilon_8 = \mu_2 + \lambda_5, \quad \varepsilon_9 = \lambda_2 + \frac{\alpha_2\rho_1}{\rho}.$$

The principal determinants of the systems (18) are different from zero, because of the uniqueness of the problem solution. $\gamma_m^{(j)}$ and $\eta_m^{(j)}$ are the constant are accordingly Fourier coefficients of $f_1^{(j)}(z)$ and $f_2^{(j)}(z)$ functions.

$W_2^{(i)}$ is constructed by form of the following series:

$$W_2^{(i)}(x) = \sum_{m=0}^{\infty} \{\alpha_m^{(i)} x f_{3m}(x) + \beta_m^{(i)} r^2 \operatorname{grad} f_{3m}(x)\}, \tag{19}$$

where the values of $\alpha_m^{(i)}$ and $\beta_m^{(i)}$ constants are sought from the system of equation:

$$\begin{gathered}
2(b_1 - \lambda_5)\alpha_0^{(1)} + 2(d + \lambda_5)\alpha_0^{(2)} = \gamma_1, \;\; 2(d + \lambda_5)\alpha_0^{(1)} + 2(b_2 - \lambda_5)\alpha_0^{(2)} = \gamma_2, \\
[2(b_1 - \lambda_5) + (b_1 - 2\lambda_5)m]\alpha_m^{(1)} + 2b_1 m \beta_m^{(1)} + \\
+ [2(d + \lambda_5) + (d + 2\lambda_5)m]\alpha_m^{(2)} + 2dm\beta_m^{(2)} = \gamma_1, \\
[2(d + \lambda_5) + (d + 2\lambda_5)m]\alpha_m^{(1)} + 2dm\beta_m^{(1)} + \\
+ [2(b_2 - \lambda_5) + (b_2 - 2\lambda_5)m]\alpha_m^{(2)} + 2b_2 m \beta_m^{(2)} = \gamma_2, \\
(a_1 + 2\lambda_5)\alpha_m^{(1)} + (c - 2\lambda_5)\alpha_m^{(2)} + 2(\mu_1 m - \lambda_5)\beta_m^{(1)} + 2(\mu_3 m + \lambda_5)\beta_m^{(2)} = 0, \\
(c - 2\lambda_5)\alpha_m^{(1)} + (a_2 - 2\lambda_5)\alpha_m^{(2)} + 2(\mu_3 m + \lambda_5)\beta_m^{(1)} + 2(\mu_2 m - \lambda_5)\beta_m^{(2)} = 0, \\
m = 1, 2, \ldots .
\end{gathered}$$

Conditions: $f^{(i)} \in C^3(S)$ and $f_3 \in C^1(S)$ ensure absolutely and uniformly convergence of series obtained for $W_1^{(i)}$ and $W_2^{(i)}$, and also of series (5).

Acknowledgement

The designated project has been fulfilled by financial support of Georgian National Science Foundation (Grant #GNSF/ST06/3-033). Any idea in this publication is possessed by the author and may not represent the opinion of Georgian National Science Foundation itself.

References

1. Green A.E. and Steel T.R. Constitutive equations for interacting continua. Inst J. Eng. Sci. 4(1966), No.4, 483–500.

2. Burchuladze T.B., Svanadze M.Zh. Potential method in the linear theory of binary mixtures of thermoelastic solids. J.Thermal Stress, 23 (2000), 601 -626.

3. Natroshvili D.G., Jagmaidze A. J. and Svanadze M. Zh. Some problems of the linear theory of elastic mixtures. (Russian) Tbilisi, 1986.

4. Michlin S.G. A course in mathematical physics. Nauka, Moscow, 1968.

5. Basheleishvili M. O. Two-dimensional boundary value problems of statics of the theory of elastic mixtures. Mem. Differential Equations Math. Phys. 6(1995), 59–105.

6. Giorgashvili L.G., Toradze D. I. and Tsagareli I.I. Solution of the first boundary value problems of statics of the disconnected theory of thermoelastic mixture for a disc. Rep. of Enlarged Sess. of the Sem. of I. Vekua Inst. of Appl. Math. 20(2005), No. 1-3, 32–36.